WriteUP

In [ ]:
```
from numpy import *
from matplotlib.pyplot import *
import util
import utils
import dr
import datasets
import runClassifier
from softmax import *
```

PART1 PCA

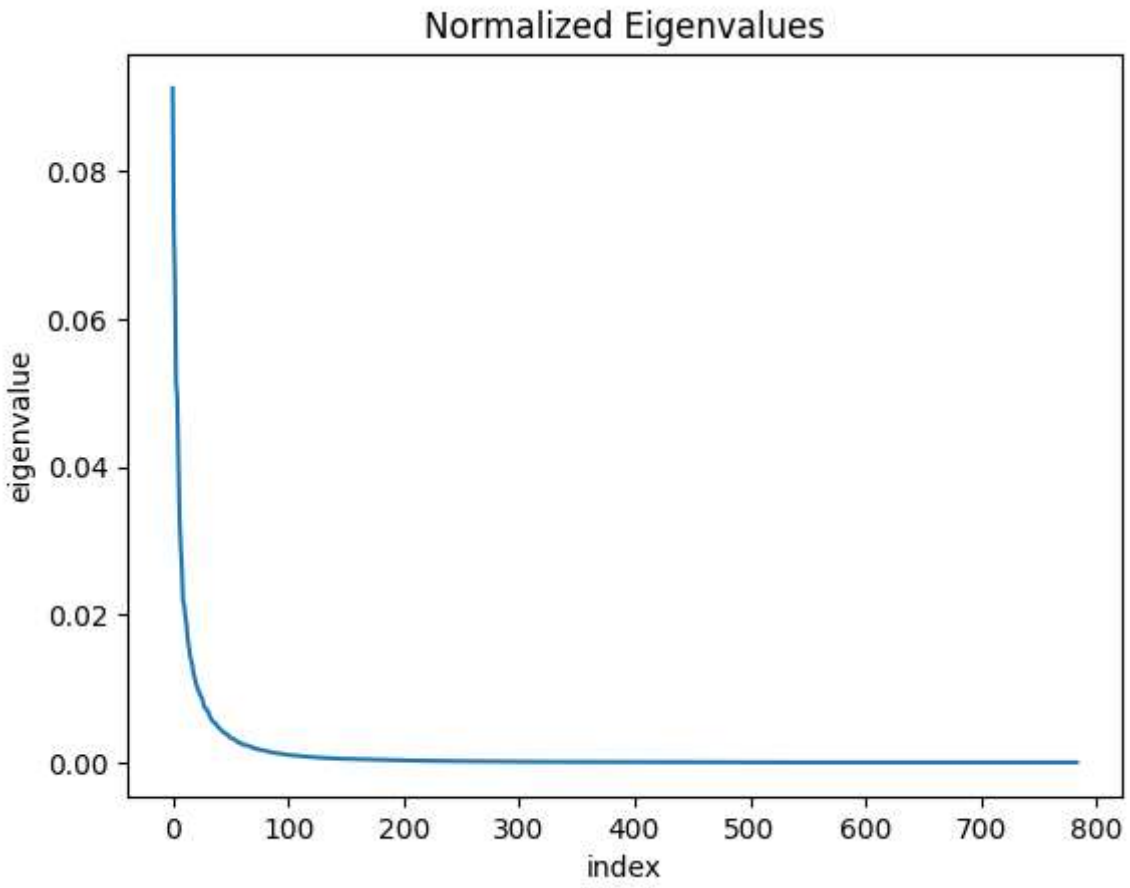
Qpca1: Implementation

see dr.py

Qpca2

In [ ]:
```
(X, Y) = datasets.loadDigits()
(P, Z, evals) = dr.pca(X, 784)

normalized_evals = evals/sum(evals)
eval_index = range(0, len(normalized_evals))

plot(eval_index, normalized_evals)
ylabel('eigenvalue')
xlabel('index')
title('Normalized Eigenvalues')

cumsum = cumsum(normalized_evals)
print (argmax(cumsum > 0.9))
print (argmax(cumsum > 0.95))
```

```
81
135
```



To determined how many eigenvalues are needed to account for 90% and 95%

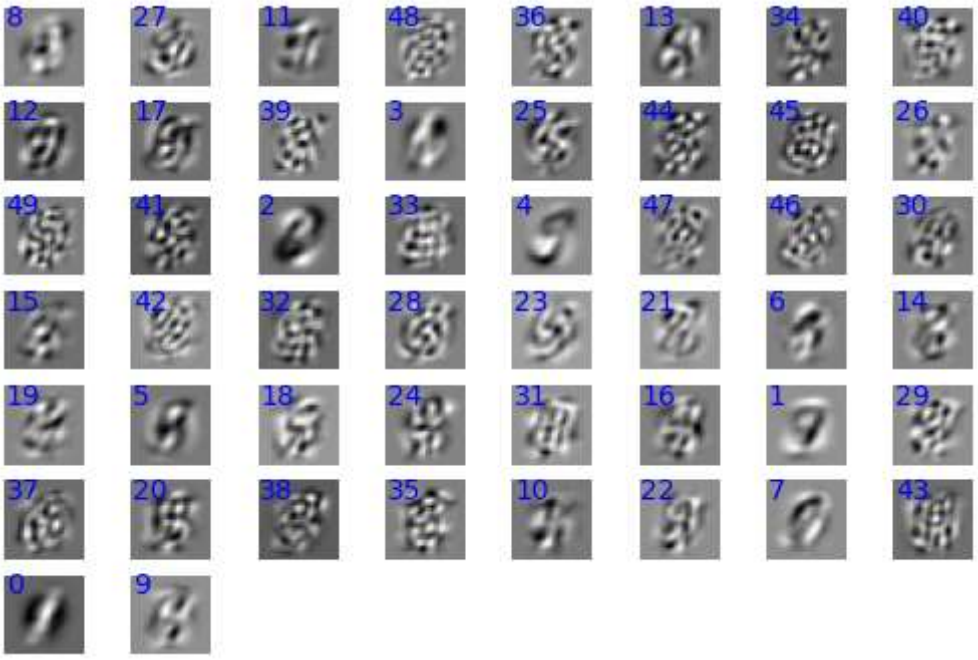
we take sum of eigen values and divide eigen values by sum of eigen values to obtain normalized eigen vals we compute cumulative sum of normalized values. Then we use argmax on obtained value to find first occurence where we accounted 90% and 95% respectively.

To have accounted for 90% of the variance, 81 eigenvectors were needed to be included. To have accounted for 95% of the variance, 135 eigenvectors were needed to be included.

Qpca3

In [ ]:
```
util.drawDigits(Z.T[:50,:], arange(50))
print (argmax(cumsum > 0.83))
```

```
50
```



They don't seem to represent digits with exception of few vaguely representing a digit. Projecting dataset onto top 50 eigenvector represents about 83% varaince in the data missing few details. They make up basic structure of digits but individually wouldn't resemble a digit

PART II Softmax Regression

QSR1

(1)
Given probability
$P[y=i]=\frac{e^{\vec{w_i}\cdot\vec{x}}}{\sum_j e^{\vec{w_j}\cdot\vec{x}}}$

Sum of probabilities:
$\sum_i P[y=i]=\sum_i \frac{e^{\vec{w_i}\cdot\vec{x}}}{\sum_j e^{\vec{w_j}\cdot\vec{x}}}=\frac{\sum_i e^{\vec{w_i}\cdot\vec{x}}}{\sum_j e^{\vec{w_j}\cdot\vec{x}}}$

Notice that numerator and denominator is equal now, so

$\frac{\sum_i e^{\vec{w_i}\cdot\vec{x}}}{\sum_j e^{\vec{w_j}\cdot\vec{x}}}=1$

(2)

W: is the weight matrix in which each row i is the weight vector $\vec{w}_i$
Assuming input features have D dimension thus $\vec{w}$ is a D dimensional vector and there are N amount of classes,

W has dimension $N \times D$

X: is the input matrix in which each column j is the single example $\vec{w}_j$.
Assuming input features have D dimension, and M examples then
X has $D \times M$ dimension

WX: is the dot product of WX. Given $n \times m$ matrix and $m \times n$ matrix resulting matrix of dot product is $n \times n$
Since W has dimension $N \times D$ and X has $D \times M$ dimension
WX has $N \times M$ dimension

QSR2:Implementation

see softmay.py

QSR3

(1) Our probability is $P[y = i] = \frac{e^{WX[i]}}{\sum_j e^{WX[j]}}$

For ease of read, we'll let Max = max(W_X) Subtradct both numerator and denominator W_X by Max

$$P[y = i] = \frac{e^{WX[i]-Max}}{\sum_j e^{WX[j]-Max}}$$

Now, This equation is eqaul to orignial equation since dividing both denominator and numerator by same amount e^{-Max} which would not alter the result gives our original equation.

$$P[y = i] = \frac{\frac{e^{WX[i]-Max}}{e^{-Max}}}{\frac{\sum_j e^{WX[j]-Max}}{e^{-Max}}} = \frac{e^{WX[i]}}{\sum_j e^{WX[j]}}$$

Therefore, W_X = W_X - np.max(W_X) does not affect the predicted probabilities

(2) Since our equation is exponentially gorwing from the entries in W_X, value can grow large very quickly. Too big value may cause overflow which would lead to inaccuracte and instable result.

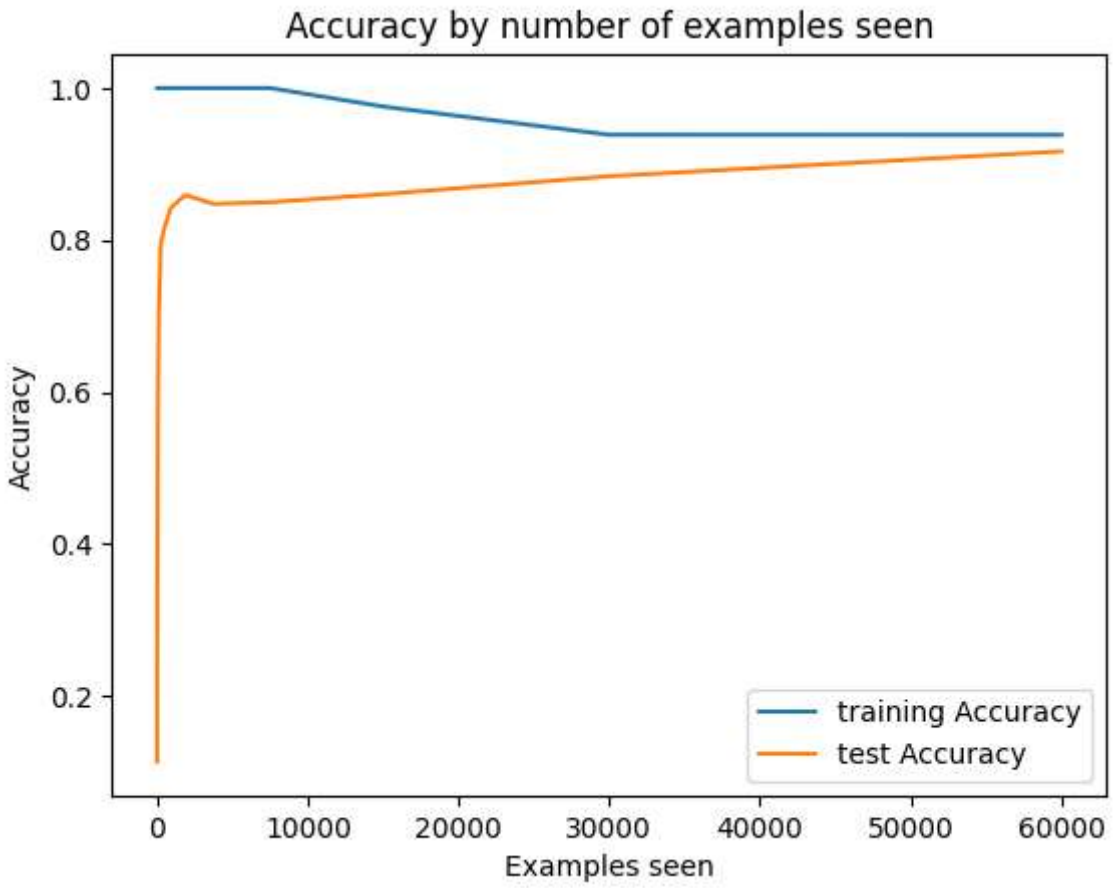
Subtracting maximum value as seen in (1) does not alter the result and helps keep the value more manageable than usiing original W_X.

QSR4

In [ ]:
```
exSize = 28*28
numClasses = 10
sm = SoftmaxRegression(numClasses, exSize)
X, Y = utils.loadMNIST('data/train-images.idx3-ubyte', 'data/train-labels.idx1-ubyt
testX, testY = utils.loadMNIST('data/t10k-images.idx3-ubyte', 'data/t10k-labels.idx

size,trainAcc,testAcc =runClassifier.learningCurve(sm,numClasses,exSize,X,Y,testX,t
```

In [ ]:
```
plot(size,trainAcc, label ='training Accuracy')
plot(size,testAcc, label = 'test Accuracy')
title("Accuracy by number of examples seen")
xlabel("Examples seen")
ylabel("Accuracy")
legend()
```

Out[ ]:
```
<matplotlib.legend.Legend at 0x1bc079f7dd0>
```



at the start, model overfits the training data with perfect accuracy on training and low accuracy on test. As the number of examples seen grow, the accuracy difference between training and test closes and by the end, there doesn't seem to be clear overfitting nor underfitting.

PART III NN

Qnn1.3

After running run_nn.py with default values: atch size 128, learning rate of 0.01, 20 Epochs result is:
loss:0.09871 accuracy:0.94770

Qnn 1.4

One reason to use random numbers to initialize weight is to break symmetry. Bides that are side-by-side in a hidden layer connected to same inputs should have different weights for the model to update weight. Otherwise, each neuron in the layer will compute the same output during forward pass and backpropagation receiving same gradient as well updating weights in the same way. This causes maintaining symmetricity thorughout training process causing the model to be stuck failing to make any changes to the network weights. Initializing with random weights help break symmetry and resolve this issue.

Qnn2

(3) I have decided to implement new optimzer for NN. I have implemented Adamsolver optimzer instead. Initial beta 1= 0.9, beta 2 = 0.999, epsilon = 1e-8

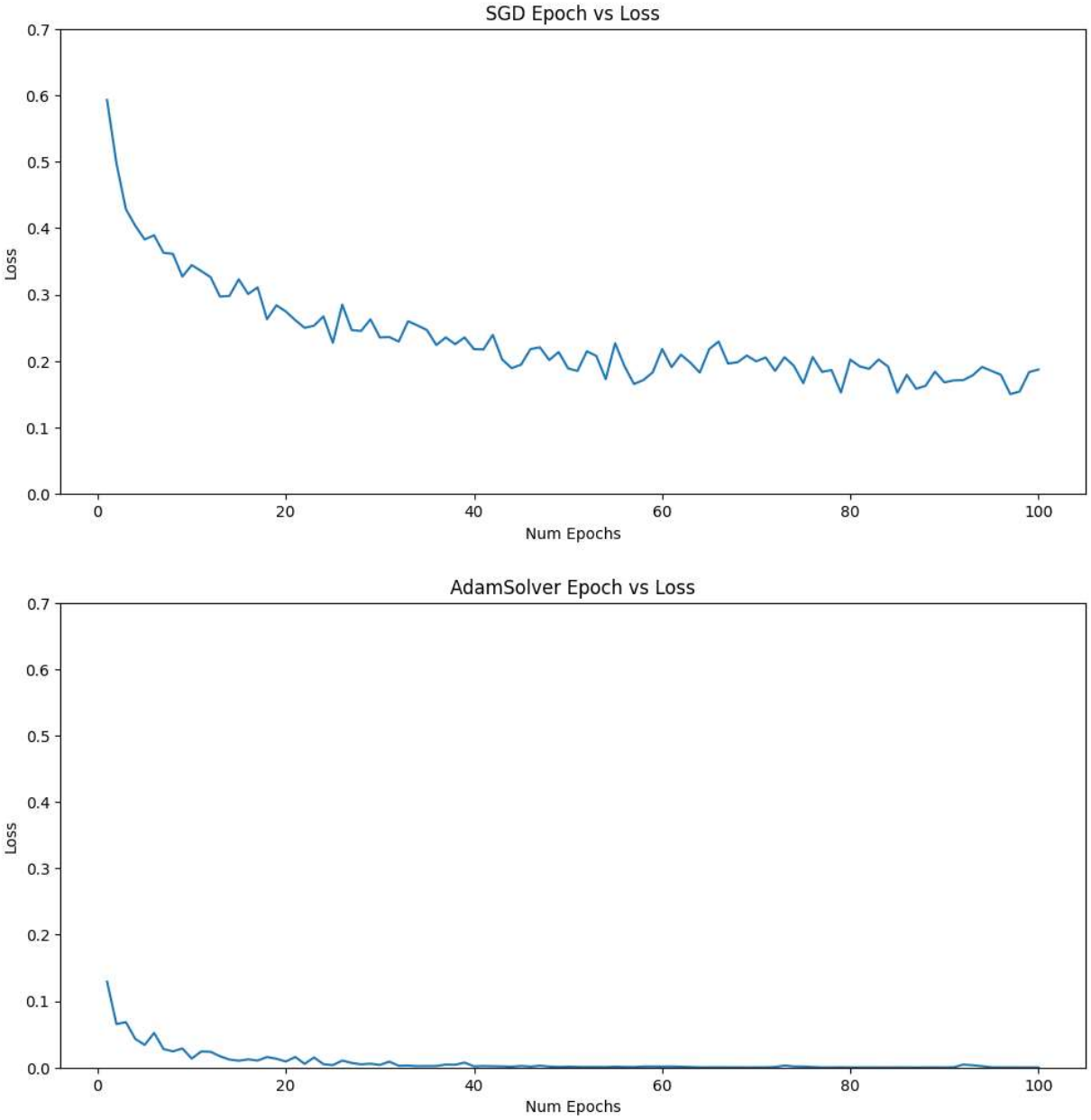
Using same values from Qnn1.3 didn't show much difference between two so I have increased batch size to exaggerate the difference over epochs and one with adamsolver seemed that 0.01 learning rate to be too high as it would overshoot so learning rate was reduced to 0.001 and number of epochs were increased to compensate for bigger batchsize and lower learning rate.

I have run NN with SGD and Adamsolver with learning rate of 0.001, batchsize 512 over 100 epoch. Using modified version of run_nn.py named run_nn2.py to return numpy array of epochs and loss instead of printing to terminal.

In [ ]:
```
#see run_nn2.py for details
#run_nn2.main takes learning rate, max epcoch, batch size,input dim, output dim as
import run_nn2

sgdEpoch,sgdLoss,adamEpoch,adamLoss = run_nn2.main(0.001,100,512,784,10)
```

In [ ]:
```
#plotting
import matplotlib.pyplot as plt

fig,(ax1,ax2) = plt.subplots(2,figsize=(10, 10))
fig.tight_layout(h_pad=5)


ax1.plot(sgdEpoch,sgdLoss)
ax2.plot(adamEpoch,adamLoss)

ax1.set_title('SGD Epoch vs Loss')
ax2.set_title('AdamSolver Epoch vs Loss')
```

```
ax1.set(xlabel='Num Epochs', ylabel='Loss')
ax2.set(xlabel='Num Epochs', ylabel='Loss')

ax1.set_ylim(0,0.7)
ax2.set_ylim(0,0.7)

plt.show()
```



From the graph, it seems that Adamsolver converges faster(less number of epochs) at same sample numbers of input dimension=784. Trying various different input dim brough about similar result.

Seems that adamsolver is better at optimizing this task than SGD.