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2/3/18
CS 302

Ex 1.8

1. a.) $\sum_{i=0}^{\infty} \frac{1}{4^i}$

$$S = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{4}{4}-\frac{1}{4}} = \boxed{\frac{4}{3}}$$

b.) $\sum_{i=0}^{\infty} \frac{i}{4^i}$

$$S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \cdots$$

$$4S = 1 + \frac{2}{4} + \frac{3}{4^2} + \cdots$$

$$3S = 1 + \frac{1}{4} + \frac{1}{4^2} + \cdots$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\boxed{S = \frac{4}{9}}$$

Ex: 2.1

2.) 3.) 1st loop = size N
2nd loop = size N
iterate n^3 times

$c * n^3 = O(n^3)$

4.) c constants

$\sum_{i=1}^{n} \left(\frac{n}{2}\right)$

$1 + 2 + \cdots + n$

$\frac{n}{2}$ $\boxed{O(n^2)}$

$n \cdot \frac{n}{2} = \frac{n^2}{2}$

2) 5.) n times outer loop
middle bar = $\frac{n^2}{2}$

$\frac{n^2}{4}$

Inner $= n \cdot \frac{n^2}{2} \cdot \frac{n^2}{4} = \frac{n^5}{8}$

$\frac{c}{8} \cdot n^5 = \boxed{O(n^5)}$

2.) 3.) Ex 2.1

$$\boxed{\frac{2}{N},\ 37,\ \sqrt{N},\ N,\ N\log\log N,\ N\log N,\ N\log N^2,\ N\log^2 N,\ N^{1.5},\ N^2,\ N^2\log N,\ N^3,\ 2^{\frac{N}{2}},\ 2^N}$$

2^n always grows more quickly than polynomial functions

$N\log N^2 = 2N\log N$
$= O(N\log N)$
$N\log N = O(N\log N)$

1.) b) exercise 1.12

a) $\sum_{i=1}^{N} (2i-1) = N^2$

$= \sum_{i=1}^{N} 2i - \sum_{i=1}^{N} 1 = 2\left(\frac{N(N+1)}{2}\right) - N$

$= N(N+1) - N = N^2 + N - N = \boxed{N^2}$

b) $\sum_{i=1}^{N} i^3 = \left(\sum_{i=1}^{N} i\right)^2$

$\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$

$\sum_{i=1}^{N} i^3 = \left(\frac{N(N+1)}{2}\right)^2$

$\frac{(k(k+1))^2}{2} + (k+1)^3$
$= \frac{(k+1)^2(k+2)^2}{2}$

$\boxed{\left(\frac{(k+1)(k+2)}{2}\right)^2}$

Q.E.D

Ex. 2.11

(4.) a.) linear

input size	operations	exec time
100	100	$0.5\,ms = 100 \times t_{op}$
500	500	$x\,ms = 500 \times t_{op}$

~~4 2 3 ms~~ $\frac{x}{100} = \frac{60,000}{0.5} = \boxed{12,000,000}$

b.) $O(N \log N)$

$100 \log 100$ $\quad 0.5\,ms = 100 \log 100 \times t_{op}$

$500 \log 500$ $\quad x\,ms = 500 \log 500 \times t_{op}$

~~x = 3.3 ms~~ $\frac{x \log x}{100 \log 100} = \frac{60,000}{0.5} = \boxed{3,656,807}$

c.) quadratic

$\frac{x^2}{100^2} = \frac{60,000}{0.5} = \boxed{34,641}$

d.) $\frac{x^3}{1000^3} = \frac{60,000}{0.5} = \boxed{4,932}$

Ex. 2.14

(5.) $O(N)$ because the for loop iterates $N+1$ times, with a constant amount of work

(6.) $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} 1$

run time $= n^3 + n$

$\boxed{O(n^3)}$

Ex 2.25

(7.) (a.) $150 N \log_2 N = A$

$N^2 = B$

$\boxed{\text{Program A}}$

(b) Program B