

Qualifying Exam

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1.

- a. Using the poisson distribution, $p(X=0)$ given $\lambda = 3.2$. probability that a particular month will have no accidents = 4%

```
dpois(0,3.2)
```

```
0.0407622
```

- b. mean and the variance of the Poisson distribution are both equal to λ .

Expected Value = $\lambda \times 1 = 3.2$

variance = $\lambda = 3.2$

2.

a) $p(y \leq 2) = p(y = 0) + p(y = 1) + p(y = 2) = 0.005 + 0.010 + 0.035 = 0.05$

b) $p(y = 1, y = 2, y = 3, y = 4) = 1 - p(y = 0) = 1 - 0.005 = 0.995$

c) $E(y) = \sum y \cdot p(y) = 0 \cdot (0.005) + 1 \cdot 0.010 + 2 \cdot 0.035 + 3 \cdot 0.050 + 4 \cdot 0.900 = 3.83$

d) $E(y^2) = \sum y^2 \cdot p(y)$

$$= 0^2 \cdot (0.005) + 1^2 \cdot 0.010 + 2^2 \cdot 0.035 + 3^2 \cdot 0.050 + 4^2 \cdot 0.900 = 15$$

So, the variance, $\sigma^2 = E(y^2) - E(y)^2$

$$= 15 - (3.83)^2 = \underline{0.3311}$$

And standard deviation,

$$\sigma = \sqrt{0.3311} = \underline{0.575413}$$

3.

```
mu = 70
sigma = 3
z socre for 64
(64-70)/3

z = -2

z socre for 76
(76-70)/3

z = 2

#between -2 and +2
pnorm(2) - pnorm(-2)

0.9544997
```

What % of males will be between 64 and 76 inches tall = 95.44 %

4.

```
a)Sample mean
values <-c(13.3,14.5,15.3,15.3,14.3,14.8,15.2,14.9,14.6,14.1)
mean(values)

14.63

b)sample variance
var(values)

0.389

sample standard deviation
sqrt(var(values))

0.6236986
```

c)

$H_0 : \mu_0 = 14.9$

$H_1 : \mu_0 \neq 14.9$

```
xbar <- 14.63          # sample mean
mu0 <- 14.90          # hypothesized value
s <- sqrt(0.389)      # sample standard deviation
n <- 10               # sample size
```

```
test_statistic <- (xbar-mu0)/(s/sqrt(n))
test_statistic = -1.368954
```

```
alpha = .01
df <- n-1
t.half.alpha <- qt(1-alpha/2,df=n-1)
c(-t.half.alpha,t.half.alpha)
```

Confidence Interval :

(-3.249836 , 3.249836), If test statistics is between this interval we can not reject the null hypothesis.

Or using P values :

```
pval <- 2*pt(test_statistic,df=n-1)
```

pval = 0.2042047 > alpha therefor can not reject the null hypothesis.

d.

```
e <- qt(0.9,df=n-1)*s # Margin of Error
e = 0.8625932
```

```
c(xbar-e,xbar+e)
```

Confidence Interval is (13.76741 15.49259)

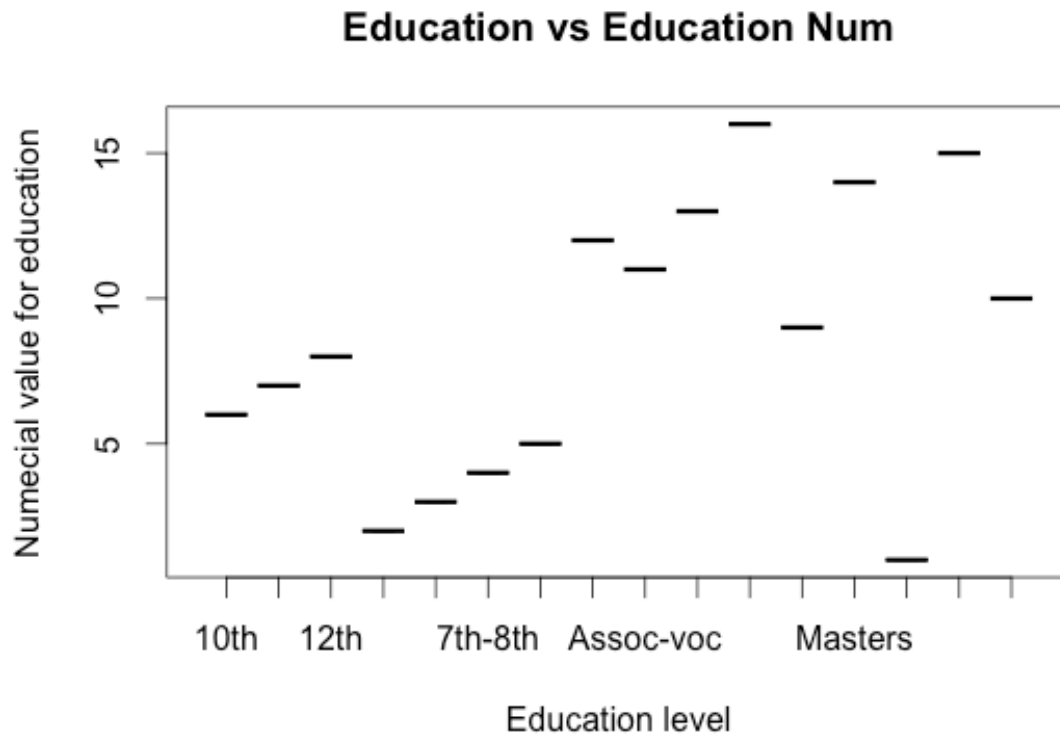
#true mean is between this confidence interval so it has not been changed.

5)

1. c
2. b
3. c
4. c
5. a

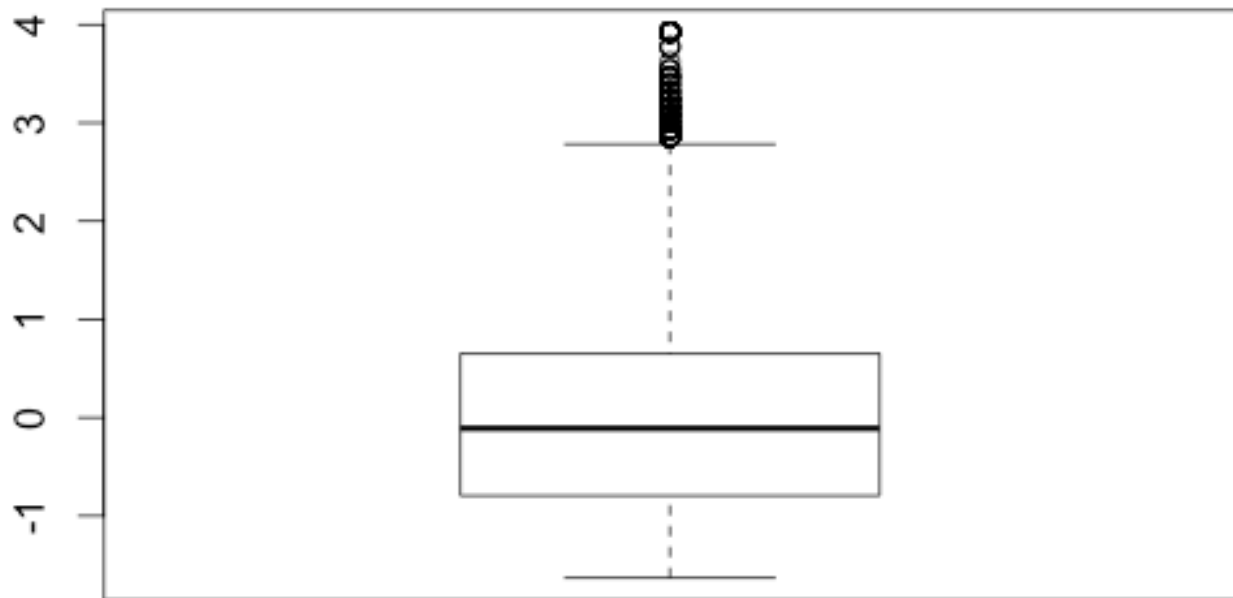
6)

1) By observing the data set the categorical predictor “education” and the continuous predictor “education_num” represents the same information. So education_num was removed initially.

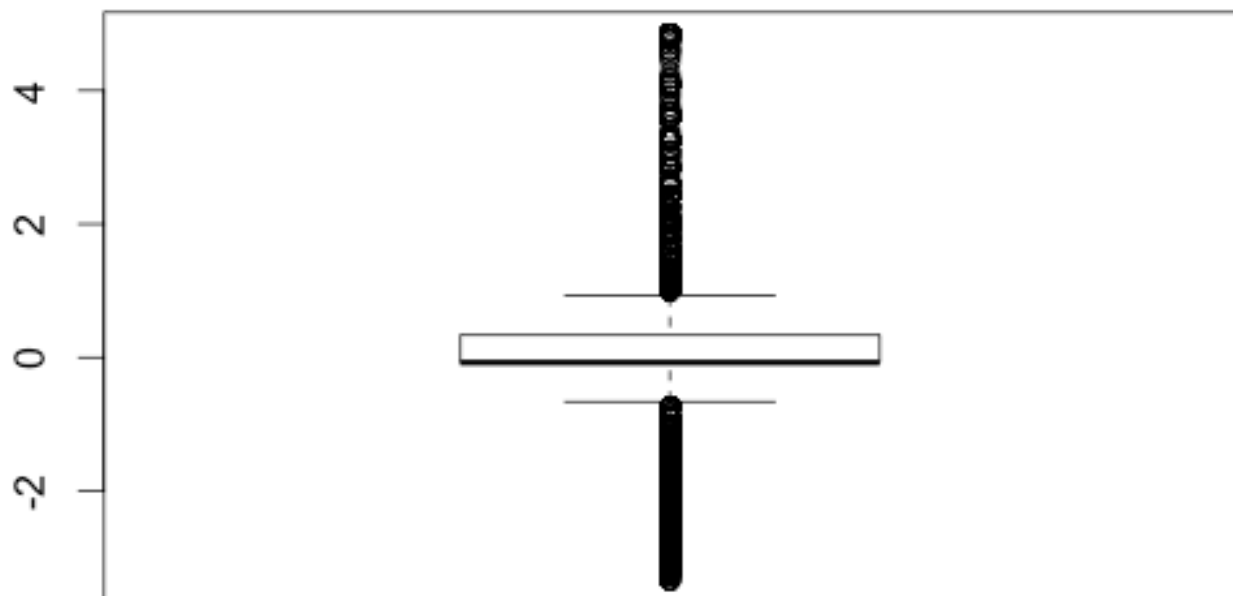


2. age and hr_per_week Continuous predictors were choose to scale. This applies a normal transformation. Each value minus its mean over the sample standard deviation. Following is the boxplots for these continuous predictors.

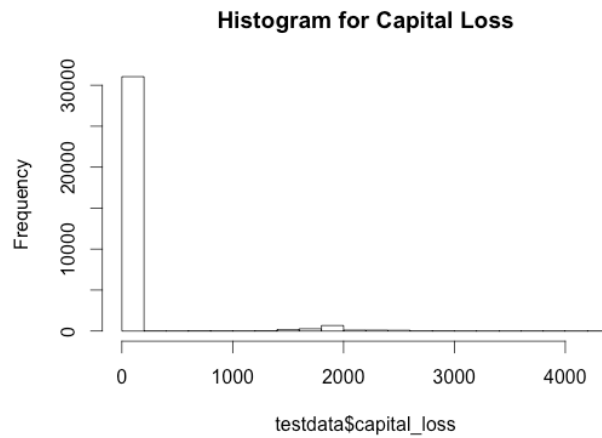
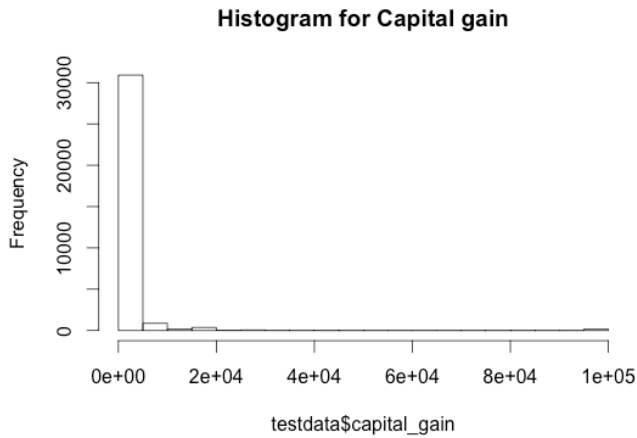
Box plot for age



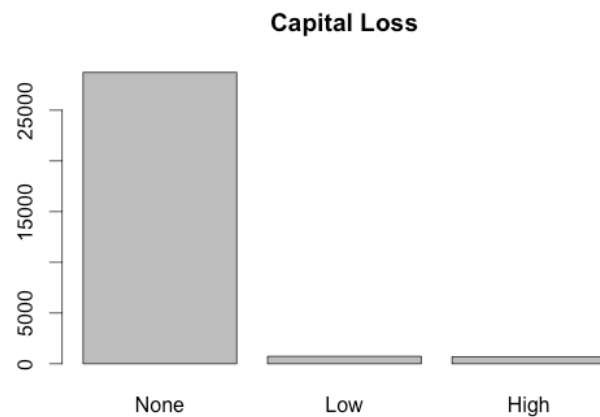
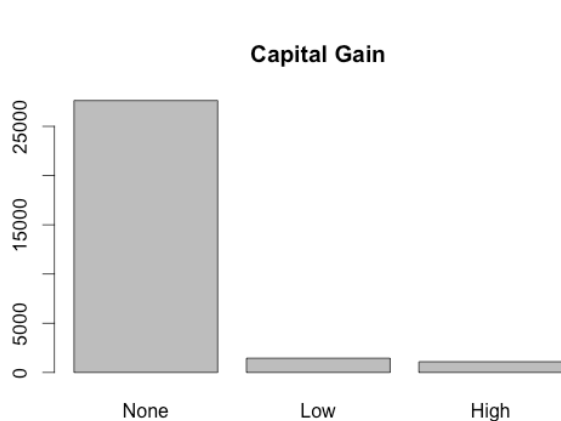
Box plot for hours per week



3. The capital gain and capital loss predictors are extremely skewed. No transformation could correct this. So the predictors were converted to categorical variables with (None,Low,High) factors.



Because of the high skewers numerical transformation would not have been appropriate so Converted to Categorical variables. For both variables, none means they don't play the market. Low means they have some investments. High means they have significant investments.



4. `nearzerovar()` function return following predictors are degenerate.

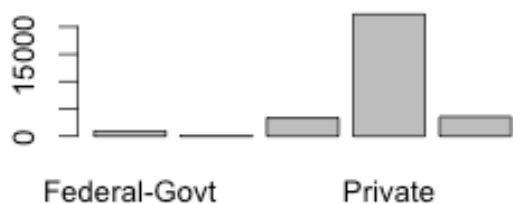
[1] "capital_gain"

[1] "capital_loss"

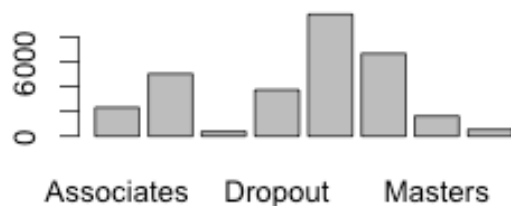
[1] "country"

Some predictors were recotagorized to include in to a broader category to reduce the number of factores in a predictor. (occupation,marital status, ect)

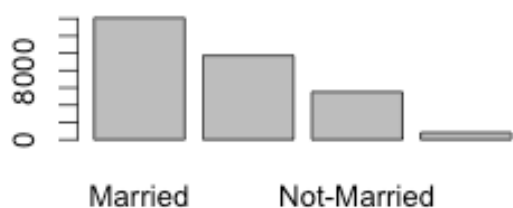
type_employer



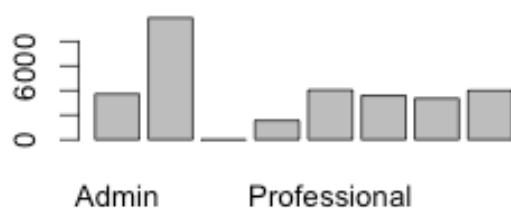
education



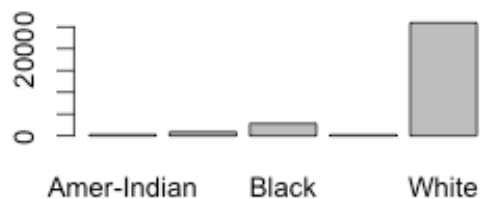
marital



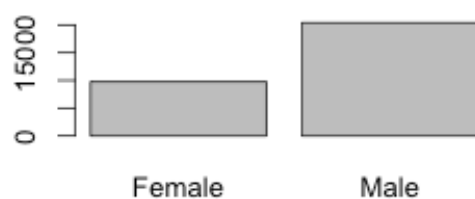
occupaion



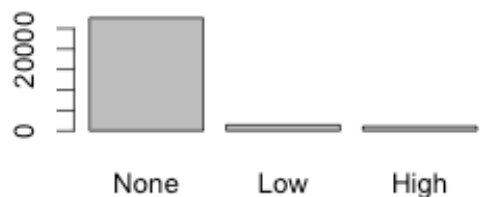
race



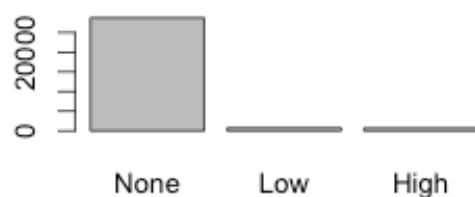
sex

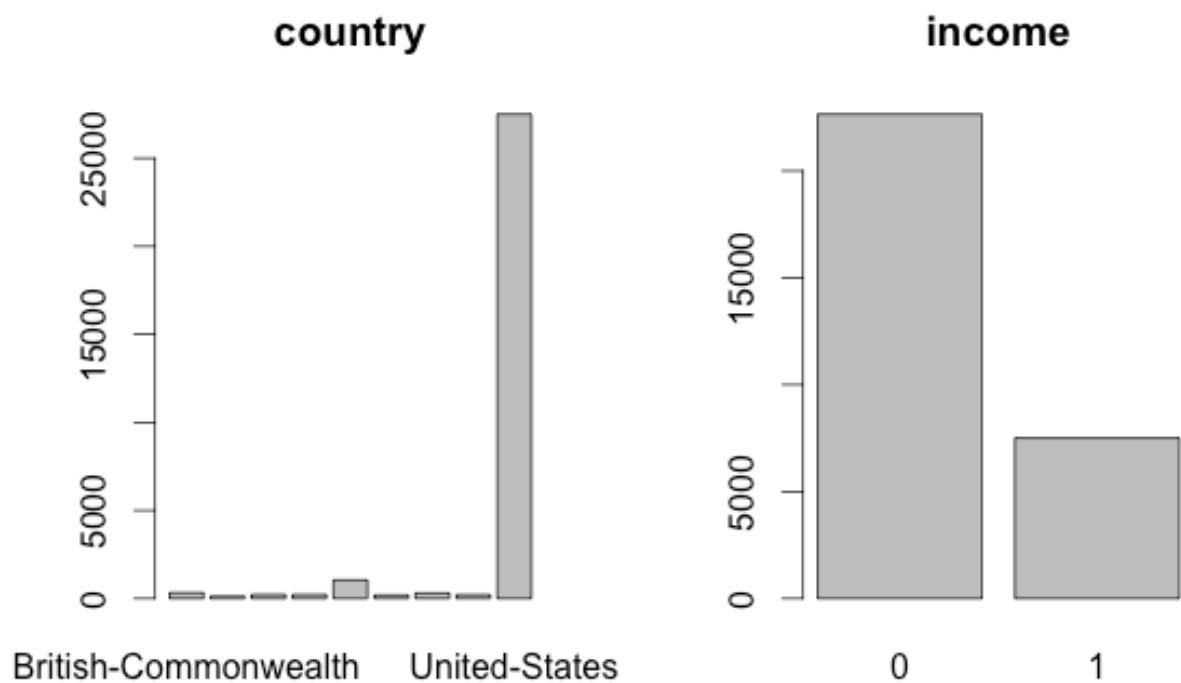


capital gain

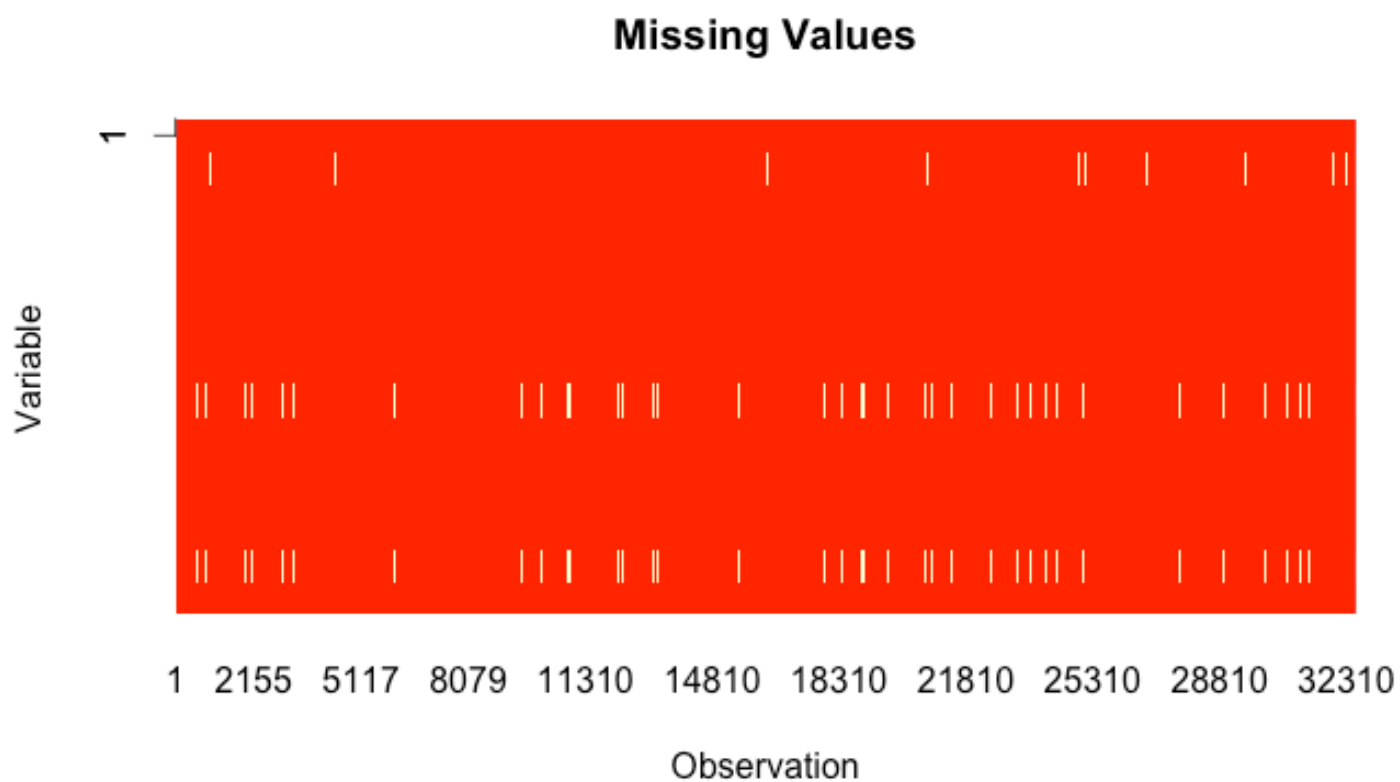


capital loss





5. Missing value visualization



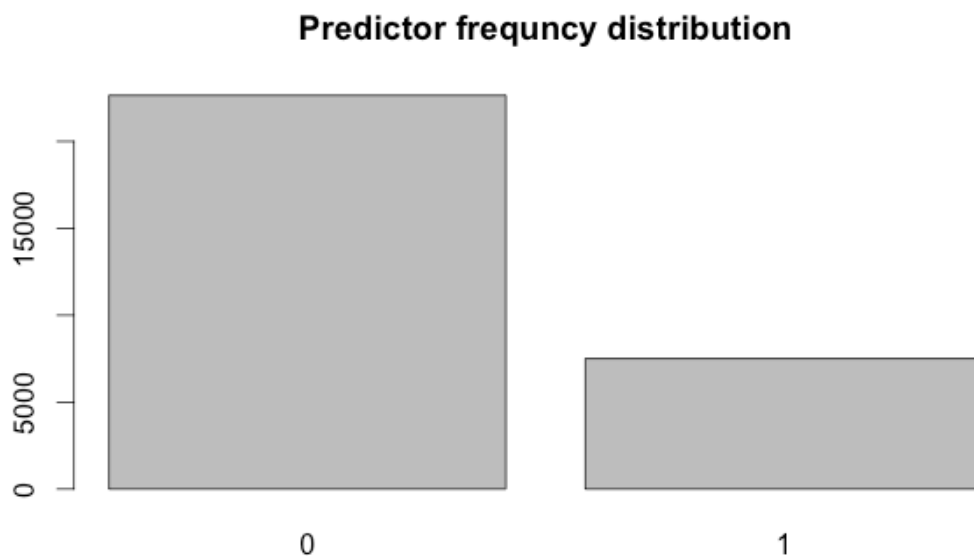
Predictor	Overall Number of missing values
Type_employer	1836
Occupation	1843
country	583

Yes. In low income ($\leq 50,000$) data points the number of missing values are high. Because low income people may not be able to give information more confidently as high income people. So there can be many missing values in a low income data point.

6. `Na.omit()` function is used to omit the data with missing values because it does not affect much as there are still more than 30,000 datapoints with complete data. Only about 2000 datapoints had to be removed because of the missing data.

7. ROC or Kappa statistic should be used. Accuracy may not be the best statistic because of the income class imbalance.

8. Response class is imbalanced.



Stratified sampling should be used to split the dataset into testing and training sets. The `createDataPartition()` function is used with $p=0.75$, which splits the dataset into 75% training and 25% testing set.

9. Preprocessing steps:

i. Remove high correlated predictors

ii. Merge factors in categorical predictors so it will reduced the number of dummy variables created. For example the country predictors can be re categorized according the larger geographical region such as (Asia, Europe, South America, Africa, ect)

iii. Remove data points with missing values

iv. Remove no information predictors (fnlwgt) is just a number with no relevance to the income, such as an ID number.

v. Remove zero variance predictors

30162 rows and 10 categorical Predictors are remaining after all the preprocessing. This dataset was split to following training and testing set.

22622 for training set

7540 for testing set

To be used in some models, Dummy variables were created for all the categorical predictors using the same dataframe(30162 rows and 10 column) mentioned above. Then number of predictors increased to 28 predictors.

10. Linear Models

Model	Tuning Para.	AUC	Sensitivity	Specificity
Logistic reg	-----No---	0.87996	0.91717	0.55448
LDA	-----No---	0.86991	0.91458	0.54237
PLSDA	Ncome = 10	0.86886	0.9301	0.50191
GLMNet	alpha = 0 and lambda = 0.1	0.86384	0.93842	0.45082
Nearest S C	Threshold =0	0.84545	0.94131	0.39741

Important predictors from Linear model - PLSDA

age 0.08234

marital.Married 0.07472

relationship.Husband 0.06400

marital.Never-Married 0.04421

occupation.Blue-Collar 0.03361

occupation.White-Collar 0.03196
 education.Bachelors 0.03138
 education.HS-grad 0.03087
 sex.Male 0.02983
 sex.Female 0.02983
 relationship.Not-in-family 0.02857
 education.Dropout 0.02854
 occupation.Professional 0.02814
 relationship.Own-child 0.02513
 marital.Not-Married 0.02473
 occupation.Service 0.02161
 education.Masters 0.01998
 relationship.Unmarried 0.01714
 type_employer.Private 0.01630
 education.HS-Graduate 0.01203

11.

Model	Tuning Para	ROC	Sensitivity	Specificity
MDA	Subclasses = 1	0.87640	0.91652	0.55499
NNet	Size = 1, decay =0.1	0.8872	0.90826	0.57705
FDA	degree = 1 and nprune = 17.	0.8763	0.9184	0.5478
SVM	C=8	0.8793	0.9235	0.5818
KNN	K=9	0.8809	0.8974	0.5715
NaiveBayes	Laplace = 2	0.8723	0.8652	0.5253

12. Best models based on AUC

1. Neural Network , 2. KNN 3. SVM – from Non Linear models

1. Logistic Reg 2. LDA 3 . PLSDA – from Linea modelss

13.

From Linear best models are Logisted ,LDA, PLSDA

From Non linear best models are NeuralNet,

model	Accuracy	Sensitivity	Specificit	Kappa
Logistic	0.8309	0.9202	0.5615	0.5158
LDA	0.8256	0.9163	0.5519	0.501
PLSDA	0.8236	0.9302	0.5019	0.4781
Nnet	0.8524	0.9129	0.5992	0.6212
KNN	0.8413	0.9135	0.5832	0.5419
SVM	0.8245	0.9161	0.5124	0.524

Based on the Kappa statistic Neural Network model is the best to classify the income.

14. Important predictors

maritalNever-Married 100.000

maritalNot-Married 83.698

educationBachelors 61.242

educationMasters 61.242

educationProf-School 54.926

age 48.643

educationDoctorate 42.061

occupationBlue-Collar 39.170

occupationWhite-Collar 36.761

maritalWidowed 30.950

hr_per_week 22.221

educationDropout	14.719
sexMale	11.332
relationshipWife	8.994
raceAsian	0.000
occupationMilitary	0.000
raceWhite	0.000
occupationSales	0.000
occupationOther-Occupations	0.000
relationshipNot-in-family	0.000

Generalized Linear Model

22622 samples

9 predictor

2 classes: '0', '1'

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results

ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
0.8799647	0.9171745	0.5544847	0.003771528	0.003718161	0.01107291

Linear Discriminant Analysis

22622 samples

28 predictor

2 classes: '0', '1'

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results

ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
0.8699113	0.9145844	0.5423738	0.004330018	0.0034694	0.01161766

Partial Least Squares

22622 samples

28 predictor

2 classes: '0', '1'

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results across tuning parameters:

ncomp	ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
1	0.8210978	0.9254815	0.3017200	0.005172884	0.004257385	0.011524530
2	0.8484582	0.9169108	0.5011230	0.004623632	0.003693251	0.009662471
3	0.8556016	0.9229951	0.5017484	0.005319495	0.002918410	0.011519707
4	0.8613771	0.9274217	0.4864534	0.005105521	0.002653747	0.012626050
5	0.8632581	0.9286461	0.4915139	0.005045462	0.003048491	0.012534031
6	0.8648489	0.9273558	0.5007249	0.005025836	0.003253502	0.011862911
7	0.8666984	0.9302190	0.4949538	0.004959947	0.003485078	0.012873540
8	0.8679223	0.9303132	0.4993035	0.005013923	0.002984934	0.012224660
9	0.8681823	0.9301342	0.4976262	0.005143483	0.003351018	0.012741545
10	0.8688623	0.9301436	0.5019190	0.005076225	0.003489609	0.013201262

ROC was used to select the optimal model using the largest value.

The final value used for the model was ncomp = 9.

glmnet

22622 samples

28 predictor

2 classes: '0', '1'

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results across tuning parameters:

alpha	lambda	ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
0.0	0.1	0.8638485	0.9384224	0.450859986	0.005655341	0.0043027220	0.012347593
0.0	0.2	0.8610042	0.9563551	0.375977257	0.005745578	0.0034350049	0.010700393
0.0	20.0	0.8480020	1.0000000	0.000000000	0.005945144	0.0000000000	0.000000000
0.2	0.1	0.8604895	0.9608288	0.340810235	0.005702182	0.0035831254	0.012370141
0.2	0.2	0.8531465	0.9989451	0.009182658	0.005966442	0.0007356016	0.005218438
0.2	20.0	0.5000000	1.0000000	0.000000000	0.000000000	0.0000000000	0.000000000
0.6	0.1	0.8454274	1.0000000	0.000000000	0.006374152	0.0000000000	0.000000000
0.6	0.2	0.7587508	1.0000000	0.000000000	0.005908302	0.0000000000	0.000000000
0.6	20.0	0.5000000	1.0000000	0.000000000	0.000000000	0.0000000000	0.000000000
0.8	0.1	0.8067229	1.0000000	0.000000000	0.016570798	0.0000000000	0.000000000
0.8	0.2	0.7575690	1.0000000	0.000000000	0.005723933	0.0000000000	0.000000000
0.8	20.0	0.5000000	1.0000000	0.000000000	0.000000000	0.0000000000	0.000000000

ROC was used to select the optimal model using the largest value.

The final values used for the model were alpha = 0 and lambda = 0.1.

Nearest Shrunk Centroids

22622 samples

28 predictor

2 classes: '0', '1'

Pre-processing: centered, scaled

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results across tuning parameters:

threshold	ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
0	0.8454513	0.9413139	0.397412935	0.004319150	0.0039207738	0.009172483
1	0.8453774	0.9468707	0.370206112	0.004381660	0.0037334138	0.008593492
2	0.8452393	0.9541323	0.338678038	0.004470150	0.0035947885	0.009051073
3	0.8450614	0.9622227	0.299587775	0.004512389	0.0030314698	0.009592952
4	0.8447973	0.9746645	0.230362473	0.004561640	0.0026193308	0.008477597
5	0.8444082	0.9863998	0.140639659	0.004629258	0.0025697198	0.008070591
6	0.8437068	0.9958276	0.045088842	0.004727403	0.0009908936	0.008072573
7	0.8424795	0.9995950	0.002501777	0.004913761	0.0003755082	0.001437952
8	0.8396445	1.0000000	0.000000000	0.005264669	0.0000000000	0.000000000
9	0.8346362	1.0000000	0.000000000	0.005436800	0.0000000000	0.000000000
10	0.8289169	1.0000000	0.000000000	0.005691125	0.0000000000	0.000000000
11	0.8190368	1.0000000	0.000000000	0.006370865	0.0000000000	0.000000000
12	0.8092706	1.0000000	0.000000000	0.006857796	0.0000000000	0.000000000
13	0.8062404	1.0000000	0.000000000	0.005685198	0.0000000000	0.000000000
14	0.8027894	1.0000000	0.000000000	0.006408375	0.0000000000	0.000000000
15	0.7925135	1.0000000	0.000000000	0.007596168	0.0000000000	0.000000000
16	0.7691202	1.0000000	0.000000000	0.008435752	0.0000000000	0.000000000

17	0.7664731	1.0000000	0.0000000000	0.005018473	0.0000000000	0.0000000000
18	0.7664731	1.0000000	0.0000000000	0.005018473	0.0000000000	0.0000000000
19	0.7664731	1.0000000	0.0000000000	0.005018473	0.0000000000	0.0000000000
20	0.7664731	1.0000000	0.0000000000	0.005018473	0.0000000000	0.0000000000
21	0.7664731	1.0000000	0.0000000000	0.005018473	0.0000000000	0.0000000000
22	0.7571205	1.0000000	0.0000000000	0.005208846	0.0000000000	0.0000000000
23	0.7571205	1.0000000	0.0000000000	0.005208846	0.0000000000	0.0000000000
24	0.7571205	1.0000000	0.0000000000	0.005208846	0.0000000000	0.0000000000
25	0.7571205	1.0000000	0.0000000000	0.005208846	0.0000000000	0.0000000000

ROC was used to select the optimal model using the largest value.

The final value used for the model was threshold = 0.

Neural Network

22622 samples

9 predictor

2 classes: '0', '1'

No pre-processing

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results across tuning parameters:

	size	decay	ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
1	0.000	0.8705316	0.8970944	0.5891969	0.014219352	0.015329727	0.03285999	
1	0.001	0.8787454	0.9099788	0.5737313	0.006186793	0.004157792	0.01368856	
1	0.100	0.8802671	0.9082647	0.5770576	0.003869122	0.004125598	0.01144904	
1	1.000	0.8802317	0.9053826	0.5840512	0.003832171	0.004103522	0.01116490	

ROC was used to select the optimal model using the largest value.

The final values used for the model were size = 1 and decay = 0.1.

Flexible Discriminant Analysis

22622 samples

9 predictor

2 classes: '0', '1'

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results across tuning parameters:

nprune	ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
2	0.3292055	1.0000000	0.0000000	0.004894496	0.000000000	0.0000000
9	0.8477668	0.9327431	0.4535608	0.005429629	0.009875514	0.0322599
17	0.8763237	0.9184083	0.5478038	0.004233990	0.005295729	0.0110436

Tuning parameter 'degree' was held constant at a value of 1

ROC was used to select the optimal model using the largest value.

The final values used for the model were degree = 1 and nprune = 17.

k-Nearest Neighbors

22622 samples

28 predictor

2 classes: '0', '1'

Resampling: Repeated Train/Test Splits Estimated (25 reps, 0.75%)

Summary of sample sizes: 16968, 16968, 16968, 16968, 16968, 16968, ...

Resampling results across tuning parameters:

k	ROC	Sens	Spec	ROC SD	Sens SD	Spec SD
3	0.8233105	0.8877796	0.5502203	0.005330046	0.005281699	0.01379943
5	0.8428730	0.8939393	0.5613362	0.004777924	0.005070641	0.01581545
9	0.8580986	0.8974523	0.5715991	0.004465693	0.005023055	0.01416908

ROC was used to select the optimal model using the largest value.

The final value used for the model was k = 9.

Naive Bayes Classifier for Discrete Predictors

Call:

```
naiveBayes.default(x = trainX, y = trainY)
```

A-priori probabilities:

trainY

0 1

0.751083 0.248917

Conditional probabilities:

age

trainY [,1] [,2]

0 -0.1348400 1.0285740

1 0.4130562 0.7764739

type_employer

trainY Federal-Govt Not-Working Other-Govt Private Self-Employed

0 0.0250720970 0.0005296922 0.1054676005 0.7698781708 0.0990524395

1 0.0458177944 0.0000000000 0.1287515539 0.6515716569 0.1738589948

education

trainY Associates Bachelors Doctorate Dropout HS-grad HS-Graduate Masters Prof-School

0 0.075922547 0.127596963 0.003884409 0.155258666 0.361838620 0.237949503 0.031722677
0.005826614

1 0.081690641 0.283786184 0.035872847 0.030190020 0.214526727 0.177943527 0.121115255
0.054874800

marital

trainY Married Never-Married Not-Married Widowed

0 0.33753163 0.40839268 0.22023424 0.03384145

1 0.85277926 0.06233351 0.07423193 0.01065530

occupation

trainY Admin Blue-Collar Military Other-Occupations Professional Sales Service White-Collar

0 0.1421929257 0.3660761580 0.0002354188 0.0460243658 0.0971102348 0.1157083162
0.1430168913 0.0896356895

1 0.0674835731 0.2214526727 0.0001775884 0.0644645711 0.2413425679 0.1250221985
0.0184691884 0.2615876399

relationship

trainY Husband Not-in-family Other-relative Own-child Unmarried Wife

0 0.298040139 0.302925078 0.037137308 0.196633512 0.133305868 0.031958095

1 0.754395312 0.108684070 0.005150062 0.008346652 0.029834843 0.093589061

race

trainY Amer-Indian Asian Black Other White

0 0.011005827 0.028073686 0.110999941 0.009475605 0.840444941

1 0.004439709 0.033386610 0.048481620 0.003019002 0.910673060

sex

trainY Female Male

0 0.3850862 0.6149138

1 0.1505949 0.8494051

hr_per_week

trainY [1] [2]

0 -0.1347207 1.0011172

1 0.4015488 0.8871906