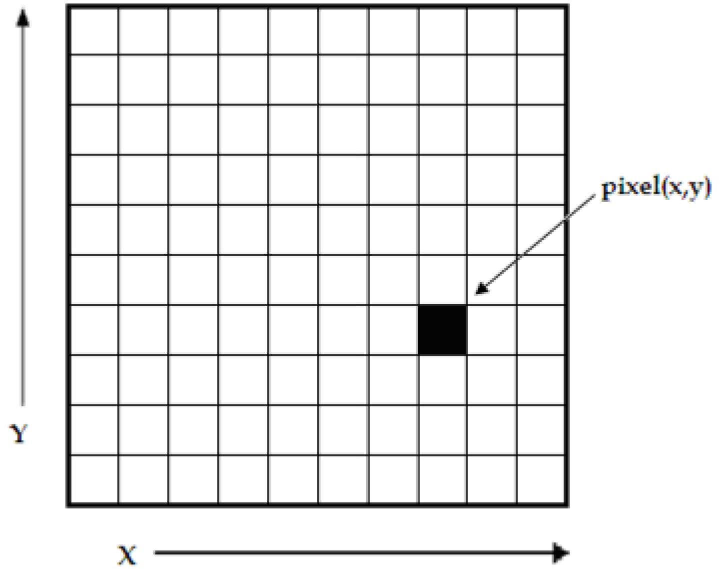
Advanced Robotics

Chap. 7 Visual sensory systems of robots

7.1 Modeling of digital images



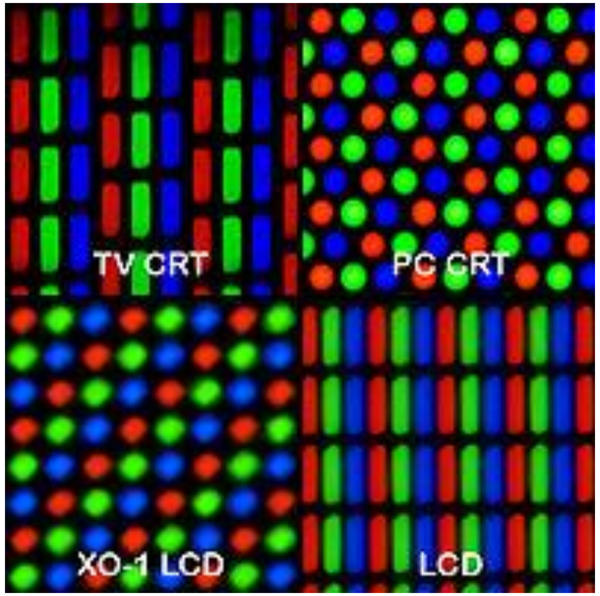


Image: chromatic information => RGB
geometric information => location of image

Chromatic modeling

- Color: chrominance + luminance
- RGB color space

$$I_R = \{r(i,j), \quad 1 \le i \le r_x, \quad 1 \le j \le r_y\}$$, red color

$$I_G = \{g(i,j), \quad 1 \le i \le r_x, \quad 1 \le j \le r_y\}$$, green color

$$I_B = \{b(i,j), \quad 1 \le i \le r_x, \quad 1 \le j \le r_y\}$$, blue color

(r_x, r_x): image resolution

- Representation of intensity image

$$I_I = \{I(i,j), \qquad 1 \le i \le r_x, \qquad 1 \le j \le r_y\}$$

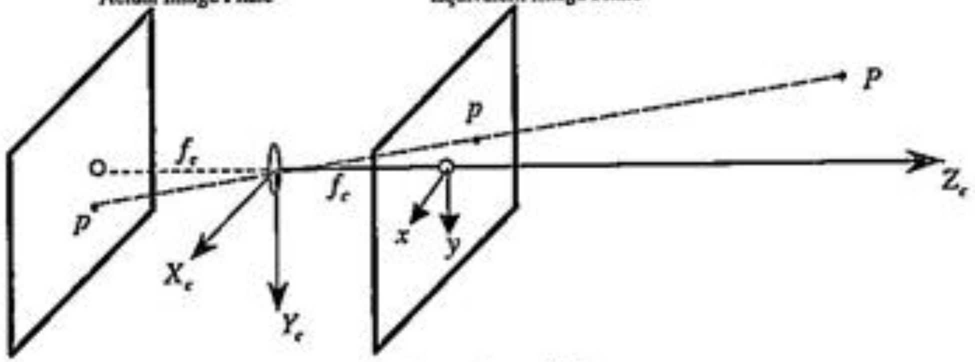
where

$$I(i,j) = 0.3\, r(i,j) + 0.59 g(i,j) + 0.11 b(i,j)$$

Geometric modeling

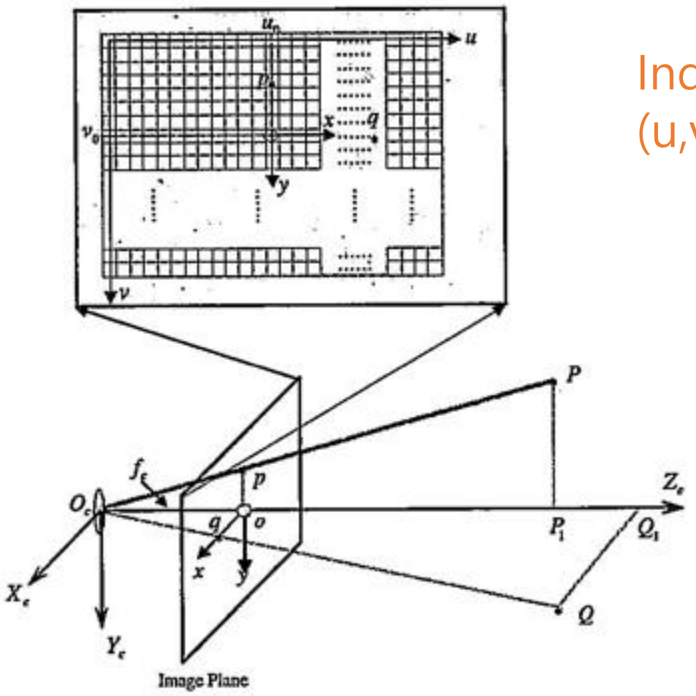
- Treat optical lens as a small hole-pin-hole

(a) Pin-hole camera model



(b) Equivalence of perspective projection

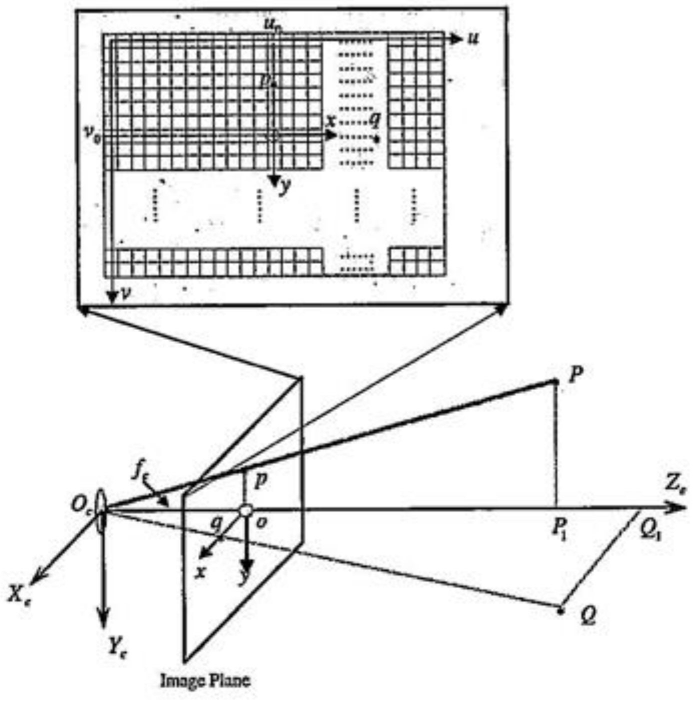
- Image coordinates & Index coordinates



Index coordinates
(u,v)

Image coordinates
(x,y)

Relationship between image coordinates and index coordinates



Perspective projection from 3D space to 2D space

$$x = f_c \frac{c_X}{c_Z}$$

$$y = f_c \frac{c_Y}{c_Z}$$

For a point **P** and p

$\Delta P P_1 O_c$ is similar to $\Delta p o O_c$

So

$\frac{c_Y}{y} = \frac{c_Z}{f_c}$

or $y = f_c \frac{c_Y}{c_Z}$

For a point Q and q

$\Delta Q Q_1 O_c$ is similar to $\Delta q o O_c$

So

$\frac{c_X}{x} = \frac{c_Z}{f_c}$

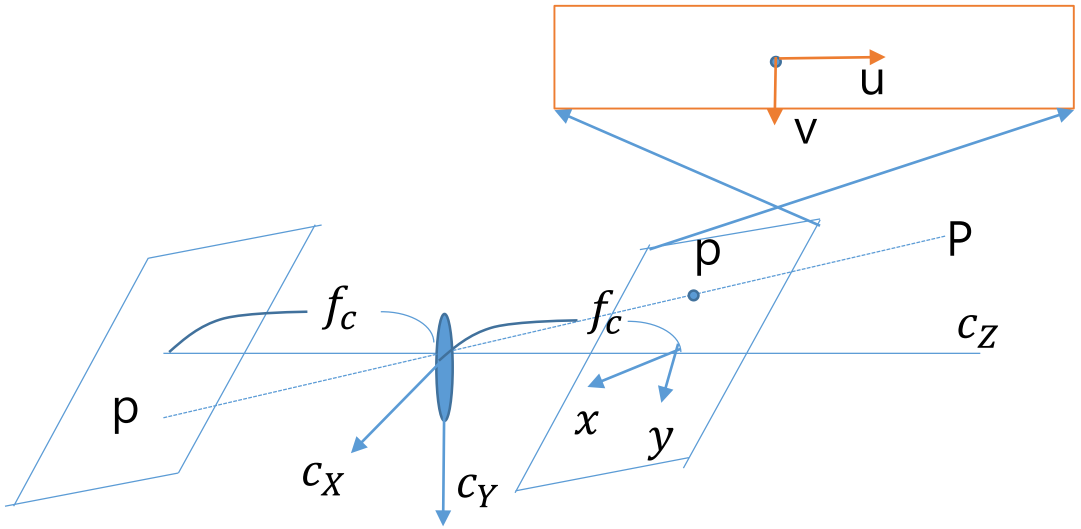
or $x = f_c \frac{c_X}{c_Z}$

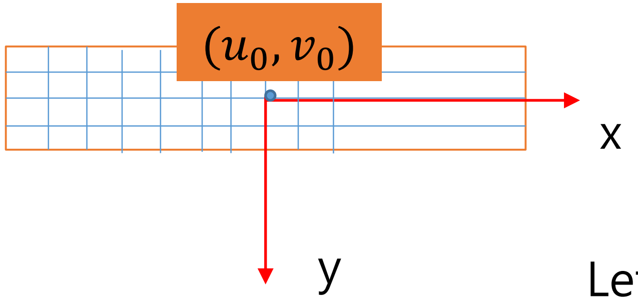
Relationship between image coordinates and index coordinates

Using scale factor s

$$s\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f_c & 0 & 0 & 0 \\ 0 & f_c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}$$

: forward projective mapping





Let $pixel\ size\ be\ (D_x, D_y)$

$$\frac{x}{D_x} = u - u_0,$$
$$\frac{y}{D_y} = v - v_0,$$

Relationship between image coordinates and index coordinates

Or $$u = u_0 + \frac{x}{D_x}$$
$$v = v_0 + \frac{y}{D_y}$$

Here $\frac{x}{D_x}$=number of digitization on horizontal axis

$\frac{y}{D_y}$=number of digitization on vertical axis

Thus,

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

: image coordinates $\longleftrightarrow$ index coordinates

Relationship between image coordinates and index coordinates

Since

$$s\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f_c & 0 & 0 & 0 \\ 0 & f_c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}$$

So

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = s\begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix} s\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} f_c & 0 & 0 & 0 \\ 0 & f_c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{f_c}{D_x} & 0 & u_0 & 0 \\ 0 & \frac{f_c}{D_y} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{f_c}{D_x} & 0 & u_0 & 0 \\ 0 & \frac{f_c}{D_y} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}, \quad \text{where} \quad f_x = \frac{f_c}{D_x},\ f_y = \frac{f_c}{D_y}$$

$$= {}^{I}p_c \begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}$$

Given $\begin{pmatrix} c_X \\ c_Y \\ c_Z \end{pmatrix}$, $then$ $\begin{pmatrix} u \\ v \end{pmatrix}$ is determined

However, given $\begin{pmatrix} u \\ v \end{pmatrix}$

$\begin{pmatrix} c_X \\ c_Y \\ c_Z \end{pmatrix}$ can not be determined

(s is not known)

${}^{I}p_c$: Perspective projection of coordinates from camera frame to index coordinates

Chap. 8 Visual perception system of robots
8.1 Introduction
Time domain transform

- RGB color space

$$I_R = \{r(i,j), \quad 1 \le i \le r_x, \quad 1 \le j \le r_y\}, \text{ red color}$$
$$I_G = \{g(i,j), \quad 1 \le i \le r_x, \quad 1 \le j \le r_y\}, \text{ green color}$$
$$I_B = \{b(i,j), \quad 1 \le i \le r_x, \quad 1 \le j \le r_y\}, \text{ blue color}$$

(r_x, r_x): image resolution

- Representation of intensity image

$$I_I = \{I(i,j), \qquad 1 \le i \le r_x, \qquad 1 \le j \le r_y\}$$

where

$$I(i,j) = 0.3\, r(i,j) + 0.59 g(i,j) + 0.11 b(i,j)$$

8.3 Image processing
Time domain transform

For NTSC TV

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = s \begin{pmatrix} 0.3 & 0.59 & 0.11 \\ 0.6 & -0.27 & -0.32 \\ 0.21 & -0.52 & -0.31 \end{pmatrix} \begin{pmatrix} Y \\ I \\ Q \end{pmatrix}$$

Y: Luminance

I: Hue

Q: Saturation

$$I = -u\sin(33^o) + vcos(33^o)$$

$$Q = u\cos(33^o) + vsin(33^o)$$

$$Y = 0.3R + 0.59G + 0.11B$$

$$v = 0.877\,(R - Y)$$

$$u = 0.493(B - Y)$$

Chromaticity

8.3 Image processing
Spatial domain transform

$$I_I^{out}(v_1, u_1) = I_I^{in}(v, u), \quad v \in [1, r_y],\ u \in [1, r_x]$$

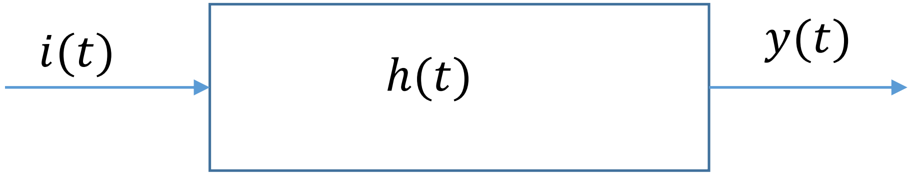
Where

$$s\begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

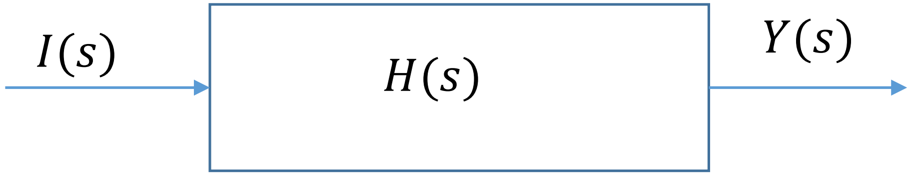
If $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\Rightarrow$ Image Rotation

8.3 Image processing
Image filtering

- Dynamic system



Time domain



Laplace domain

$$Y(s) = H(s)I(s)$$

$I(s)$ = Input image $\quad$ $Y(s)$ = filtered image $\quad$ $H(s)$ = filter

8.3 Image processing
Image filtering

- Convolution

$$y(x) = h(x) * i(x) = \int_{-\infty}^{x} h(\alpha) i(x-\alpha) d\alpha$$
$$= \int_{-\infty}^{x} i(\alpha) h(x-\alpha) d\alpha$$

$h(x)$: convolution kernel

- In a image plane

$$I_I^{in} = \{ I_I^{in}(v,u), v \in [1, r_y], u \in [1, r_x] \}$$
$$I_I^{out} = \{ I_I^{out}(v,u), v \in [1, r_y], u \in [1, r_x] \}$$

8.3 Image processing

Image filtering

- Introducing discrete convolution kernel

$$h_k = \{h_k(m,n), m \in [1, k_y], n \in [1, k_x]\ \}$$

with

$$\begin{cases} h_k(m,n) \neq 0, & if\ \forall m \in [1, k_y], \forall n \in [1, k_x] \\ h_k(m,n) = 0\ , & otherwise \end{cases}$$

Result image by kernel

$$I_I^{out}(v,u) = h_k(m,n) * I_I^{in}(v,u)$$
$$= \sum_{v_1=1}^{v} \sum_{u_1=1}^{u} \{h_k(v - v_1, u - u_1) I_I^{in}(v_1, u_1)$$

8.3 Image processing
Image filtering

Let $u - u_1 = n, v - v_1 = m,$

Then

$1 \le u - u_1 = n \le k_x,\ 1 \le v - v_1 = m \le k_y$

So

$$u - k_x \le u_1 \le u - 1,\ \ v - k_y \le v_1 \le v - 1$$

- Output image

$$I_I^{out}(v,u) = h_k(m,n)^{*} I_I^{in}(v,u)$$
$$= \sum_{m=1}^{k_y} \sum_{n=1}^{k_x} \{h_k(m,n) I_I^{in}(v-m, u-n)\}$$

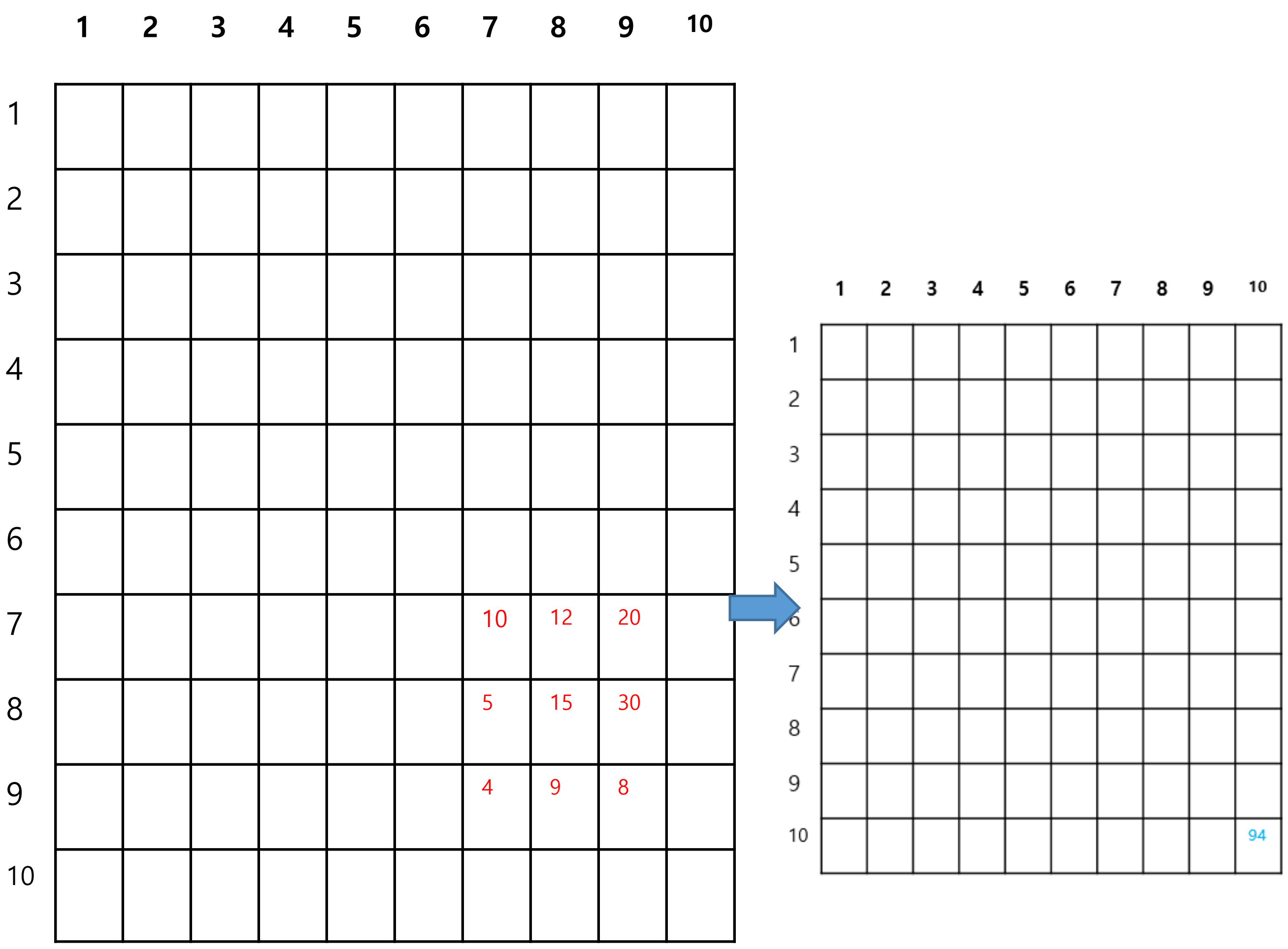
8.3 Image processing
Image filtering

- E.g.

  convolution kernel

$$h(m,n) = \frac{1}{6}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix},\ k_x = 3, k_y = 3$$

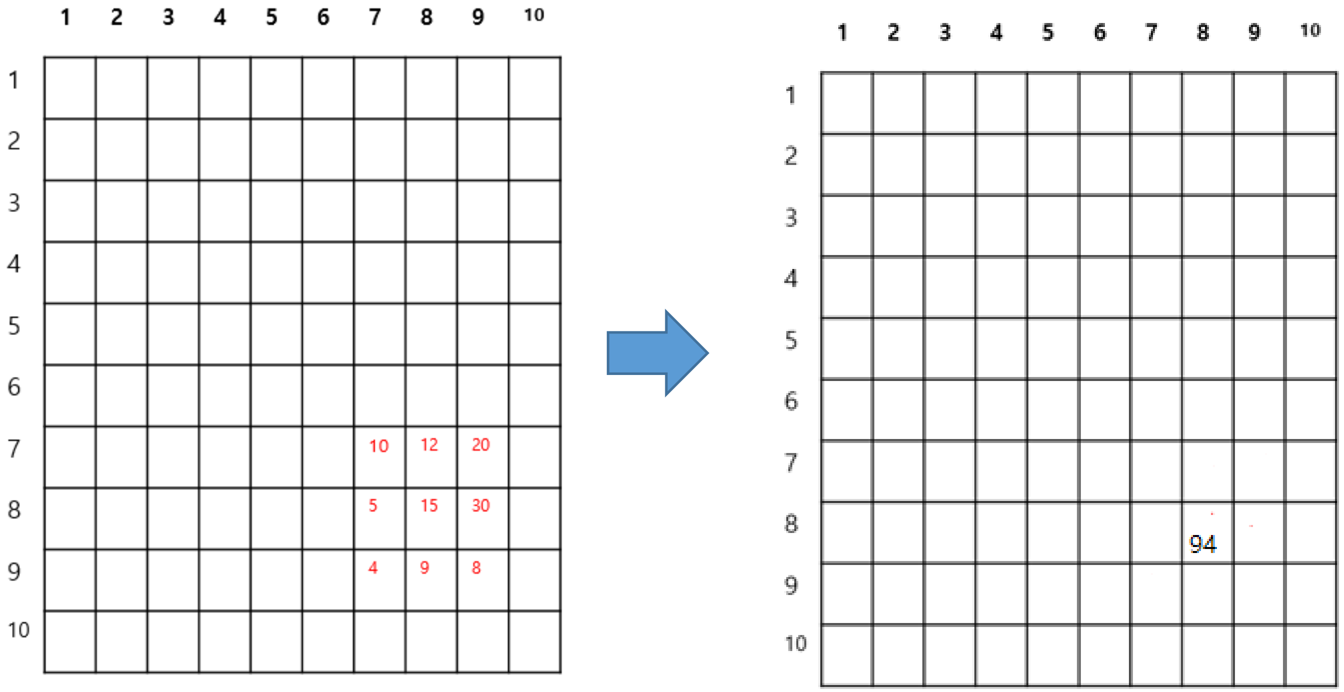
$$I_I^{out}(10,10) = h_k(m,n) {}^* I_I^{in}(v,u)$$
$$= \sum_{m=1}^{3}\sum_{n=1}^{3}\{h_k(m,n) I_I^{in}(v-m,u-n\ )$$
$$= h_k(1,1) I_I^{in}(10-1,10-1) +$$
$$h_k(1,2) I_I^{in}(10-1,10-2) +$$
$$h_k(1,3) I_I^{in}(10-1,10-3) + \ldots\ldots$$
$$+\ h_k(3,3) I_I^{in}(10-3,10-3) + \ldots\ldots = 94$$



8.3 Image processing
Image filtering

- Original input image

Output image



For output centered on input image

$$I_I^{out}(v,u) = h_k(m,n)*I_I^{in}(v,u)$$
$$=\sum_{m=1}^{k_y}\sum_{n=1}^{k_x}\{h_k(m,n)I_I^{in}\left(v-m+\frac{k_y}{2}+1, u-n+\frac{k_x}{2}+1\right)$$

8.3 Image processing
Image filtering

- Derivative of convolutions

$$Y(s) = H(s)I(s)$$

- Laplace transform of derivative

$£\left(f(x)\right) \triangleq \int_0^\infty e^{-sx} f(x)dx$: Laplace transform

since $£\left(\frac{df(x)}{dx}\right) = sF(s)$

So
$sY(s) = sH(s)I(s) = H(s)s\,I(s)$

By inverse Laplace transform
$\frac{df(x)}{dx} = h(x) * \frac{d\,i(x)}{dx}$, $i(x)$: input image, $\frac{df(x)}{dx}$: image derivative
$= \frac{dh(x)}{dx} * i(x)$: convolution for image derivative

8.3 Image processing

Image filtering

- E.g. $\xrightarrow{\quad\quad} n$

$$h(m,n) = \frac{1}{6}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \ \downarrow m \quad \text{is given}$$

Compute the horizontal directional derivative

$$\frac{dh(m,n)}{dn} = \frac{1}{6}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

By convolution

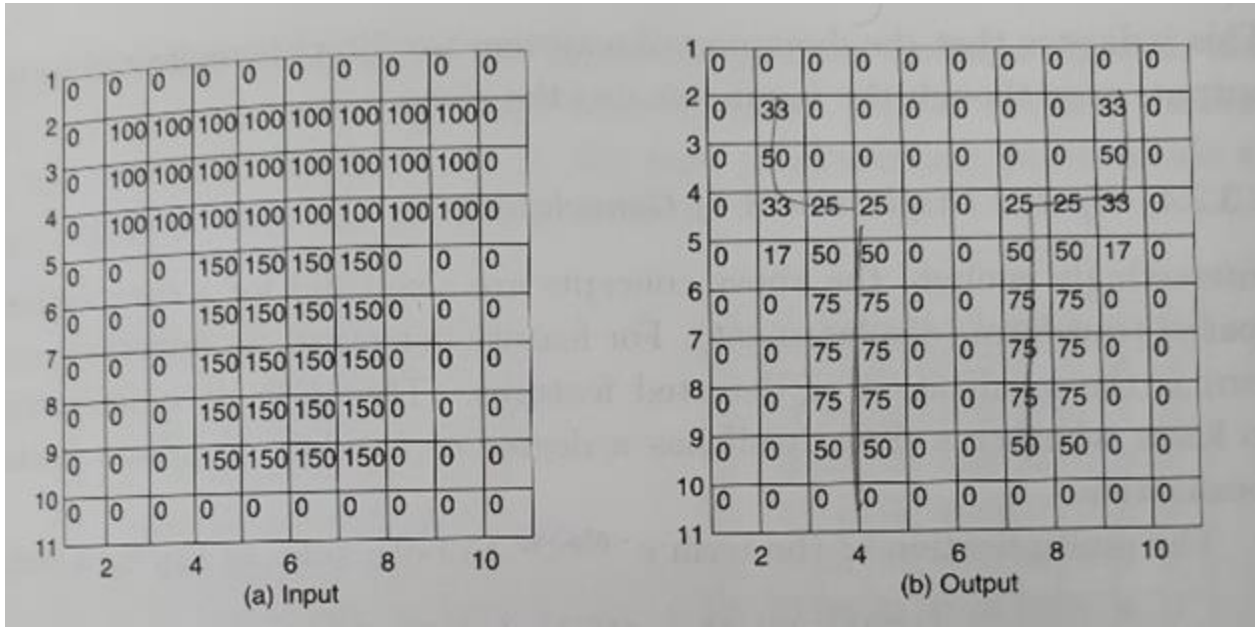
$$\frac{df(x)}{dx} = \frac{dh(x)}{dx} * i(x)$$

$$\frac{dI_I^{out}}{dn} = \sum_{m=1}^{k_y} \sum_{n=1}^{k_x} \{\frac{dh(m,n)}{dn} I_I^{in}(v-m, u-n)\}$$

8.3 Image processing
Image filtering

Result

Used for feature extraction



(a) Input

	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	100	100	100	100	100	100	100	100	0
3	0	100	100	100	100	100	100	100	100	0
4	0	100	100	100	100	100	100	100	100	0
5	0	0	0	150	150	150	150	0	0	0
6	0	0	0	150	150	150	150	0	0	0
7	0	0	0	150	150	150	150	0	0	0
8	0	0	0	150	150	150	150	0	0	0
9	0	0	0	150	150	150	150	0	0	0
10	0	0	0	0	0	0	0	0	0	0

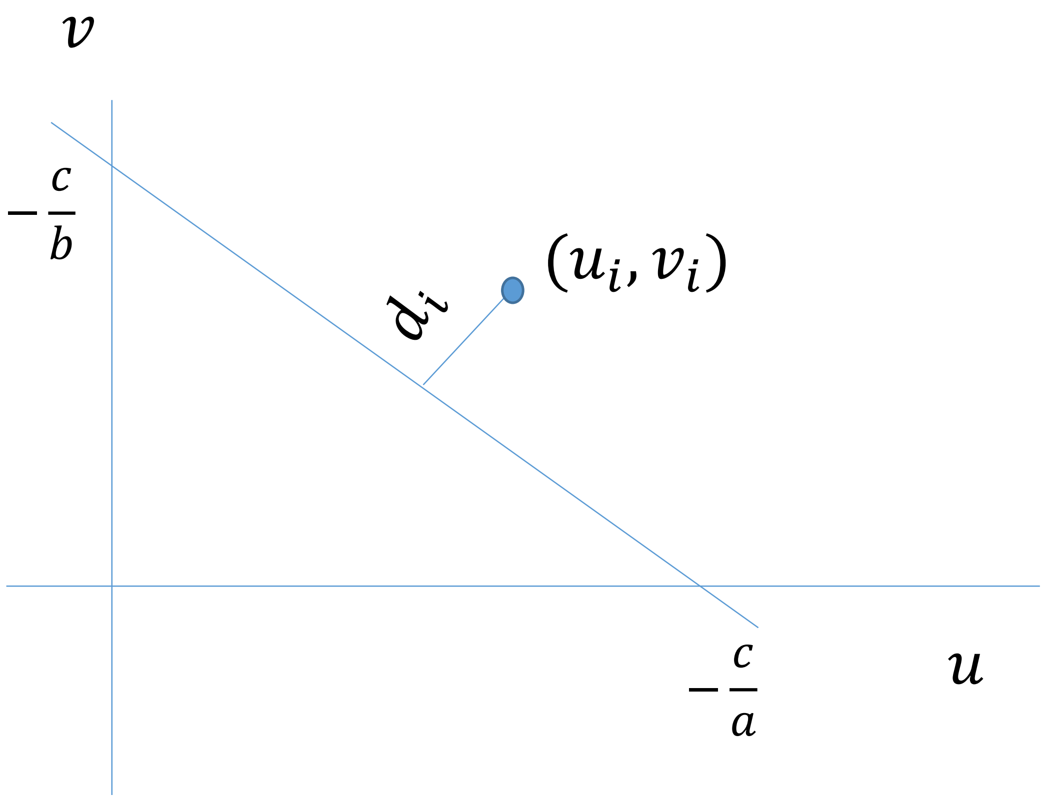
(b) Output

	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	33	0	0	0	0	0	0	33	0
3	0	50	0	0	0	0	0	0	50	0
4	0	33	25	25	0	0	25	25	33	0
5	0	17	50	50	0	0	50	50	17	0
6	0	0	75	75	0	0	75	75	0	0
7	0	0	75	75	0	0	75	75	0	0
8	0	0	75	75	0	0	75	75	0	0
9	0	0	50	50	0	0	50	50	0	0
10	0	0	0	0	0	0	0	0	0	0

Cited from Ming Xie, Fundamentals of Robotics, World Scientific

8.4 Image contour splitting

- Arbitrary shape image -> set of simple curves

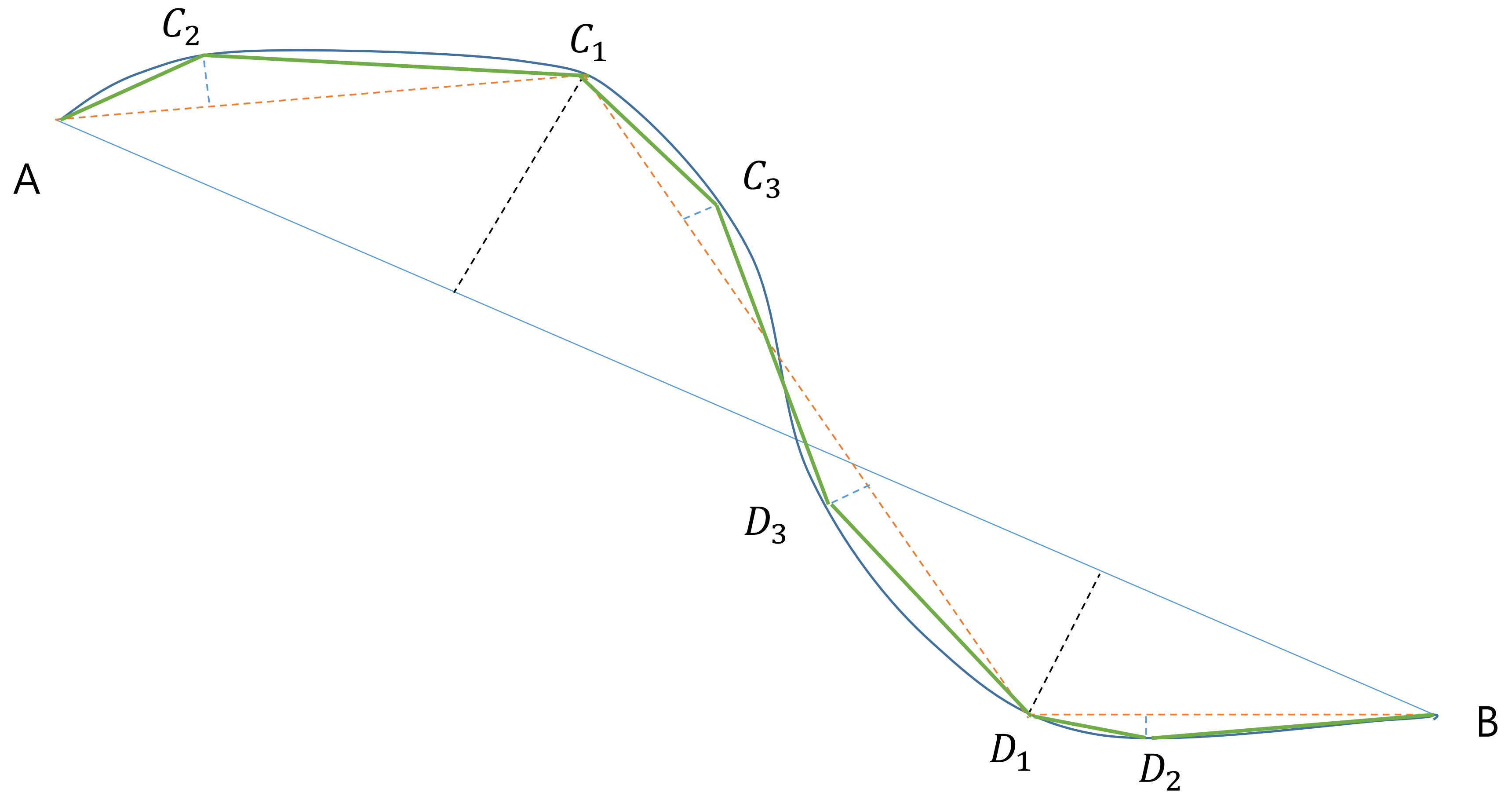


$$au + bv + c = 0$$

$$d_i = \frac{||au_i + bv_i + c||}{\sqrt{a^2 + b^2}}$$

8.4 Image contour splitting

- Successive linear curves



8.4 Image contour splitting

- Procedures for successive linear curves

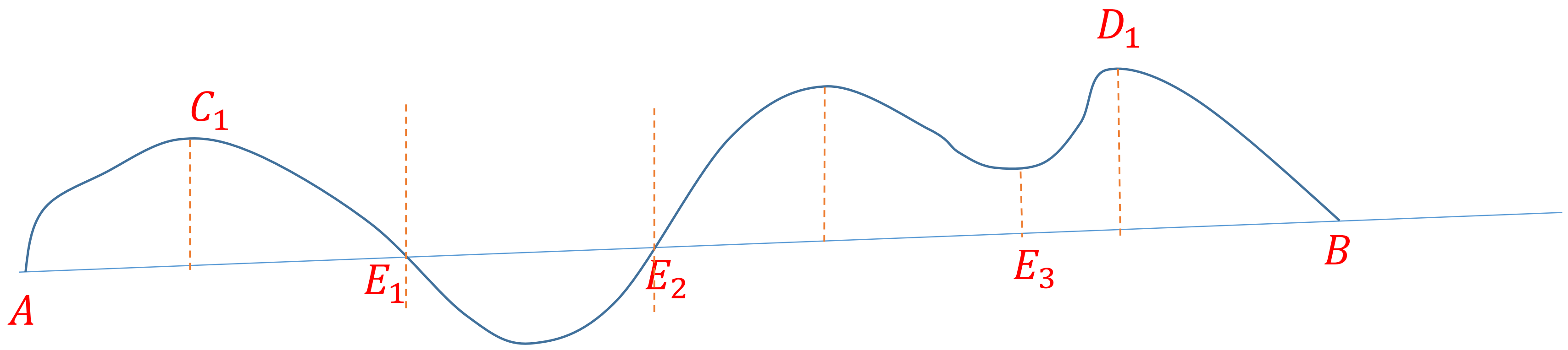
1) Find the local maximum edge ($d_i > d_0$) nearest to one end point of the contour: C_1
2) Find the local maximum edge nearest to other end point of the contour: D_1
3) Repeat until no splitting occurs

Final linear curves

$AC_2 \rightarrow C_2C_1 \rightarrow C_1C_3 \rightarrow C_3D_3 \rightarrow D_3D_1 \rightarrow D_1D_2 \rightarrow D_2B$

8.4 Image contour splitting

- Procedures for successive nonlinear curves



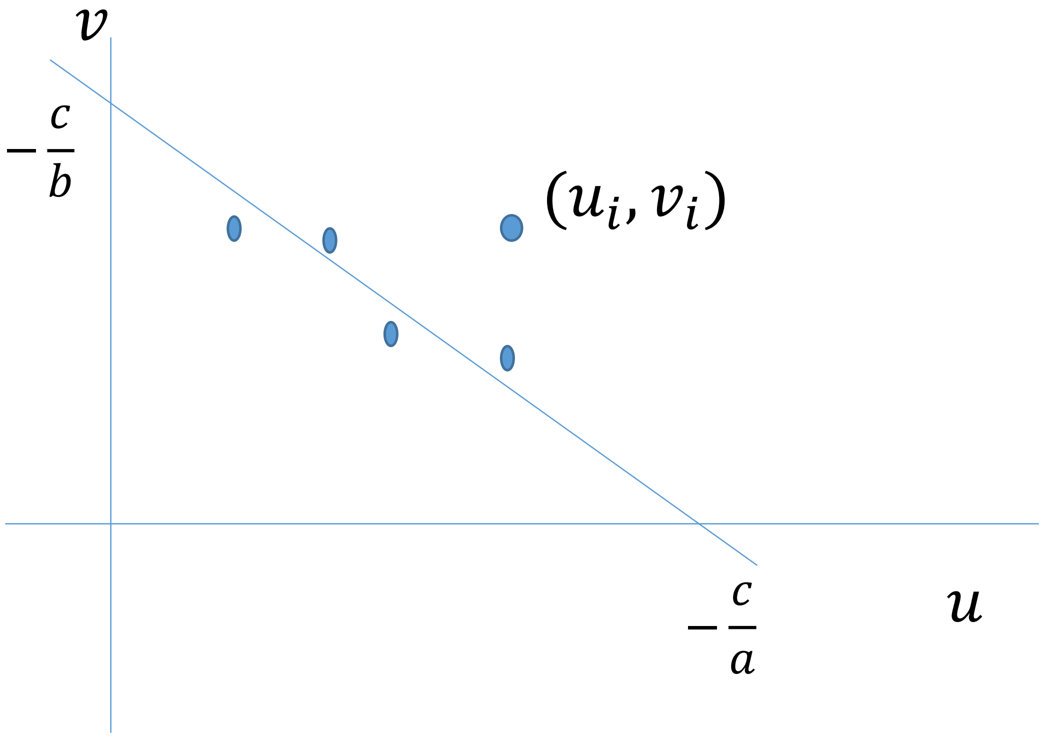
1) Find the local maximum edges ($d_i > d_0$) nearest to one end point of the contour: C_1, D_1
2) Between two local maximum edges, find the edge nearest to the approximation line: E_1, E_2
3) Contour E_2B is further split at edge E_3
4) Repeat until no splitting occurs

Final nonlinear curves
$AE_1 \rightarrow E_1E_2 \rightarrow E_2E_3 \rightarrow E_3B$

8.5 Curve fitting

- For a simple contour, approximate with linear or curve function

1) Linear segment approximation



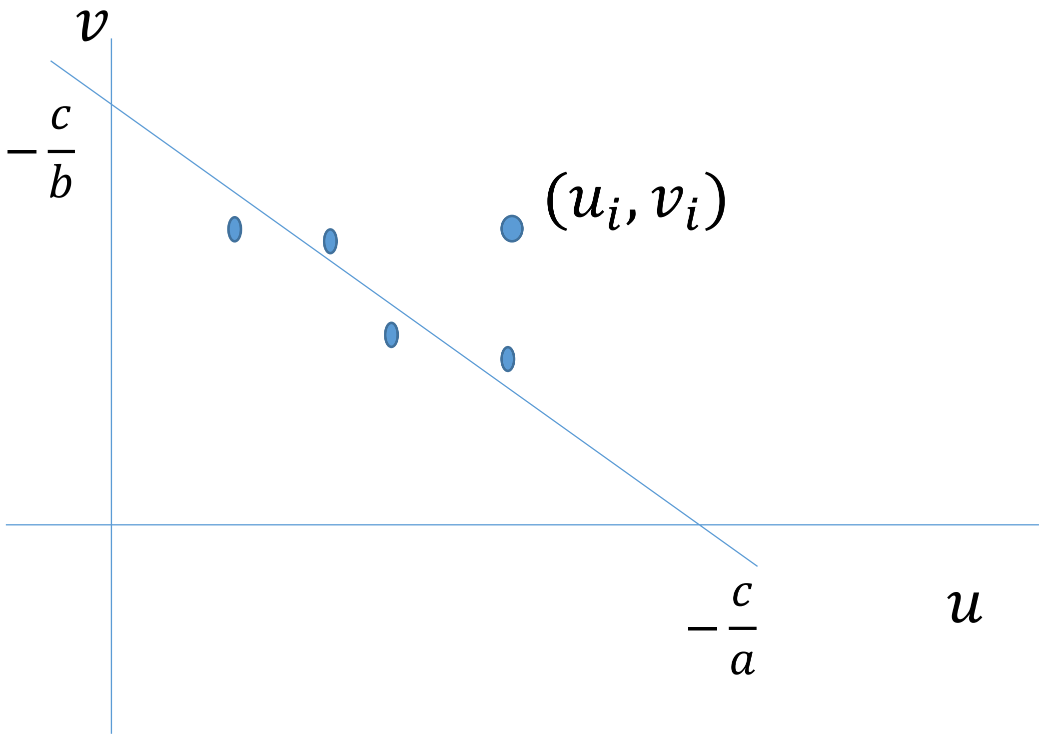
Input: a simple contour in the form of a list of edges:

$$C = \{(u_i, v_i), i = 1,2, \dots . n\}$$

Output: parameters of equation describing the contour

8.5 Curve fitting

1) Linear segment approximation



Line equation:

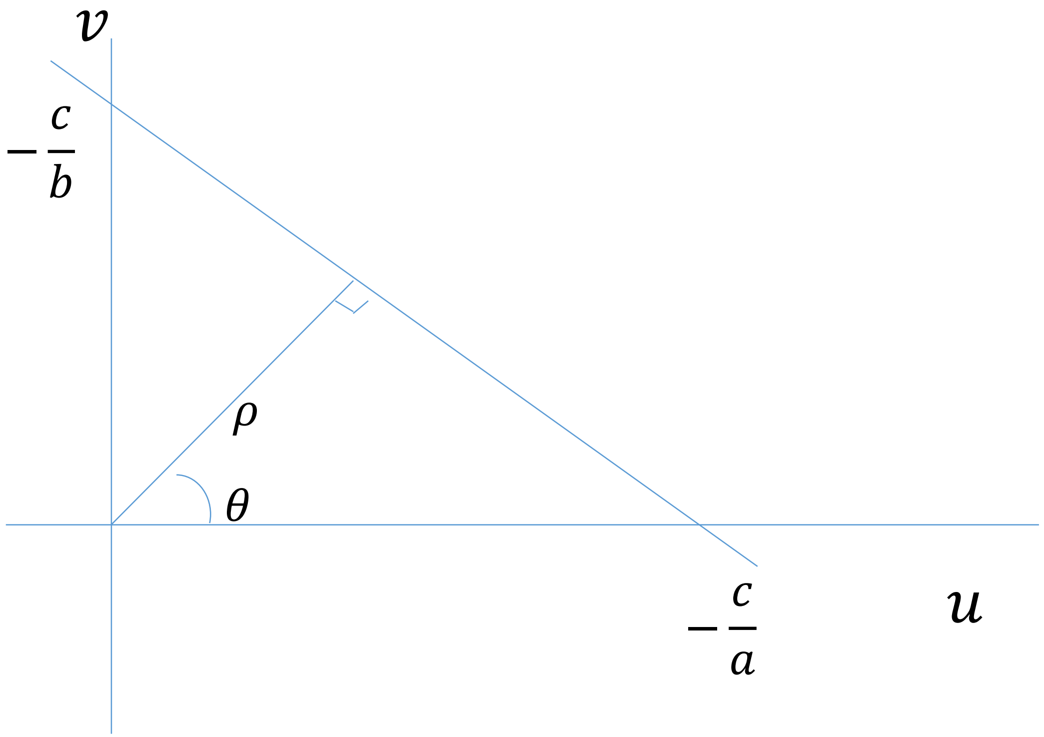
$$au + bv + c = 0$$

Dividing by $\sqrt{a^2 + b^2}$

$$\frac{a}{\sqrt{a^2+b^2}}u + \frac{b}{\sqrt{a^2+b^2}}v + \frac{c}{\sqrt{a^2+b^2}} = 0$$

8.6 Curve fitting

1) Linear segment approximation



Line equation:

$$\frac{a}{\sqrt{a^2+b^2}}u + \frac{b}{\sqrt{a^2+b^2}}v + \frac{c}{\sqrt{a^2+b^2}} = 0$$

Then

$$u\, cos\theta + v\, sin\theta + \rho = 0$$

where

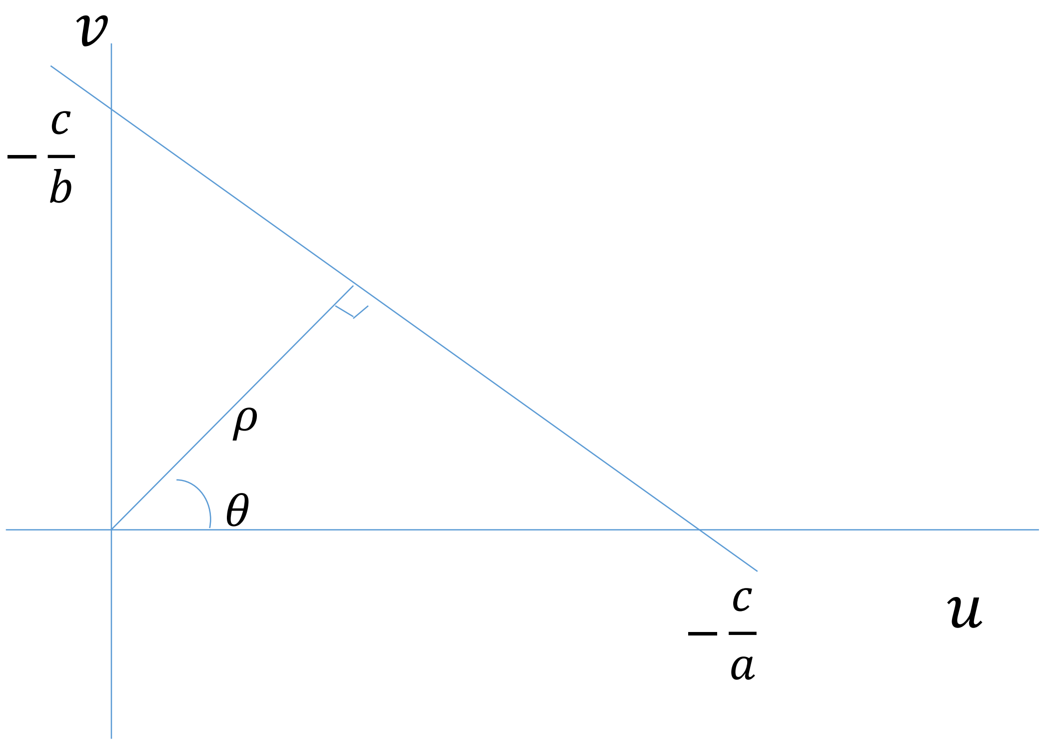
$$cos\theta = \frac{-a}{\sqrt{a^2+b^2}}$$

$$sin\theta = \frac{-b}{\sqrt{a^2+b^2}}$$

$$\rho = \frac{c}{\sqrt{a^2+b^2}}$$

8.5 Curve fitting

1) Linear segment approximation



Line equation:

$$u\ cos\theta + v\ sin\theta + \rho = 0$$

Proof:

$$cos\theta = \frac{\rho}{-\frac{c}{a}} = \frac{-\rho a}{c}$$

$$sin\theta = \frac{\rho}{-\frac{c}{b}} = \frac{-\rho b}{c}$$

By

$$cos^2\theta + sin^2\theta = 1$$

$$\left(\frac{-\rho a}{c}\right)^2 + \left(\frac{-\rho b}{c}\right)^2 = 1$$

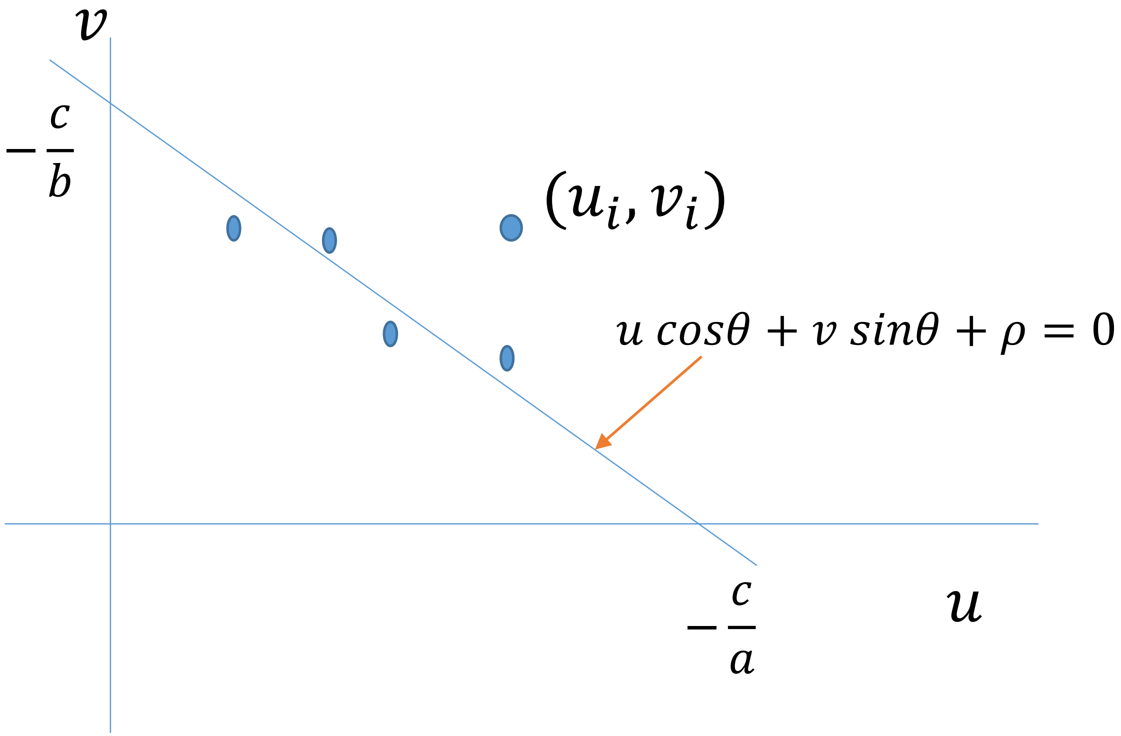
Thus,

$$\rho = \frac{c}{\sqrt{a^2 + b^2}}$$

8.5 Curve fitting

- Approximation line: error function

$$L = \sum_{i=1}^{n} (u_i cos\theta + v_i sin\theta + \rho)^2$$



Optimal solution: Minimizing the error function L

$$\frac{\partial L}{\partial \rho} = 0, \qquad \frac{\partial L}{\partial \theta} = 0$$

For $\frac{\partial L}{\partial \rho} = 0$:

$$\frac{\partial L}{\partial \rho} = 2\sum_{i=1}^{n} (u_i cos\theta + v_i sin\theta + \rho) = 0$$

8.5 Curve fitting

$$\frac{\partial L}{\partial \rho} = 2\sum_{i=1}^{n}(u_i cos\theta + v_i sin\theta + \rho) = 0$$

Thus,

$$n\rho = -\sum_{i=1}^{n} u_i cos\theta - \sum_{i=1}^{n} v_i sin\theta$$

$\therefore\ \rho \triangleq -\bar{u}cos\theta - \bar{v}sin\theta$

where

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i,\ \ \bar{v} = \frac{1}{n}\sum_{i=1}^{n} v_i,$$

Substituting ρ into L:

$$L = \sum_{i=1}^{n}(\overline{u_i} cos\theta + \overline{v_i} sin\theta)^2$$

where

$\overline{u_i} = u_i\text{-}\bar{u},\ \overline{v_i} = v_i\text{-}\bar{v}$

8.5 Curve fitting

For $\frac{\partial L}{\partial \theta} = 0$:

$\sum_{i=1}^{n}\{(\overline{u}_i cos\theta + \overline{v}_i sin\theta)\ (-\overline{u}_i sin\theta + \overline{v}_i cos\theta)\} = 0$

Then

$(B - A) sin\theta cos\theta + C(cos^2\theta - sin^2\theta) = 0$

where

$$A = \sum_{i=1}^{n} \overline{u}_i{}^2, \qquad B = \sum_{i=1}^{n} \overline{v}_i{}^2, C = \sum_{i=1}^{n} (\overline{u_i} \cdot \overline{v_i})$$

Since

$cos^2\theta - sin^2\theta = cos2\theta$

$sin\theta cos\theta = \frac{1}{2}\sin(2\theta)$

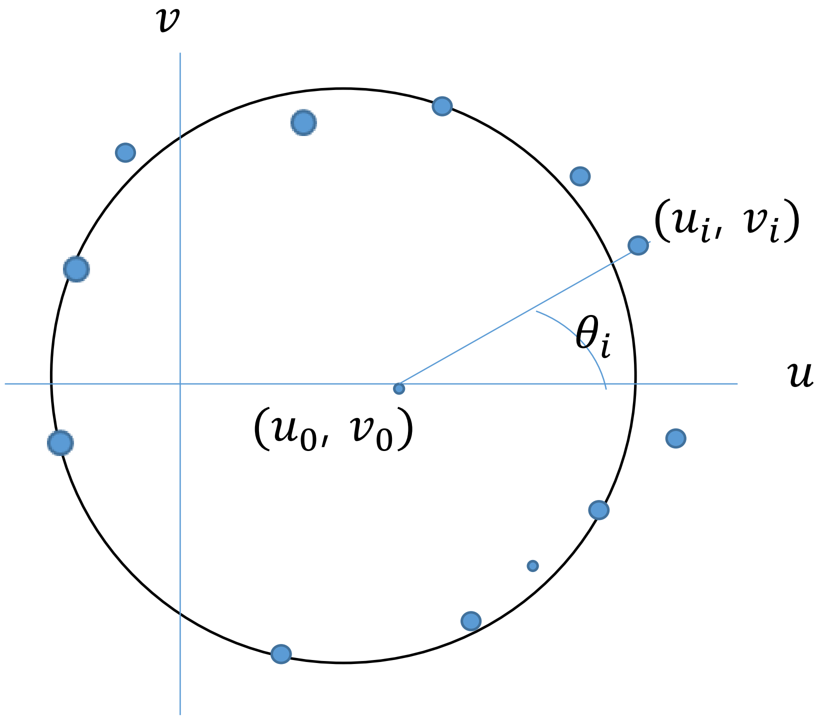
Finally,

$\frac{1}{2}(\mathrm{B} - \mathrm{A})\sin(2\theta) + C\ cos2\theta = 0;$

$\theta = \frac{1}{2} tan^{-1} \frac{2C}{A-B}$

8.5 Curve fitting

-Circle arc approximation



Circle equation:

$$(u-u_0)^2+(v-v_0)^2=\mathrm{r}^2$$

Also

$$u-u_0=r\cos\theta$$
$$v-v_0=r\sin\theta$$

so

$$r=\frac{u-u_0}{\cos\theta}=\frac{v-v_0}{\sin\theta}$$

Eliminating r

$$(u-u_0)\sin\theta-(v-v_0)\cos\theta=0$$

or

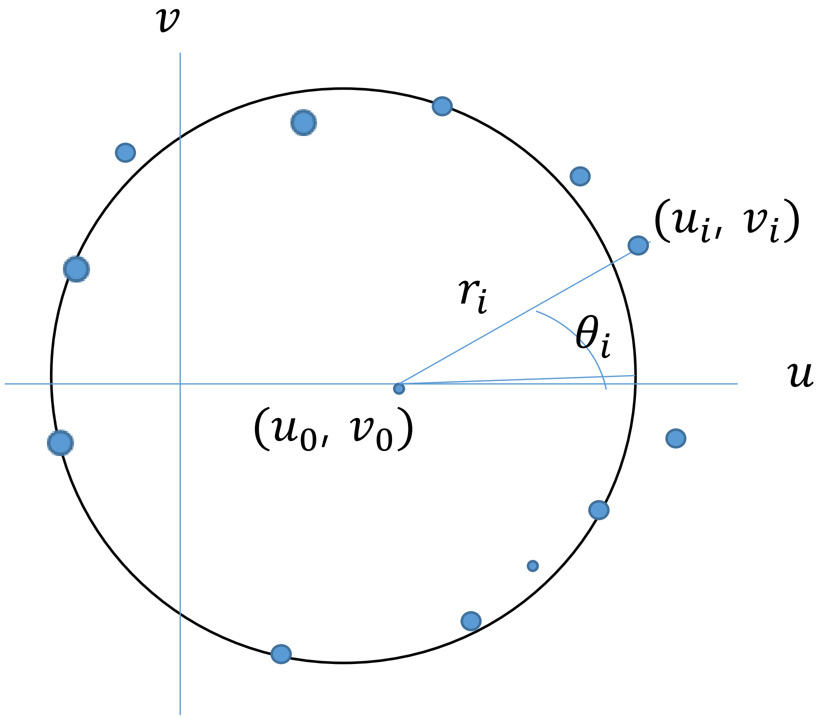
$$u_0\sin\theta-v_0\cos\theta=-v\cos\theta+u\sin\theta$$

Contour:

$$C=\{u_i, v_i, \theta_i, i=1,2,3,..n\}$$

8.5 Curve fitting

-Circle arc approximation



Error function

L= $\sum(u_0 sin\theta_i - v_0 \cos\theta_i + v_i cos\theta_i - u_i sin\theta_i)^2$

$to\ minimize\ L$

$$\frac{\delta L}{\delta u_0} = 0, \qquad \frac{\delta L}{\delta v_0} = 0 \ \Rightarrow determine\ u_0, v_0$$

when u_i, v_i $on\ the\ circle,\ L = 0$

$$nr \cong \sum_{i=1}^{n} \sqrt{(u_i - u_0)^2 + (v_i - v_0)^2}$$

$$r = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(u_i - u_0)^2 + (v_i - v_0)^2}$$

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Ellipse fitting with Least Square Method

ii) Elliptic equation

$$x^2 + b'xy + c'y^2 + d'x + e'y + f' = 0$$

$$\underbrace{\begin{bmatrix} x_1y_1 & {y_1}^2 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_ny_n & {y_n}^2 & x_n & y_n & 1 \end{bmatrix}}_{J} \underbrace{\begin{bmatrix} b' \\ c' \\ d' \\ e' \\ f' \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -{x_1}^2 \\ \vdots \\ -{x_n}^2 \end{bmatrix}}_{Y}$$

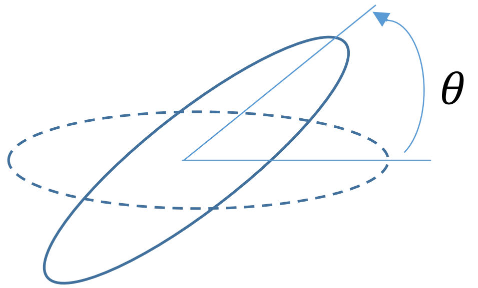
$$JX = Y$$

$$X = (J^TJ)^{-1}J^TY$$

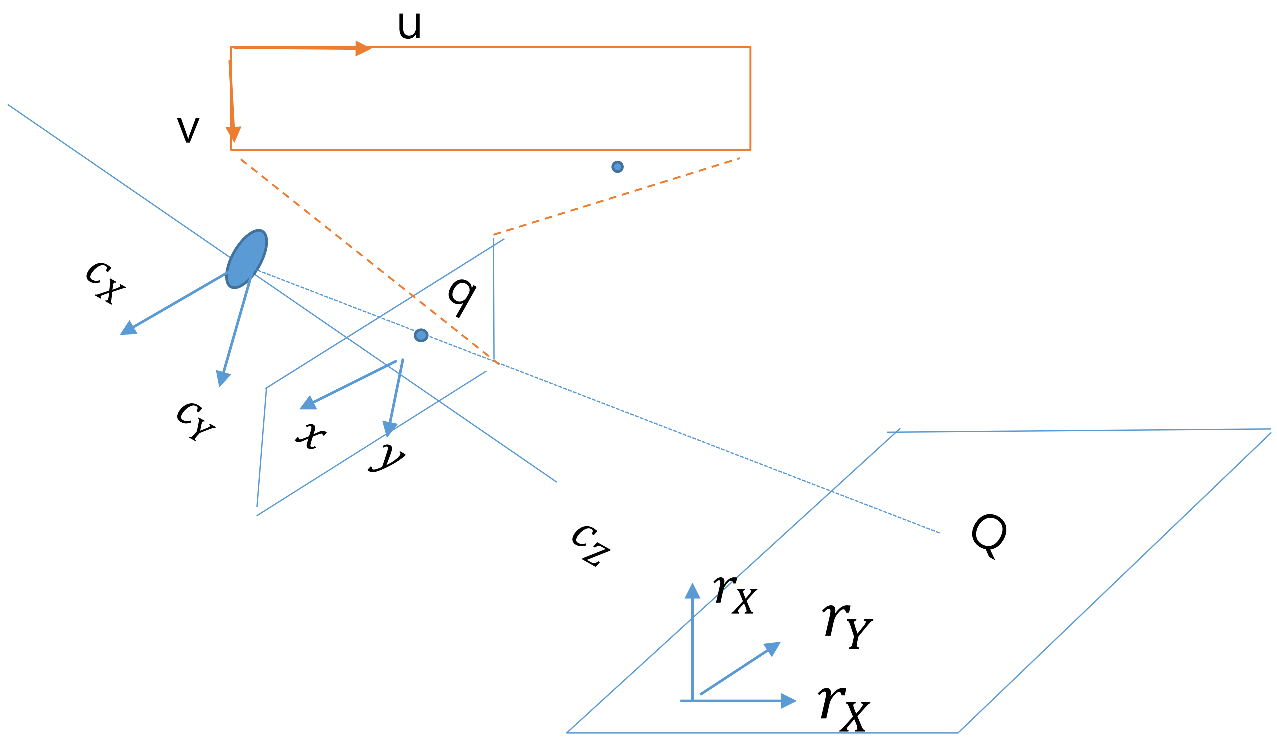
$$\frac{(x-x_1)^2}{a^2} + \frac{(y-y_1)^2}{b^2} = 1$$

(Reflect the rotation)

$$\frac{[cos(\theta)x + sin(\theta)y - x_1]^2}{a^2} + \frac{[-sin(\theta)x + cos(\theta)y - y_1]^2}{b^2} = 1$$



8.6 Geometry measurement



Relationship between index coordinates
& reference coordinates

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = {}^{I}P_c \begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix}$$

where

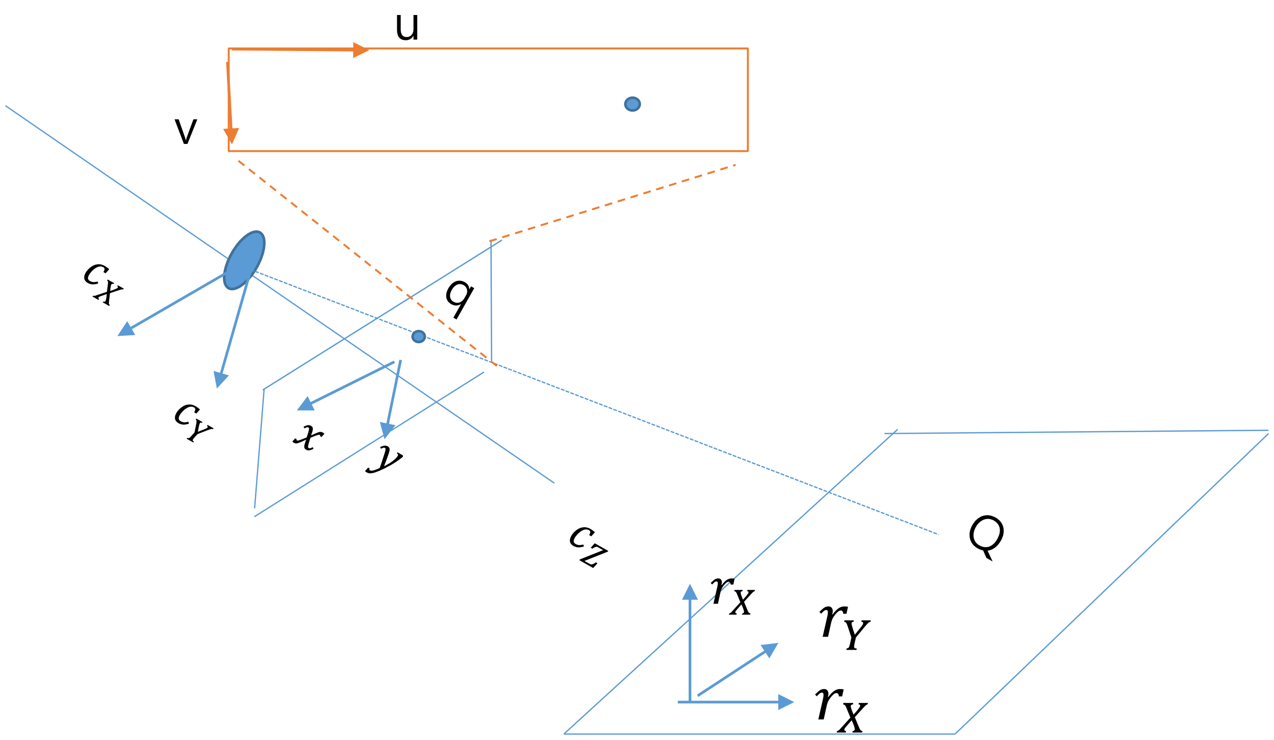
$${}^{I}P_c = \begin{pmatrix} \frac{f_c}{Dx} & 0 & u_0 & 0 \\ 0 & \frac{f_c}{Dy} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},\ {}^{I}P_c\text{: intrinsic parameters}$$

Can Q be determined from q wrt reference frame?

$$\begin{pmatrix} c_X \\ c_Y \\ c_Z \\ 1 \end{pmatrix} = {}^{c}M_r \begin{pmatrix} r_X \\ r_Y \\ r_Z \\ 1 \end{pmatrix}$$

Transformation
Matrix

8.6 Geometry measurement



f_c =focal length

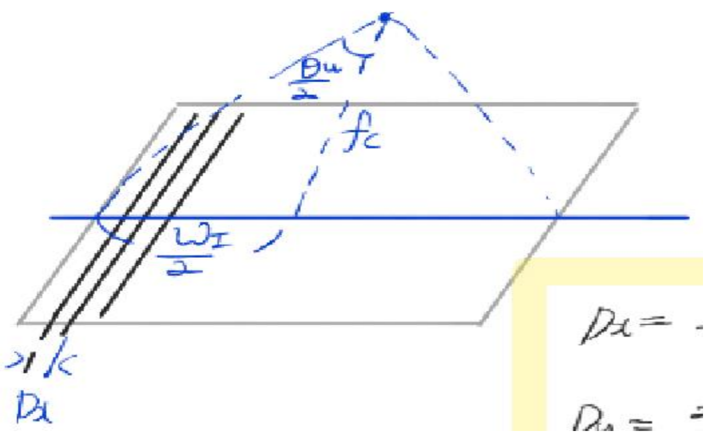
Camera equation is

$$s\begin{pmatrix}u\\v\\1\end{pmatrix} = {}^{I}P_c\begin{pmatrix}c_X\\c_Y\\c_Z\\1\end{pmatrix} = {}^{I}P_c\,{}^{c}M_r\begin{pmatrix}r_X\\r_Y\\r_Z\\1\end{pmatrix} \triangleq H\begin{pmatrix}r_X\\r_Y\\r_Z\\1\end{pmatrix}$$

H= calibration matrix

8.6 Geometry measurement

fc, Dx, Dy determination



Horizontal pixel size $\boldsymbol{Dx}$

$$\tan\frac{\theta_u}{2} = \frac{\frac{\omega_I}{2}}{f_c} = \frac{\omega_I}{2f_c} = \frac{r_x\,P_x}{2f_c}$$

$$D_x = \frac{2f_c}{r_x}\tan\left(\frac{\theta u}{2}\right)$$

θ_u: *Horizontal angle of view*

r_x : horizontal axis resolution

Similarly

vertical pixel size $\boldsymbol{Dy}$

$$D_y = \frac{2f_c}{r_y}\tan\left(\frac{\theta v}{2}\right)$$

θ_v: *Vertical angle of view*

r_y : vertical axis resolution

8.6 Geometry measurement

e.g. Determine ${}^{I}P_c$

Given $resolution : 512 \text{ x } 512,$ $focal\ length : 1cm$

$Aperture\ angle: 70° \text{ x } 70°$

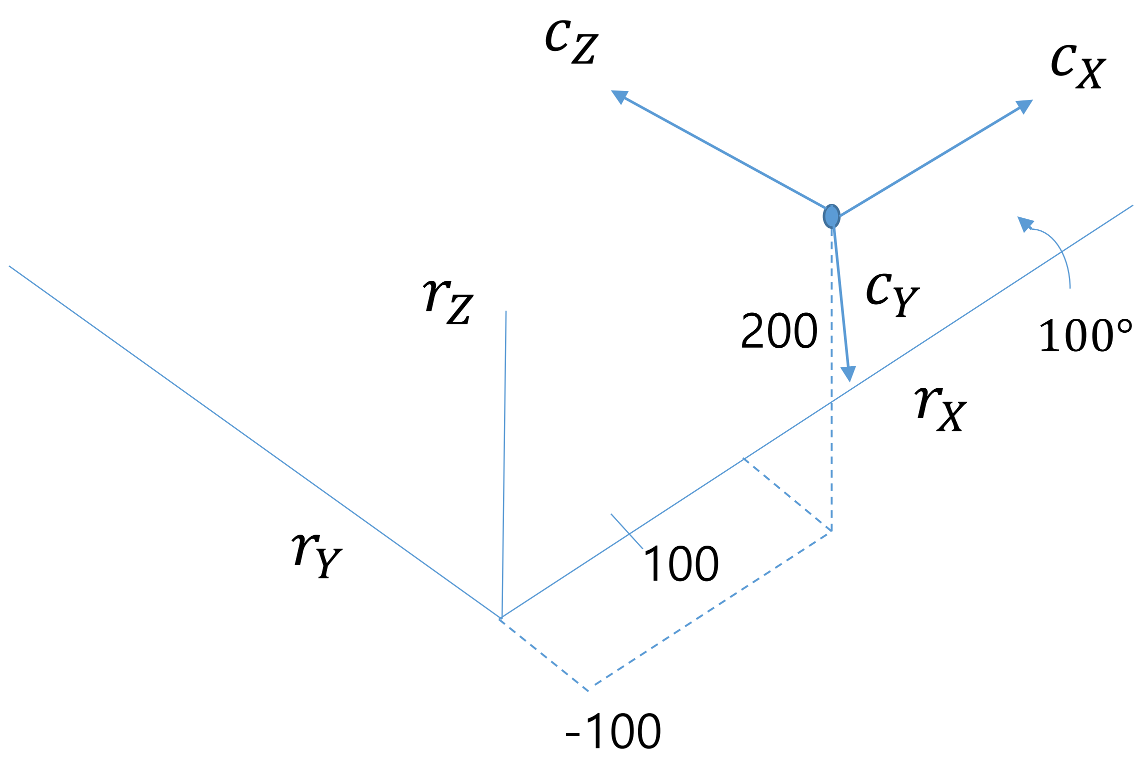
$$(u_0, v_0) = \left(\frac{\mathrm{r_x}}{2}, \frac{\mathrm{r_y}}{2}\right) = (256{,}256)$$

$${}^{I}P_c = \begin{pmatrix} \frac{f_c}{Dx} & 0 & u_0 & 0 \\ 0 & \frac{f_c}{Dy} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 365.6 & 0 & 256 & 0 \\ 0 & 365.6 & 256 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

8.6 Geometry measurement

e.g- Forward projective mapping

${}^{r}M_c$
: $camera\ frame\ is\ formed\ by\ translating\ along$
(200, -100, 200) cm
and $by\ rotating\ rot(r_X, -100°)$



$${}^{r}M_c = \begin{pmatrix} 1 & 0 & 0 & 200 \\ 0 & c\theta & -S\theta & -100 \\ 0 & S\theta & c\theta & 200 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ then, how about ${}^{c}M_r$?

$\theta = -100°$

$${}^{c}M_r = ({}^{r}M_c)^{-1}$$
$$= \begin{pmatrix} 1 & 0 & 0 & -p \cdot n \\ 0 & c\theta & s\theta & -p \cdot o \\ 0 & -s\theta & c\theta & -p \cdot a \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$n = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, o = \begin{pmatrix} 1 \\ c\theta \\ s\theta \end{pmatrix}, a = \begin{pmatrix} 0 \\ -s\theta \\ c\theta \end{pmatrix}, p = \begin{pmatrix} 200 \\ -100 \\ 200 \end{pmatrix}$$

$${}^{c}M_r = ({}^{r}M_c)^{-1} = \begin{pmatrix} 1 & 0 & 0 & -200 \\ 0 & -0.173 & -0.984 & 179.59 \\ 0 & 0.984 & -0.173 & 133.21 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

8.6 Geometry measurement

e.g- Forward projective mapping

$$H = {}^{I}P_c\,{}^{c}M_r = \begin{pmatrix} 0.036 & 0.025 & -0.004 & -3.88 \\ 0 & 0.019 & -0.04 & 9.97 \\ 0 & 0.001 & 0 & 0.133 \end{pmatrix} \times 10^4$$

Dividing by H(3,4)= 0.0133×10^4

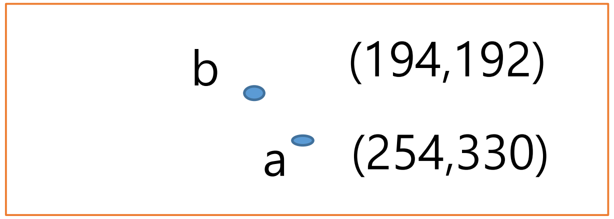
$$\mathrm{H} = \begin{pmatrix} 2.74 & 1.89 & -0.33 & -292.46 \\ 0 & 1.426 & -3.02 & 750.07 \\ 0 & 0.0074 & -0.0013 & 1 \end{pmatrix}$$

For object A = (200,400,0) in (r_X, r_Y, r_Z)

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H\begin{pmatrix} 200 \\ 400 \\ 0 \end{pmatrix} = \begin{pmatrix} 1014 \\ 1320 \\ 4 \end{pmatrix}, \quad \begin{matrix} s = 4 \\ 4u = 1014 \\ 4v = 1320 \end{matrix} \Rightarrow \begin{pmatrix} u = 253.5 \\ v = 330 \end{pmatrix} \quad index\ frame$$

8.6 Geometry measurement

e.g- Forward projective mapping

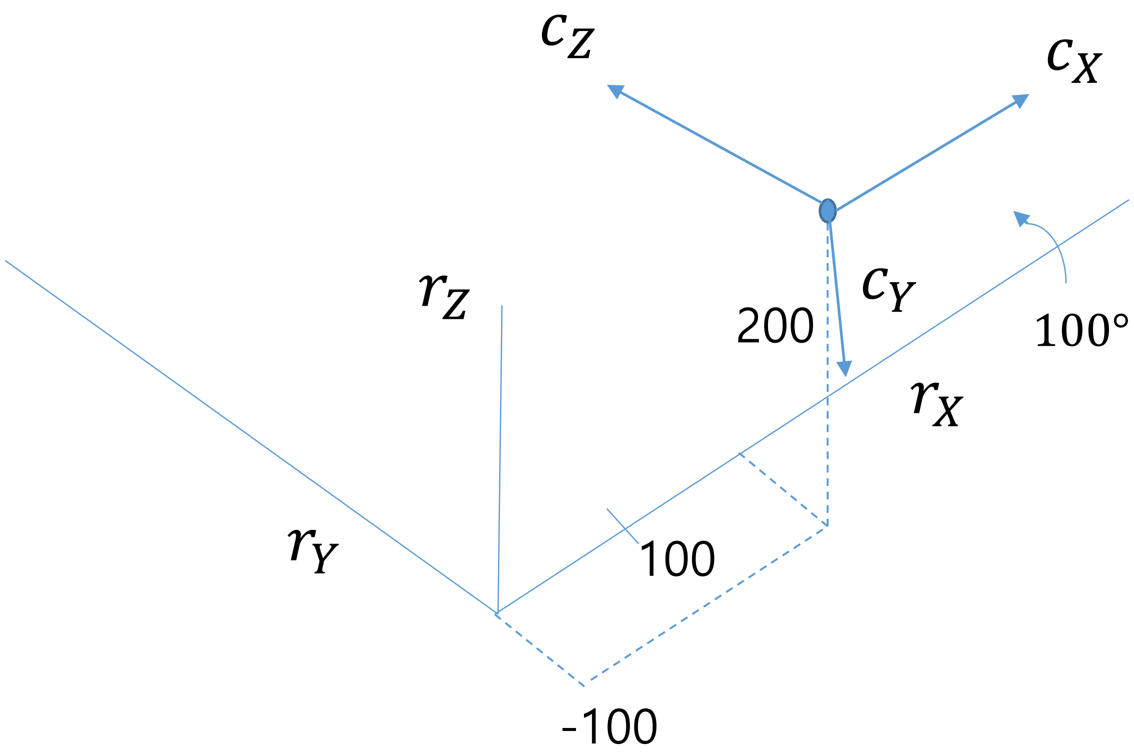


For object B = (100,500,200)

$$s\begin{pmatrix}u\\v\\1\end{pmatrix} = H\begin{pmatrix}100\\500\\200\end{pmatrix}$$

so

$$\begin{pmatrix}u\\v\\1\end{pmatrix} = \begin{pmatrix}194\\192\\1\end{pmatrix} \quad ,$$



8.6 Geometry measurement

Inverse projective mapping

Given (u, v) information, can we find the 3D information of object (r_X, r_Y, r_Z)?

$\boldsymbol{r_{X_3}}$

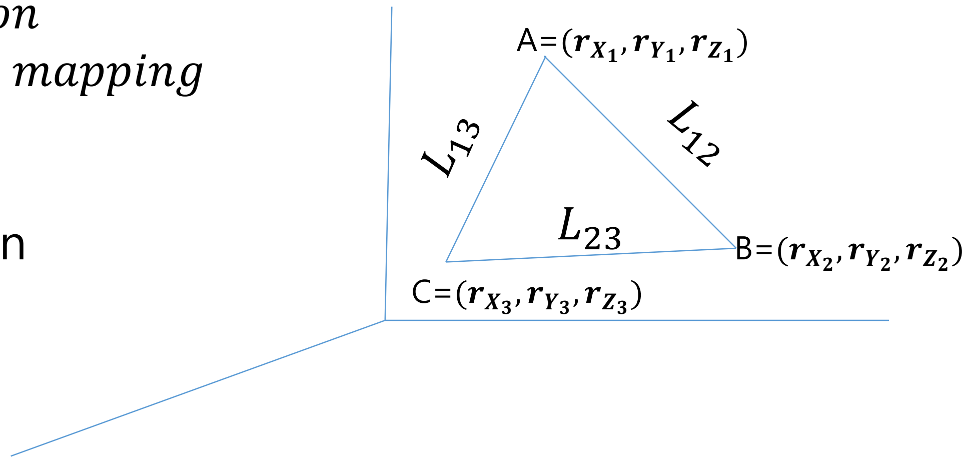
$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H\begin{pmatrix} r_X \\ r_Y \\ r_Z \\ 1 \end{pmatrix}$$

For inverse mapping 4 unknowns (s,r_X, r_Y, r_Z), 3 constrains => infinite solutions

Need to find unique solution from infinite solutions
1. Remove one unknown element, $\Rightarrow$ *2D monocular vision*
2. Add one more equation $\Rightarrow$ *model based inverse mapping*

a) model based inverse mapping

Assume 3 points (A,B,C) and relative distances are known



8.6 Geometry measurement

Inverse projective mapping

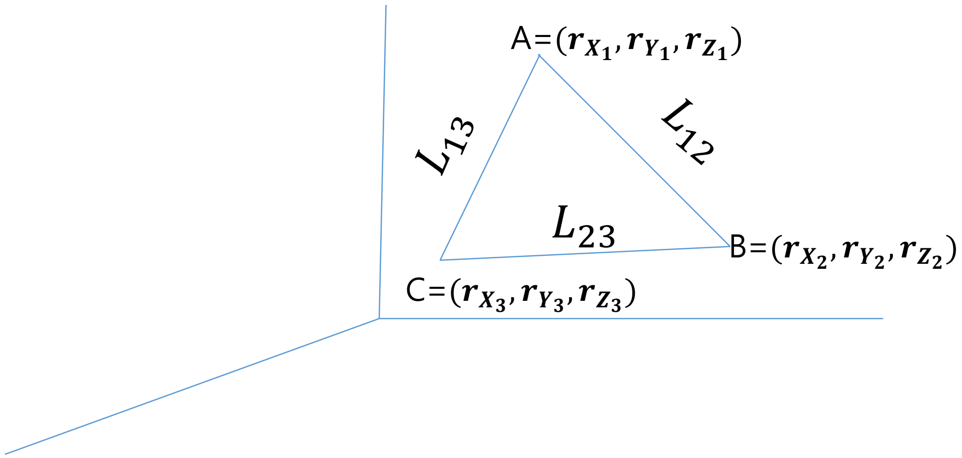
a) model based inverse mapping

Assume 3 points (A,B,C) and relative distances are known

$$S_1\begin{pmatrix}u_1\\v_1\\1\end{pmatrix}=H\begin{pmatrix}\boldsymbol{r}_{X_1}\\\boldsymbol{r}_{Y_1}\\\boldsymbol{r}_{Z_1}\\1\end{pmatrix},\qquad L_{12}=\sqrt{\left(\boldsymbol{r}_{X_1}-\boldsymbol{r}_{X_2}\right)^2+\left(\boldsymbol{r}_{Y_1}-\boldsymbol{r}_{Y_2}\right)^2+\left(\boldsymbol{r}_{Z_1}-\boldsymbol{r}_{Z_2}\right)^2}$$

$$S_2\begin{pmatrix}u_2\\v_2\\1\end{pmatrix}=H\begin{pmatrix}\boldsymbol{r}_{X_2}\\\boldsymbol{r}_{Y_2}\\\boldsymbol{r}_{Z_2}\\1\end{pmatrix},\qquad L_{23}=\sqrt{\left(\boldsymbol{r}_{X_2}-\boldsymbol{r}_{X_3}\right)^2+\left(\boldsymbol{r}_{Y_2}-\boldsymbol{r}_{Y_3}\right)^2+\left(\boldsymbol{r}_{Z_2}-\boldsymbol{r}_{Z_3}\right)^2}$$

$$S_3\begin{pmatrix}u_3\\v_3\\1\end{pmatrix}=H\begin{pmatrix}\boldsymbol{r}_{X_3}\\\boldsymbol{r}_{Y_3}\\\boldsymbol{r}_{Z_3}\\1\end{pmatrix},\qquad L_{31}=\sqrt{\left(\boldsymbol{r}_{X_3}-\boldsymbol{r}_{X_1}\right)^2+\left(\boldsymbol{r}_{Y_3}-\boldsymbol{r}_{Y_1}\right)^2+\left(\boldsymbol{r}_{Z_3}-\boldsymbol{r}_{Z_1}\right)^2}$$



L_{12}, L_{23}, L_{13} are known => By numerical method, solve the 3D information of each point.

8.6 Geometry measurement

Inverse projective mapping

b) 2-D Monocular vision

Suppose r_Z is set (known): on conveyor belt line

Let r_Z=0

Then

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H\begin{pmatrix} r_X \\ r_Y \\ 0 \\ 1 \end{pmatrix}$$

Calibration matrix H:

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & 1 \end{bmatrix}$$ by normalization

And

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & 1 \end{pmatrix}\begin{pmatrix} r_X \\ r_Y \\ 0 \\ 1 \end{pmatrix}$$

Then

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{14} \\ h_{21} & h_{22} & h_{24} \\ h_{31} & h_{32} & 1 \end{pmatrix}\begin{pmatrix} r_X \\ r_Y \\ 1 \end{pmatrix} \triangleq H'\begin{pmatrix} r_X \\ r_Y \\ 1 \end{pmatrix}$$

8.6 Geometry measurement
Inverse projective mapping

b) 2-D Monocular vision

$$\therefore \begin{pmatrix} r_X \\ r_Y \\ 1 \end{pmatrix} = s\,(H')^{-1}\begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Or

$$\rho \begin{pmatrix} r_X \\ r_Y \\ 1 \end{pmatrix} = D \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Where

$$D = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

$$\rho = d_{31}u + d_{32}v + d_{33}$$

Thus,

$$r_X = \frac{1}{\rho}(d_{11}u + d_{12}v + d_{13})$$

$$r_Y = \frac{1}{\rho}(d_{21}u + d_{22}v + d_{23})$$

8.6 Geometry measurement

Camera calibration

Finding camera's extrinsic and intrinsic parameters

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H\begin{pmatrix} r_X \\ r_Y \\ r_Z \\ 1 \end{pmatrix} = {}^{I}P_c\,{}^{c}M_r\begin{pmatrix} r_X \\ r_Y \\ r_Z \\ 1 \end{pmatrix}$$

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & 1 \end{pmatrix}\begin{pmatrix} r_X \\ r_Y \\ r_Z \\ 1 \end{pmatrix}$$

$$s = h_{31}r_X + h_{32}r_Y + h_{33}r_Z + 1$$

$$u = \frac{1}{s}(h_{11}r_X + h_{12}r_Y + h_{13}r_Z + h_{14}) = \frac{h_{11}r_X + h_{12}r_Y + h_{13}r_Z + h_{14}}{h_{31}r_X + h_{32}r_Y + h_{33}r_Z + 1}$$

$$v = \frac{1}{s}(h_{21}r_X + h_{22}r_Y + h_{23}r_Z + h_{24}) = \frac{h_{21}r_X + h_{22}r_Y + h_{23}r_Z + h_{24}}{h_{31}r_X + h_{32}r_Y + h_{33}r_Z + 1}$$

$$2\ equations\ , 11\ unknowns$$

8.6 Geometry measurement

Camera calibration

$$u = h_{11}r_X + h_{12}r_Y + h_{13}r_Z + h_{14} - uh_{31}r_X - uh_{32}r_Y - uh_{33}r_Z$$
$$v = h_{21}r_X + h_{22}r_Y + h_{23}r_Z + h_{24} - vh_{31}r_X - vh_{32}r_Y - vh_{33}r_Z$$

$$\begin{bmatrix} r_X & r_Y & r_Z & 1 & 0 & 0 & 0 & 0 & -u\,r_X & -u\,r_Y & -u\,r_Z \\ 0 & 0 & 0 & 0 & r_X & r_Y & r_Z & 1 & -v\,r_X & -v\,r_Y & -v\,r_Z \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{24} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\Rightarrow AV = B$$

2 constraints , 11 knowns

For unique solution for H, 11 constraints needed .

Least Square Method

$$\begin{pmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}=\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$A \quad X \quad B$

*pseudo inverse

$$X = pinv(A)B$$
$$= (A^T A)^{-1} A^T B$$

–Is this X really a model parameter that minimizes the sum of the residual squares?

$$\sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

➡ minimize $\|B - AX\|^2$ $\quad (B-AX)^T(B-AX)$

➡ Partial differentiation for X : $\dfrac{\partial \sum r_i^2}{\partial X}$

➡ $-2A^T(B-AX) = 0$ $\quad \therefore \; X = (A^T A)^{-1} A^T B$

8.6 Geometry measurement

Camera calibration

$$\begin{bmatrix} r_X & r_Y & r_Z & 1 & 0 & 0 & 0 & 0 & -u\,r_X & -u\,r_Y & -u\,r_Z \\ 0 & 0 & 0 & 0 & r_X & r_Y & r_Z & 1 & -v\,r_X & -v\,r_Y & -v\,r_Z \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{24} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$\Rightarrow \quad AV = B$

2 constraints , 11 knowns

For unique solution for H, 11 constraints needed .

For one pair of (u, v) , 2 constraints are made.

Thus, at least 6 pairs of (u, v) are needed.

For more than 6 pairs

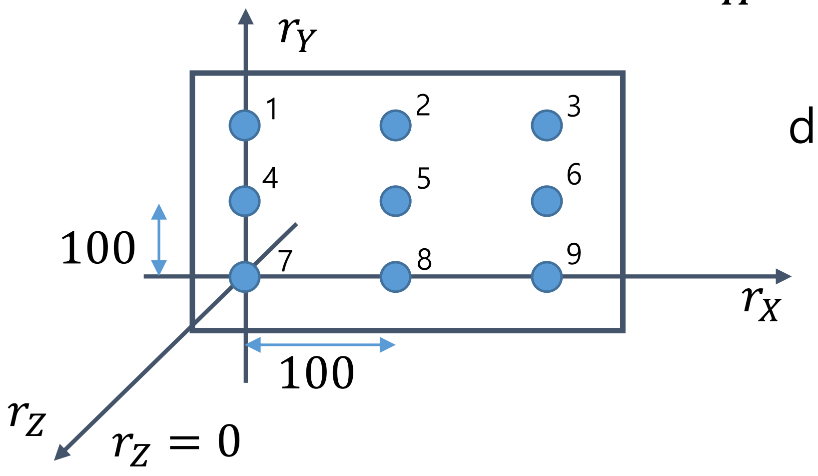
$$V = (A^T A)^{-1}(A^T B) \ or$$

$$V = A^T (AA^T)^{-1} B$$

8.6 Geometry measurement

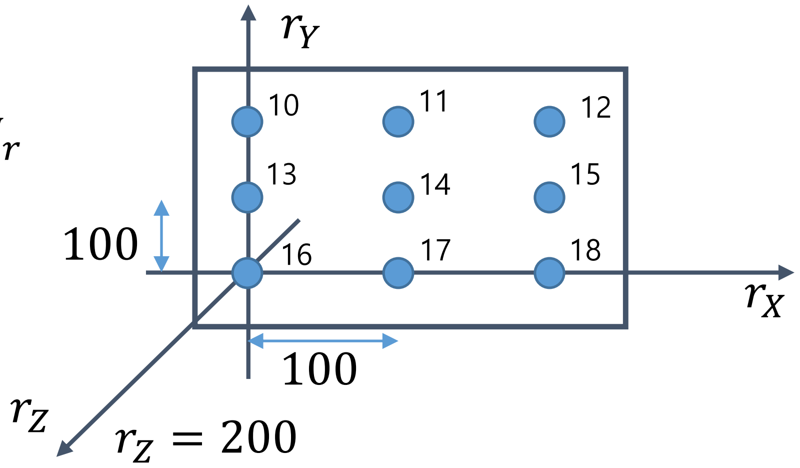
Camera calibration example

$H = {}^{I}P_{c}\,{}^{c}M_{r}$



different ${}^{c}M_{r}$

$H = {}^{I}P_{c}\,{}^{c}M_{r}$



$point$	u	v	r_X	r_Y	r_Z
1	116	38	0	200	0
2					
7					
10					
15	⋮				
18		⋮	⋮	⋮	⋮

8.6 Geometry measurement

Camera calibration example

$$H = {}^{I}P_c\,{}^{c}M_r$$

$point$	u	v	r_X	r_Y	r_Z
1	116	38	0	200	0
2					
7					
10					
15	⋮				
18		⋮	⋮	⋮	⋮

$$A = \begin{bmatrix} r_{X_i} & r_{Y_i} & r_{Z_i} & 1 & 0 & 0 & 0 & 0 & -u_i\,r_{X_i} & -u_i\,r_{Y_i} & -u_i\,r_{Z_i} \\ 0 & 0 & 0 & 0 & r_{X_i} & r_{X_i} & r_{X_i} & 1 & -v_i\,r_{X_i} & -v_i\,r_{Y_i} & -v_i\,r_{Z_i} \end{bmatrix}, \qquad i = 1,2,7,10,15,16$$

$$B = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

$$V = (A^T A)^{-1}(A^T B)\ or$$
$$V = A^T (A A^T)^{-1} B$$

$A = 12\text{x } 11$ matrix
$B = 12\text{x}1$
$V = 11\text{x}1$

$$V = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{24} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}$$

8.6 Geometry measurement

Determination of camera parameters given camera calibration matrix H

Given $H = {}^{I}P_c\, M_r,$ determine $({}^{I}P_c\, , {}^{c}M_r)$

$if\ you\ know\ H\ matrix\ via\ calibration, we\ can\ determine\ {}^{I}P_c\, , {}^{c}M_r$

$$H = {}^{I}P_c{}^{c}M_r = \begin{pmatrix} \frac{f_c}{D_x} & 0 & u_0 & 0 \\ 0 & \frac{f_c}{D_y} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \overrightarrow{R_1} & t_x \\ \overrightarrow{R_2} & t_y \\ \overrightarrow{R_3} & t_z \\ \vec{0} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} f_x & 0 & u_0 & 0 \\ 0 & f_y & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \overrightarrow{R_1} & t_x \\ \overrightarrow{R_2} & t_y \\ \overrightarrow{R_3} & t_z \\ \vec{0} & 1 \end{pmatrix}$$

$for\ \ rot(x, \theta)$
$\overrightarrow{R_1} = (1\ 0\ 0)$
$\overrightarrow{R_2} = (0 \quad c\theta \quad -s\theta)$
$\overrightarrow{R_3} = (0 \quad s\theta \quad c\theta)$

$\overrightarrow{R_1} \cdot \overrightarrow{R_2} = 0 \quad (\overrightarrow{R_1} \perp \overrightarrow{R_2})$

8.6 Geometry measurement

Determination of camera parameters given camera calibration matrix H

16 unknowns, 11 egns,

12 knowns in ${}^{c}M_{r}$,
+ 4 knowns in ${}^{I}P_{c}$

Characteristics on rotational matrix

$$\overrightarrow{R_1}\cdot\overrightarrow{R_2}=0\,,\quad \left|\overrightarrow{R_1}\right|=1$$
$$\overrightarrow{R_2}\cdot\overrightarrow{R_3}=0\,,\quad \left|\overrightarrow{R_2}\right|=1$$
$$\overrightarrow{R_3}\cdot\overrightarrow{R_1}=0\,,\quad \left|\overrightarrow{R_3}\right|=1$$

$$\overrightarrow{R_1}\cdot\overrightarrow{R_2}=0 \Rightarrow \quad (\overrightarrow{R_1}\perp\overrightarrow{R_2})$$

$$H={}^{I}P_{c}\,{}^{c}M_{r}=t_z\begin{pmatrix} f_x\dfrac{\overrightarrow{R_1}}{t_z}+u_0\dfrac{\overrightarrow{R_3}}{t_z} & f_x\dfrac{t_x}{t_z}+u_0 \\ f_y\dfrac{\overrightarrow{R_2}}{t_z}+v_0\dfrac{\overrightarrow{R_3}}{t_z} & f_y\dfrac{t_y}{t_z}+v_0 \\ \dfrac{\overrightarrow{R_3}}{t_z} & 1 \end{pmatrix}$$
$$=\begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & 1 \end{pmatrix}$$

8.6 Geometry measurement

Determination of camera parameters given camera calibration matrix H

1) $f_x \frac{\overrightarrow{R_1}}{t_z} + u_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{11} \quad h_{12} \quad h_{13})$

2) $f_y \frac{\overrightarrow{R_2}}{t_z} + v_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{21} \quad h_{22} \quad h_{23})$

3) $\frac{\overrightarrow{R_3}}{t_z} = (h_{31} \quad h_{32} \quad h_{33})$

4) $f_x \frac{t_x}{t_z} + u_0 = h_{14}$

5) $f_y \frac{t_y}{t_z} + v_0 = h_{24}$

8.6 Geometry measurement

Determination of camera parameters given camera calibration matrix H

i) Solution for t_z from 3)

$$3)\ \frac{\overrightarrow{R_3}}{t_z} = (h_{31} \quad h_{32} \quad h_{33}) \Rightarrow |\overrightarrow{R_3}| = |t_z(h_{31} \quad h_{32} \quad h_{33})| = 1$$

$$\Rightarrow \left\|\frac{\overrightarrow{R_3}}{t_z}\right\| = \sqrt{{h_{31}}^2 + {h_{32}}^2 + {h_{33}}^2} = \left\|\frac{\overrightarrow{R_3}}{t_z}\right\|$$

$$t_z = \frac{1}{\sqrt{{h_{31}}^2 + {h_{32}}^2 + {h_{33}}^2}}$$

ii) $\overrightarrow{R_3}$ from 3)

$$|\overrightarrow{R_3}| = t_z(h_{31} \quad h_{32} \quad h_{33})$$

$$1)\ f_x\frac{\overrightarrow{R_1}}{t_z} + u_0\frac{\overrightarrow{R_3}}{t_z} = (h_{11} \quad h_{12} \quad h_{13})$$

$$2)\ f_y\frac{\overrightarrow{R_2}}{t_z} + v_0\frac{\overrightarrow{R_3}}{t_z} = (h_{21} \quad h_{22} \quad h_{23})$$

$$3)\ \frac{\overrightarrow{R_3}}{t_z} = (h_{31} \quad h_{32} \quad h_{33})$$

$$4)\ f_x\frac{t_x}{t_z} + u_0 = h_{14}$$

$$5)\ f_y\frac{t_y}{t_z} + v_0 = h_{24}$$

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iii) u_0 from 1)

1) $\overrightarrow{R_3}^T\left(\frac{f_x \overrightarrow{R_1}}{t_z} + u_0 \frac{\overrightarrow{R_3}}{t_z}\right) = \overrightarrow{R_3}^T (h_{11} \quad h_{12} \quad h_{13})$

$$u_0 \frac{\overrightarrow{R_3}^T \overrightarrow{R_3}}{tz} = \overrightarrow{R_3}^T (h_{11} \quad h_{12} \quad h_{13})$$

$$u_0 = t_z \overrightarrow{R_3}^T (h_{11} \quad h_{12} \quad h_{13})$$

iv) v_0 from 2)

2) $f_y \frac{\overrightarrow{R_2}}{t_z} + v_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{21} \quad h_{22} \quad h_{23})$

$$v_0 = t_z \overrightarrow{R_3}^T (h_{21} \quad h_{22} \quad h_{23})$$

1) $f_x \frac{\overrightarrow{R_1}}{t_z} + u_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{11} \quad h_{12} \quad h_{13})$

2) $f_y \frac{\overrightarrow{R_2}}{t_z} + v_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{21} \quad h_{22} \quad h_{23})$

3) $\frac{\overrightarrow{R_3}}{t_z} = (h_{31} \quad h_{32} \quad h_{33})$

4) $f_x \frac{t_x}{t_z} + u_0 = h_{14}$

5) $f_y \frac{t_y}{t_z} + v_0 = h_{24}$

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v) f_x from 1)

$$1)\ f_x \frac{\overrightarrow{R_1}}{t_z} + u_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{11} \quad h_{12} \quad h_{13})$$

$$=> \ f_x \overrightarrow{R_1} = t_z (h_{11} \quad h_{12} \quad h_{13}) - u_0 \ \overrightarrow{R_3}$$

Thus

$$\left\| f_x \overrightarrow{R_1} \right\| = \left\| t_z (h_{11} \quad h_{12} \quad h_{13}) - u_0 \ \overrightarrow{R_3} \right\|$$

By

$$\left\| \overrightarrow{R_1} \right\| = 1, \quad => \quad f_x = \left\| t_z (h_{11} \quad h_{12} \quad h_{13}) - u_0 \ \overrightarrow{R_3} \right\|$$

vi) f_y from 2)

From $\left\| \overrightarrow{R_2} \right\| = 1$

$$f_y = \left\| t_z (h_{21} \quad h_{22} \quad h_{23}) - v_0 \ \overrightarrow{R_3} \right\|$$

$$1)\ f_x \frac{\overrightarrow{R_1}}{t_z} + u_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{11} \quad h_{12} \quad h_{13})$$

$$2)\ f_y \frac{\overrightarrow{R_2}}{t_z} + v_0 \frac{\overrightarrow{R_3}}{t_z} = (h_{21} \quad h_{22} \quad h_{23})$$

$$3)\ \frac{\overrightarrow{R_3}}{t_z} = (h_{31} \quad h_{32} \quad h_{33})$$

$$4)\ f_x \frac{t_x}{t_z} + u_0 = h_{14}$$

$$5)\ f_y \frac{t_y}{t_z} + v_0 = h_{24}$$

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vii) $\overrightarrow{R_1}$, $\overrightarrow{R_2}$ from 1) and 2)

$$\overrightarrow{R_1} = \frac{t_z}{f_x}(h_{11} \quad h_{12} \quad h_{13}) - \frac{u_0}{f_x}\overrightarrow{R_3}$$
$$\overrightarrow{R_2} = \frac{t_z}{f_y}(h_{21} \quad h_{22} \quad h_{23}) - \frac{v_0}{f_y}\overrightarrow{R_3}$$

viii) t_x, t_y from 4) and 5)

$$t_x = \frac{t_z}{f_x}(h_{14} - u_0)$$

$$t_y = \frac{t_z}{f_y}(h_{24} - v_0)$$