Visual sensory II

04/22

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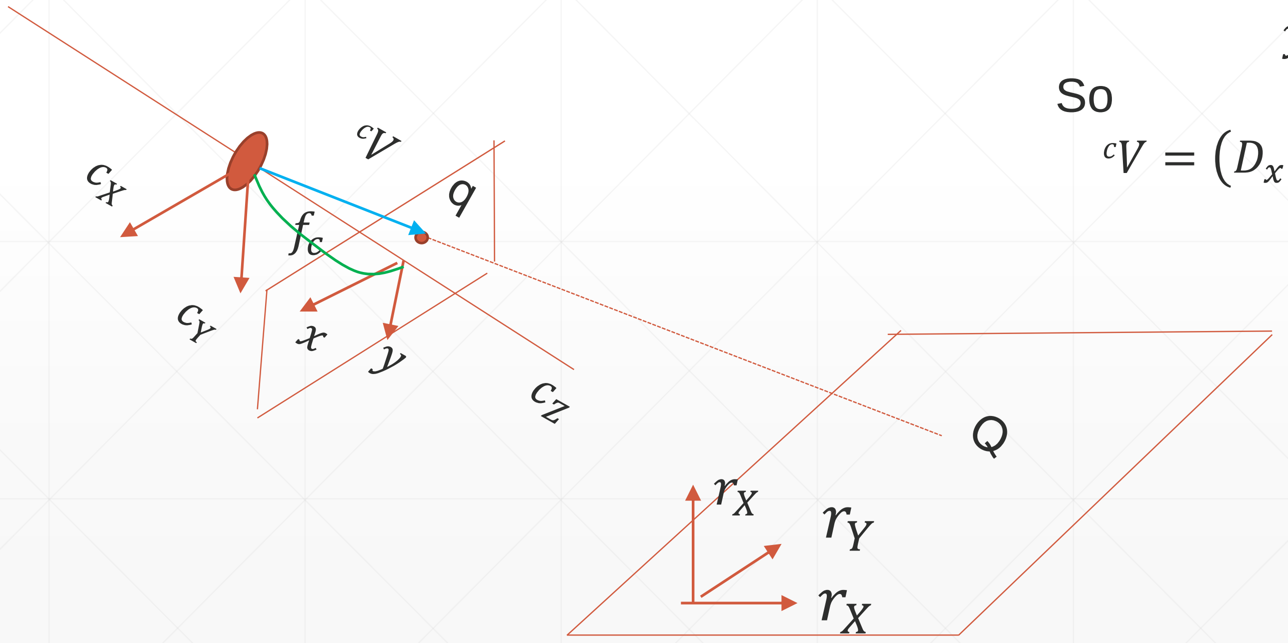
8.6 Geometry measurement

- Walking direction (vision guided walking)



For a projection q w.r.t camera frame

$$ {}^{c}V = (x, y, f_c) $$

At index coordinates

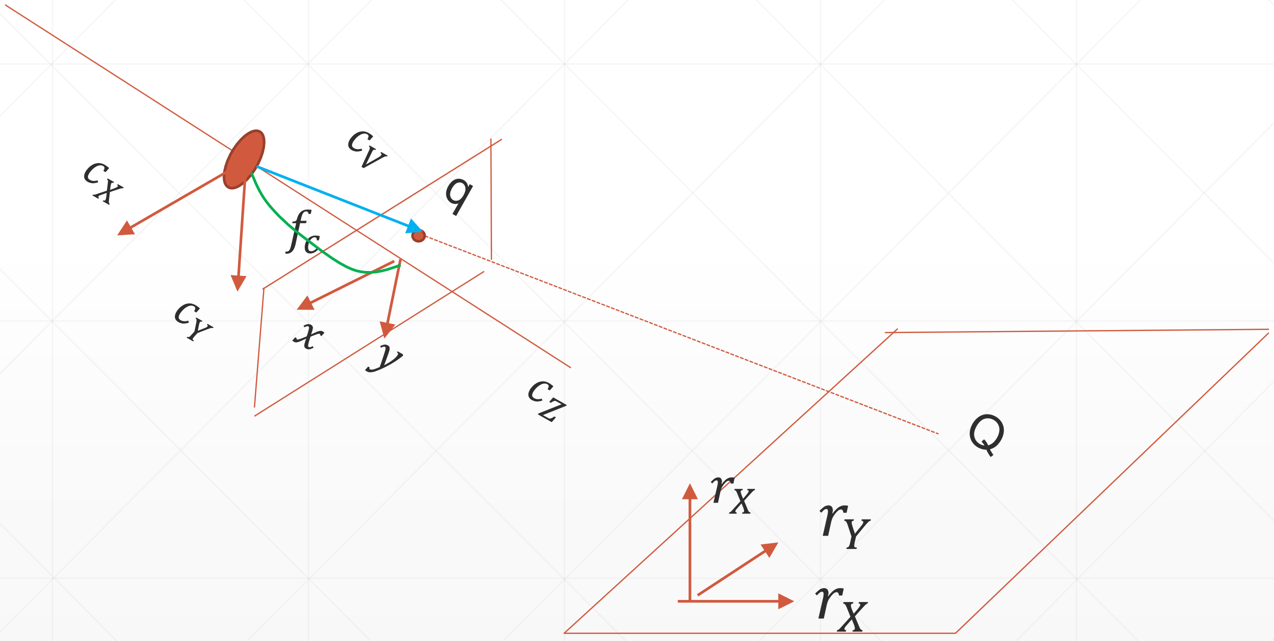
$$ x = D_x(u - u_0) $$
$$ y = D_y(v - v_0) $$

So

$$ {}^{c}V = \left(D_x(u - u_0), D_y(v - v_0), f_c\right) $$

8.6 Geometry measurement

- Walking direction (vision guided walking)



For a projection q w.r.t camera frame

$$^{c}V = (x, y, f_c)$$

At index coordinates

$$x = D_x(u - u_0)$$
$$y = D_y(v - v_0)$$

Normalizing by f_c

$$^{c}V = \left(\frac{D_x(u-u_0)}{f_c}, \frac{D_y(v-v_0)}{f_c}, 1\right)$$
$$= \left(\frac{(u-u_0)}{f_x}, \frac{(v-v_0)}{f_y}, 1\right)$$

Where

$$f_x = \frac{f_c}{D_x}, f_y = \frac{f_c}{D_y}$$

And also

$$^{r}V = {}^{r}M_c{}^{c}V = ({}^{c}M_r)^{-1}\,{}^{c}V$$

Binocular vision

$for\ a\ point\ \mathrm{A}$

$$s_1 \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

4 knowns, 3 constraints:
Need one more constraint to solve inverse projective-mapping problem

Way to solve:
If displacement vector is known

$$s_2 \begin{pmatrix} u + \Delta u \\ v + \Delta v \\ 1 \end{pmatrix} = H \begin{pmatrix} {}^{r}X + \Delta X \\ {}^{r}Y + \Delta Y \\ {}^{r}Z + \Delta Z \\ 1 \end{pmatrix}$$

5 knowns, 6 constraints
=> Point A can be uniquely determined
=> Motion stereo, dynamic monocular vision

Alternatives: Placing two cameras at two different locations
=> Binocular vision

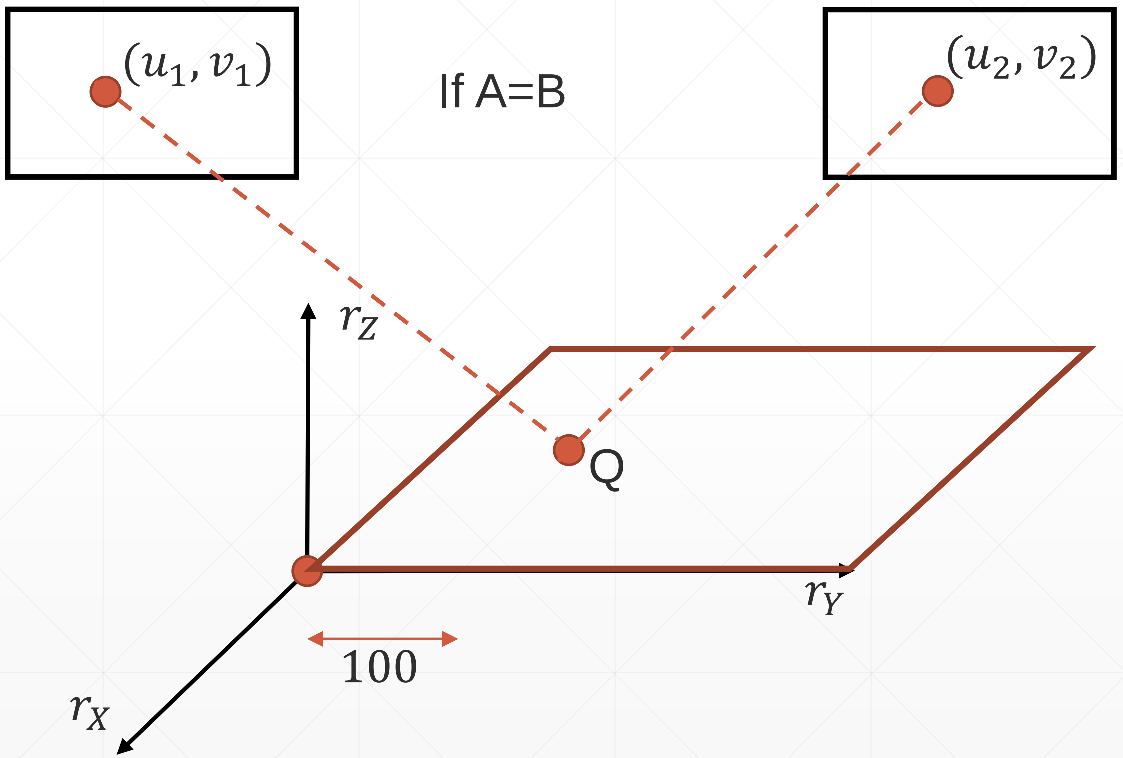
Binocular vision

for a point A

$$s_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = H_1 \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

for a point B

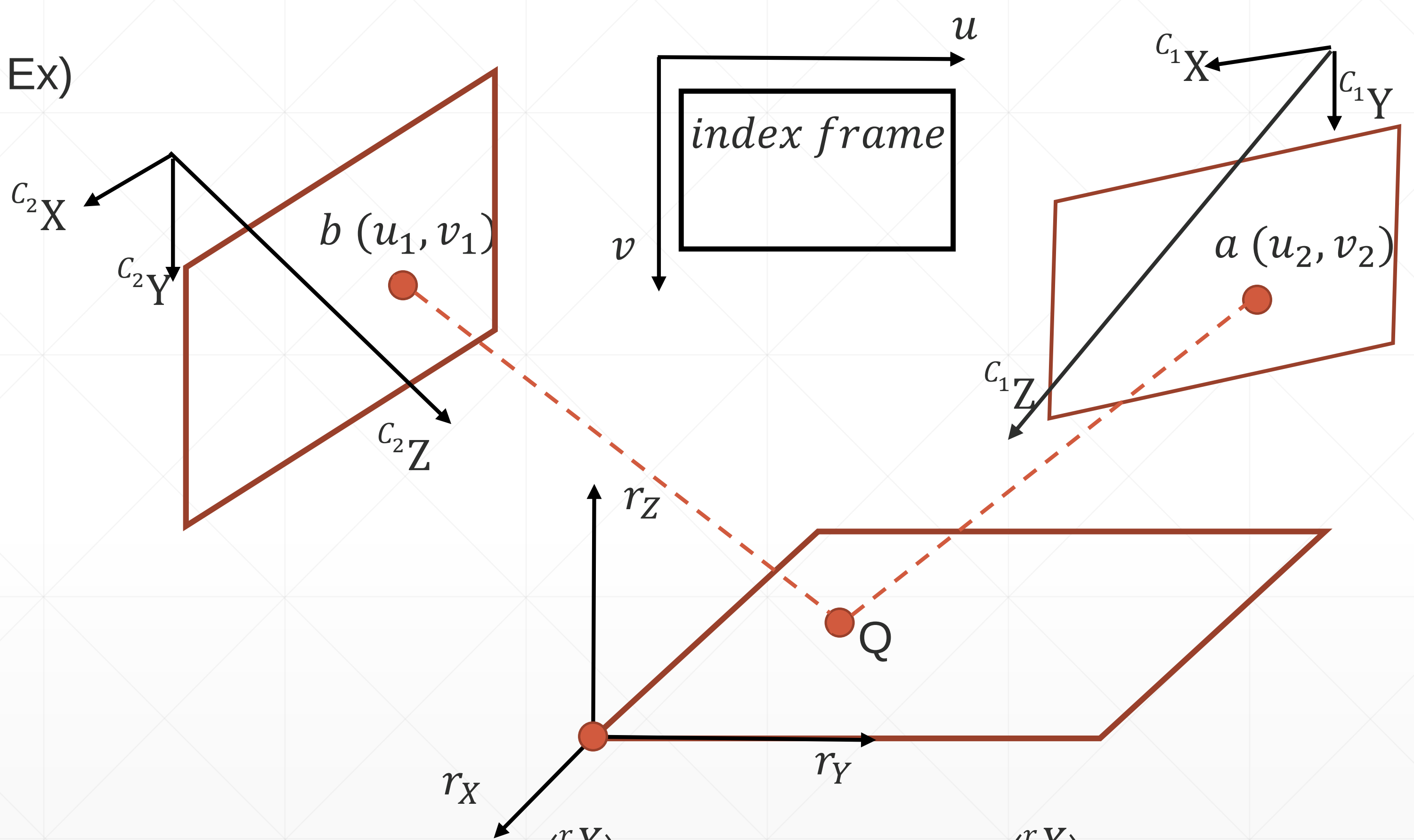
$$s_2 \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = H_2 \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$



Binocular vision

1) Forward projective-mapping

Ex)



$$s_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = \mathrm{H}_1 \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}, \quad s_2 \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = \mathrm{H}_2 \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

Binocular vision

(Forward projective-mapping)

$$s_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = \mathrm{H}_1 \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix} \quad , \quad s_2 \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = \mathrm{H}_2 \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

Given H_1, and H_2, if $\begin{pmatrix} {}^r\boldsymbol{X} \\ {}^r\boldsymbol{Y} \\ {}^r\boldsymbol{Z} \\ \boldsymbol{1} \end{pmatrix}$ is known then (u_1, v_1) and (u_2, v_2) can be determined.

Inverse projective mapping

$given \quad a = (u_1, v_1)\,, b = (u_1, v_1)$

$$determine\ Q = \begin{pmatrix} {}^r\boldsymbol{X} \\ {}^r\boldsymbol{Y} \\ {}^r\boldsymbol{Z} \\ \boldsymbol{1} \end{pmatrix}$$

Binocular vision

Let

$$H_1 = \begin{pmatrix} i_{11} & i_{12} & i_{13} & i_{14} \\ i_{21} & i_{22} & i_{23} & i_{24} \\ i_{31} & i_{32} & i_{33} & 1 \end{pmatrix} \quad , H_2 = \begin{pmatrix} j_{11} & j_{12} & j_{13} & j_{14} \\ j_{21} & j_{22} & j_{23} & j_{24} \\ j_{31} & j_{32} & j_{33} & 1 \end{pmatrix}$$

By $\mathrm{s}_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = \mathrm{H}_1 \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$

Thus

$$s_1 u_1 = i_{11}{}^rX + i_{12}{}^rY + i_{13}{}^rZ + i_{14}$$
$$s_1 v_1 = i_{21}{}^rX + i_{22}{}^rY + i_{23}{}^rZ + i_{24}$$
$$s_1 = i_{31}{}^rX + i_{32}{}^rY + i_{33}{}^rZ + 1$$

Binocular vision

Eliminating s_1

$$(i_{11} - i_{31}u_1)^r X + (i_{12} - i_{32}u_1)^r Y + (i_{13} - i_{33}u_1)^r Z = u_1 - i_{14}$$
$$(i_{21} - i_{31}v_1)^r X + (i_{22} - i_{32}v_1)^r Y + (i_{23} - i_{33}v_1)^r Z = v_1 - i_{24}$$

Also, for camera 2

$$(j_{11} - j_{31}u_2)^r X + (j_{12} - j_{32}u_2)^r Y + (j_{13} - j_{33}u_2)^r Z = u_2 - j_{14}$$
$$(j_{21} - j_{31}v_2)^r X + (j_{22} - j_{32}v_2)^r Y + (j_{23} - j_{33}v_2)^r Z = v_2 - j_{24}$$

Binocular vision

Let $A = \begin{pmatrix} i_{11} - i_{31}u_1 & i_{12} - i_{32}u_1 & i_{13} - i_{33}u_1 \\ i_{21} - i_{31}v_1 & i_{22} - i_{32}v_1 & i_{23} - i_{33}v_1 \\ j_{11} - j_{31}u_2 & j_{12} - j_{32}u_2 & j_{13} - j_{33}u_2 \\ j_{21} - j_{31}v_2 & j_{22} - j_{32}v_2 & j_{23} - j_{33}v_2 \end{pmatrix}$

Then

$$A\begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = \begin{pmatrix} u_1 - i_{14} \\ v_1 - i_{24} \\ u_2 - j_{14} \\ v_2 - j_{24} \end{pmatrix}$$

Therefore

$$X = \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = (A^T A)^{-1} A^T B \quad with \quad B = \begin{pmatrix} u_1 - i_{14} \\ v_1 - i_{24} \\ u_2 - j_{14} \\ v_2 - j_{24} \end{pmatrix}$$

Binocular vision

$E.g.$ *Given two projections,* a*, and* b

$$a = (300,200) = (u_1, v_1), \qquad b = (200,200) = (u_2, v_2)$$

And

$$H_1 = \begin{pmatrix} 2.74 & 1.89 & -0.33 & -18.45 \\ 0 & 1.410 & -3.03 & 748.91 \\ 0 & 0.007 & -0.0013 & 1 \end{pmatrix}, H_2 = \begin{pmatrix} 2.74 & 1.89 & -0.33 & -18.45 \\ 0 & 1.410 & -3.03 & 748.91 \\ 0 & 0.007 & -0.0013 & 1 \end{pmatrix}$$

Then

$$A = \begin{pmatrix} 2.74 & -0.325 & 0.057 \\ 0 & -0.062 & -2.79 \\ 2.74 & 0.414 & -0.073 \\ 0 & -0.062 & -2.776 \end{pmatrix}, \quad B = \begin{pmatrix} 318.4 \\ -548.9 \\ 438.0 \\ -548.91 \end{pmatrix}$$

$$X = \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = (A^T A)^{-1} A^T B = \begin{pmatrix} 135.2 \\ 195.8 \\ 193.3 \end{pmatrix}$$

Binocular vision

Issues for Binocular vision

a. Search for its corresponding point in other image
   => Verify all possible locations in other image

b. Different parameters for each camera must be calibrated in advance: intrinsic and extrinsic parameters

Continuous Epipolar line constraint

-Limiting the search for the correspondence to an interval along a line
-Reduction of search space

Let the index coordinates of two feature point be

$a = (u_1, v_1)$
$b = (u_2, v_2)$

Walking direction in camera 1:
Projection line passing through point a:

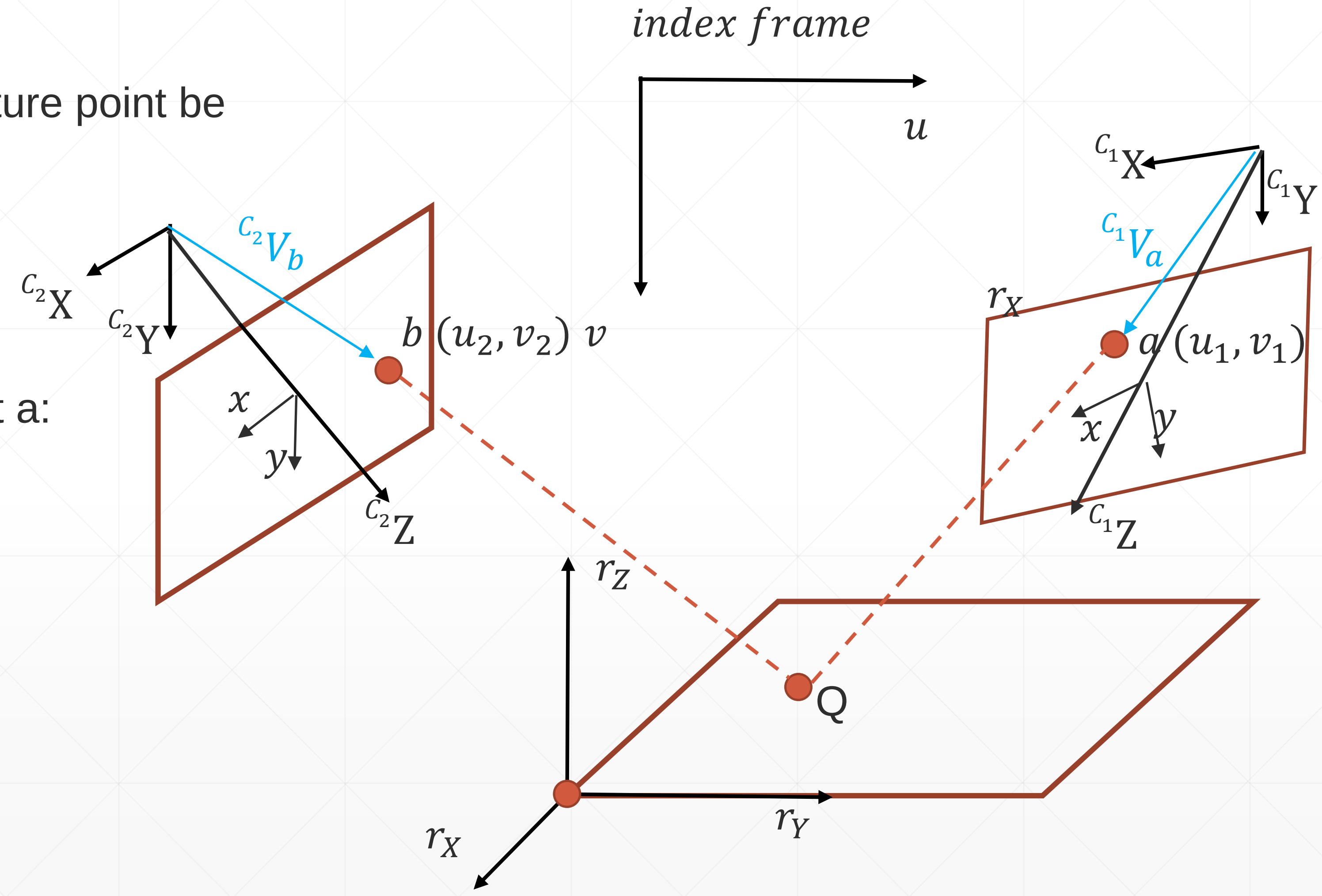
$$ {}^{C_1}V_a = \left(\frac{u_1 - u_{1,0}}{f_{1,x}}, \frac{v_1 - v_{1,0}}{f_{1,y}}, 1 \right) $$

Where

$$ f_{1,x} = \frac{f_{C1}}{D_{1,x}} \quad f_{1,y} = \frac{f_{C1}}{D_{1,y}} $$

Or

$$ {}^{C_1}V_a = \begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{f_{1,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} $$



8.6 Geometry measurement

Continuous Epipolar line constraint

Walking direction in camera 2: Projection line passing through point b:

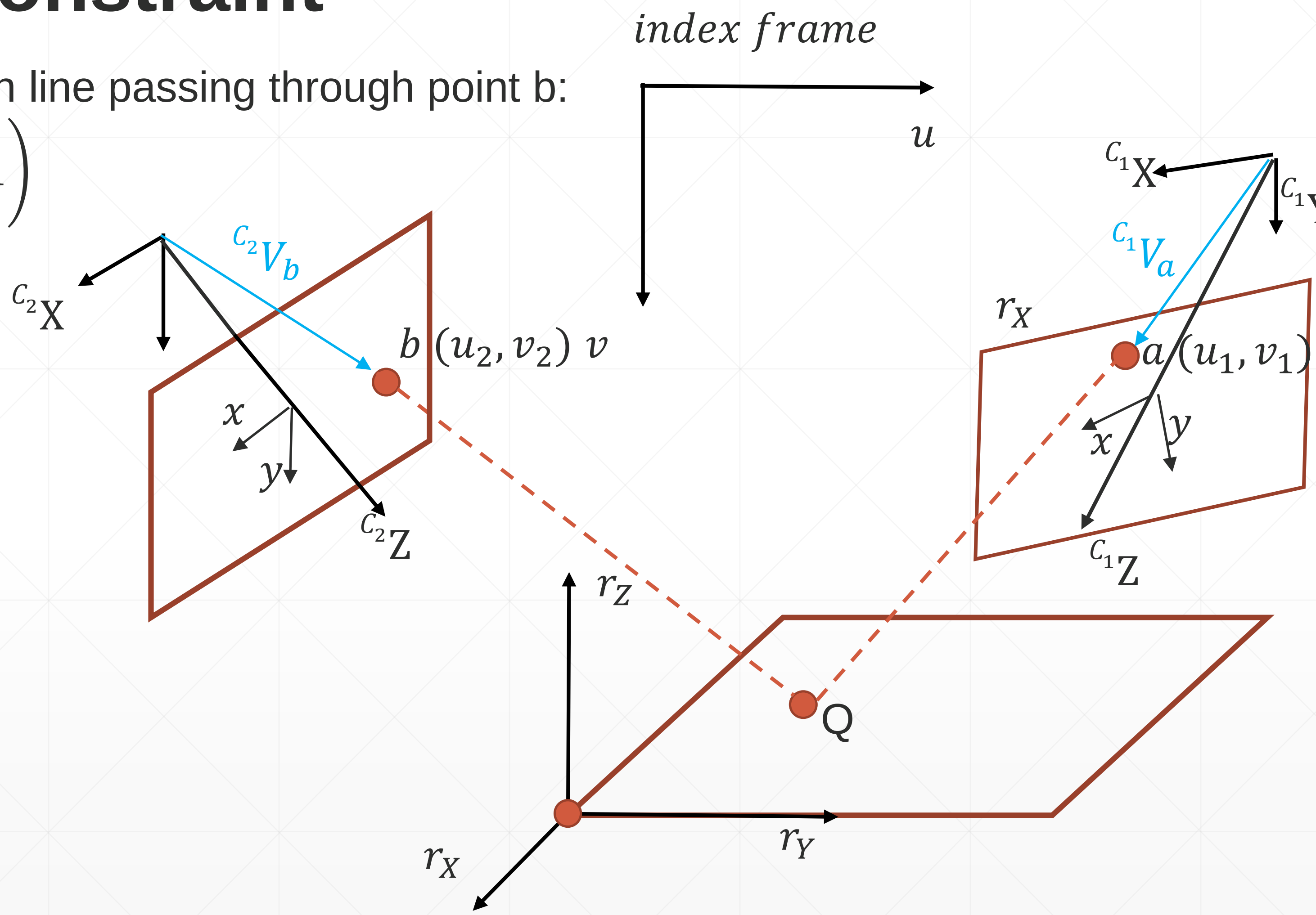
$$^{C_2}V_b = \left(\frac{u_2 - u_{2,0}}{f_{2,x}}, \frac{v_2 - v_{2,0}}{f_{2,y}}, 1 \right)$$

Where

$$f_{2,x} = \frac{f_{c2}}{D_{2,x}} \quad f_{2,y} = \frac{f_{c2}}{D_{2,y}}$$

Or

$$^{C_2}V_b = \begin{pmatrix} \frac{1}{f_{2,x}} & 0 & -\frac{u_{2,0}}{f_{2,x}} \\ 0 & \frac{1}{f_{2,y}} & -\frac{v_{2,0}}{f_{2,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix}$$



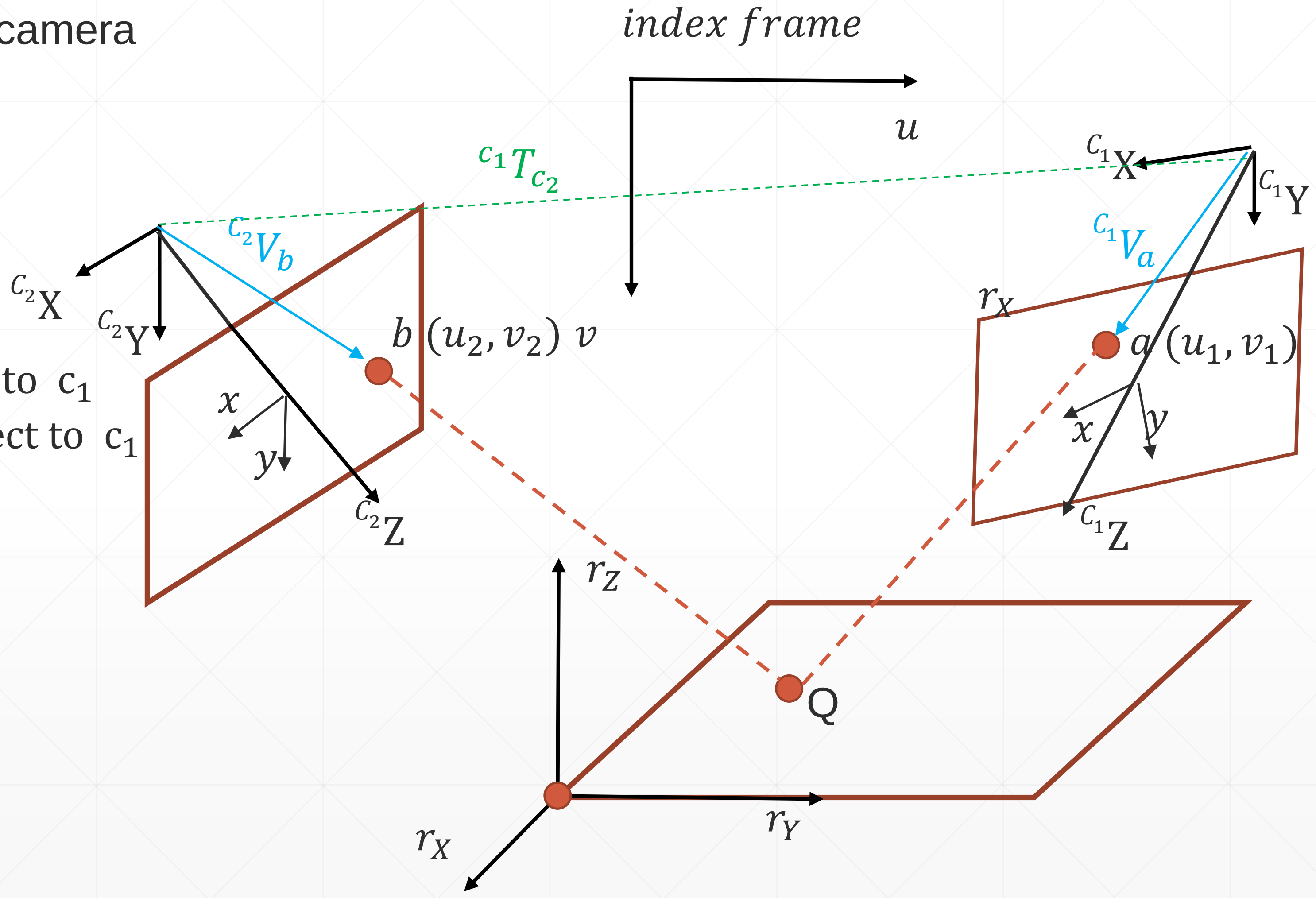
Continuous Epipolar line constraint

If a, b is projection of Q, ${}^{C_1}V_a$ and ${}^{C_2}V_b$ $will\ intersect.$
Transformation between left camera and right camera

$$ {}^{c_1}M_{c_2} = \begin{bmatrix} {}^{c_1}R_{c_2} & {}^{c_1}T_{c_2} \\ 000 & 1 \end{bmatrix} $$

Where
${}^{c_1}R_{c_2}$: rotation of camera frame c_2 with respect to c_1
${}^{c_1}T_{c_2}$: translation of camera c_2 origin with respect to c_1

$$ \begin{aligned} {}^{c_1}V_b &= {}^{c_1}T_{c_2} + {}^{c_1}R_{c_2}\,{}^{c_2}V_b \\ &\triangleq {}^{c_1}T_{c_2} + {}^{c_1}\hat{V}_b \end{aligned} $$



Continuous Epipolar line constraint

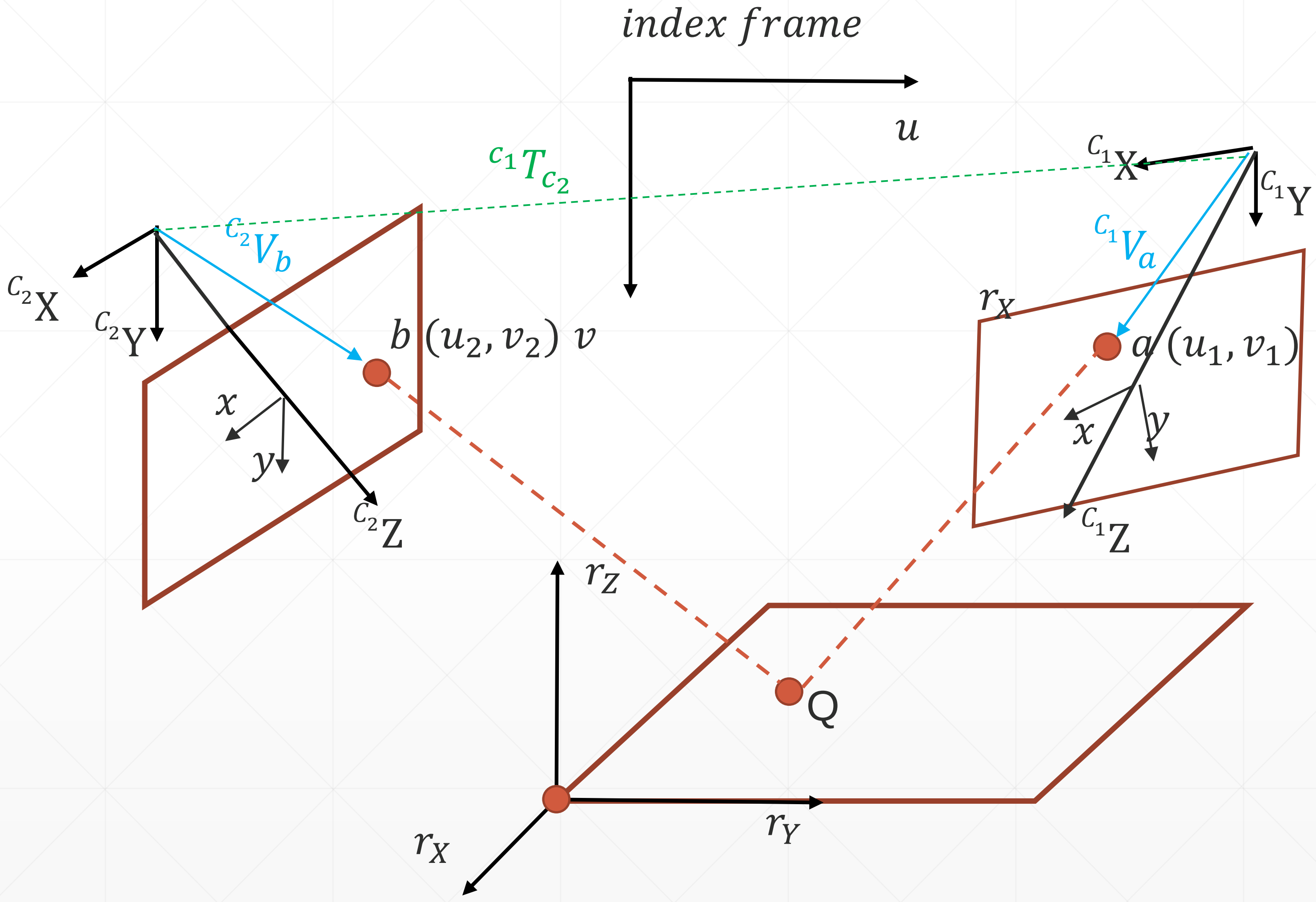
$if\ ,\ {}^{c_1}T_{c_2} = 0\ (ideal\ case)$

a and b will be superimposed
So

$${}^{c_1}V_a = {}^{c_1}V_b = {}^{c_1}R_{c_2}\,{}^{c_2}V_b$$

Therefore

$${}^{c_1}V_a = \begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{f_{1,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}$$



Continuous Epipolar line constraint

$$ {}^{c_1}V_a = {}^{c_1}V_b = {}^{c_1}R_{c_2}\, {}^{c_2}V_b $$

Therefore

$$ {}^{c_1}V_a = \begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{f_{1,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = {}^{c_1}R_{c_2} \begin{pmatrix} \frac{1}{f_{2,x}} & 0 & -\frac{u_{2,0}}{f_{2,x}} \\ 0 & \frac{1}{f_{2,y}} & -\frac{v_{2,0}}{f_{2,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} $$

Finally

$$ \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = M_{3\times 3} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} $$

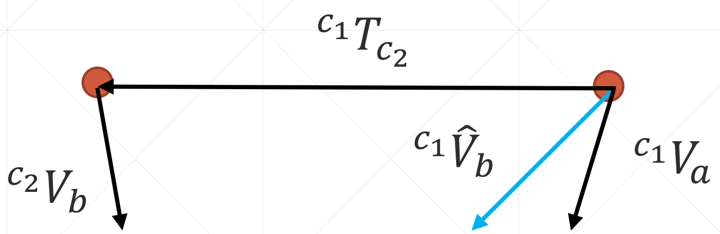
Where

$$ M_{3\times 3} = \begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{f_{1,y}} \\ 0 & 0 & 1 \end{pmatrix}^{-1} {}^{c_1}R_{c_2} \begin{pmatrix} \frac{1}{f_{2,x}} & 0 & -\frac{u_{2,0}}{f_{2,x}} \\ 0 & \frac{1}{f_{2,y}} & -\frac{v_{2,0}}{f_{2,y}} \\ 0 & 0 & 1 \end{pmatrix} $$

Continuous Epipolar line constraint

$practically$

$$^{c_1}T_{c_2} \neq 0$$



$if\ ^{c_1}V_a,\ ^{c_1}\hat{V}_b,\ ^{c_1}T_{c_2}\ are\ coplanar$

$\Rightarrow have\ intersection\ between\,(\,^{c_1}V_a,\ ^{c_2}V_b\,)$

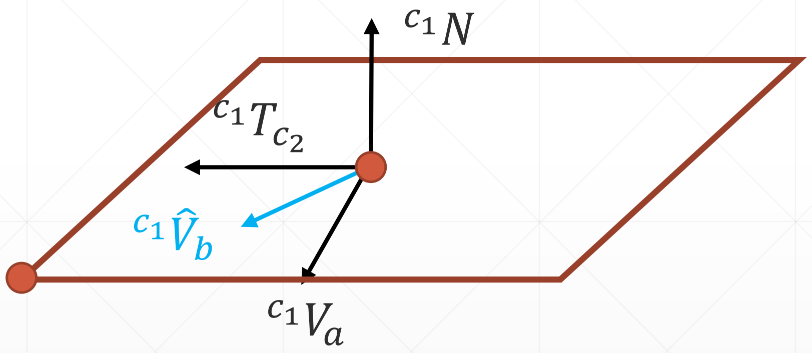
normal vector of plane N

$^{c_1}N = \ ^{c_1}T_{c_2} \times \ ^{c_1}V_a$

$^{c_1}N \perp \ ^{c_1}\hat{V}_b$

$\Rightarrow \ (^{c_1}N)^T\ ^{c_1}\hat{V}_b = 0$

$\left(\ ^{c_1}T_{c_2} \times \ ^{c_1}V_a\right)^T\ ^{c_1}\hat{V}_b = 0$



$let \quad ^{c_1}T_{c_2} = \left(t_x\ t_y\ t_z\right)^T$

Continuous Epipolar line constraint

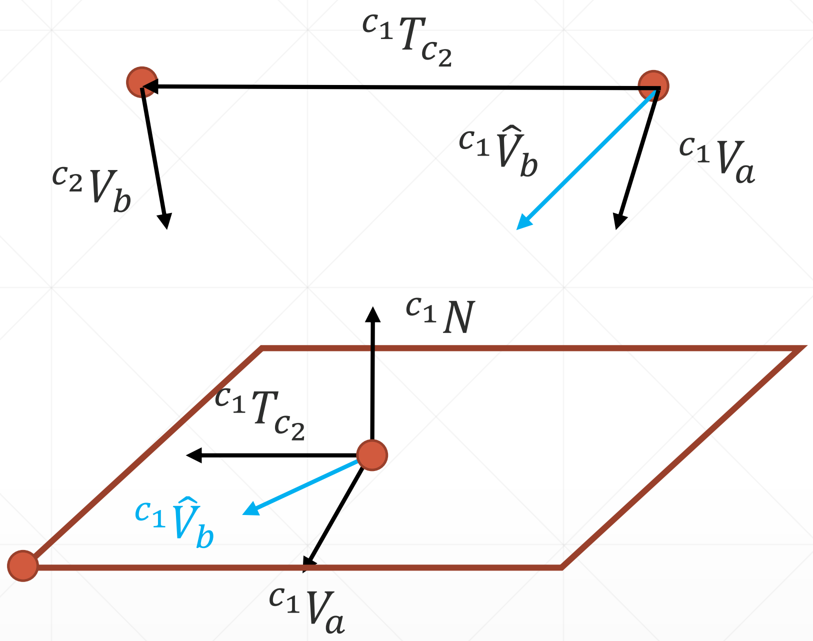
$By\, skew\ symmetric\ property$

$$S_T = S\left({}^{c_1}T_{c_2}\right) = \begin{pmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{pmatrix}$$

$$\left({}^{c_1}T_{c_2} \times {}^{c_1}V_a\right)^T {}^{c_1}\hat{V}_b = 0 => \left(S\left({}^{c_1}T_{c_2}\right) {}^{c_1}V_a\right)^T {}^{c_1}\hat{V}_b = 0$$

$hence,$ $({}^{c_1}V_a)^T {S_T}^T\, {}^{c_1}\hat{V}_b = 0$

$$\boxed{\begin{array}{c} skew\ symmetric\ property \\ a \times p = S(a)p \end{array}}$$

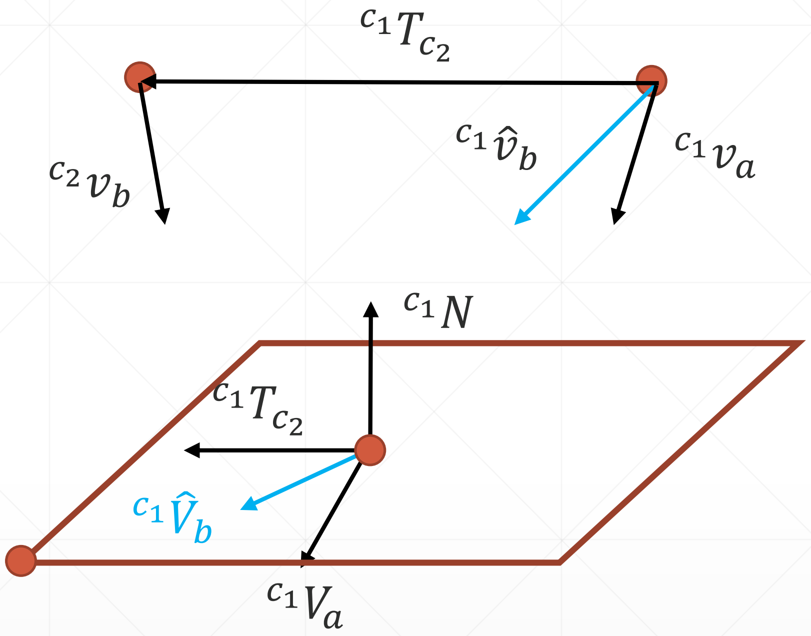


Continuous Epipolar line constraint

$$({}^{c_1}V_a)^T {S_T}^T\, {}^{c_1}\hat{V}_b = 0$$

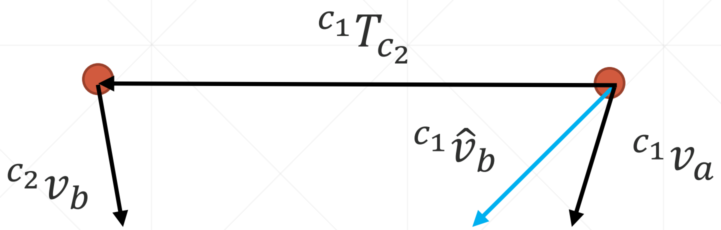
$$\left[\begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{f_{1,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}\right]^T (S_T)^T\, {}^{c_1}R_{c_2}\, {}^{c_2}V_b = 0$$

$$\boxed{\begin{pmatrix} \frac{1}{f_{2,x}} & 0 & -\frac{u_{2,o}}{f_{1,x}} \\ 0 & \frac{1}{f_{2,y}} & -\frac{v_{2,0}}{f_{2,y}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix}}$$



Continuous Epipolar line constraint

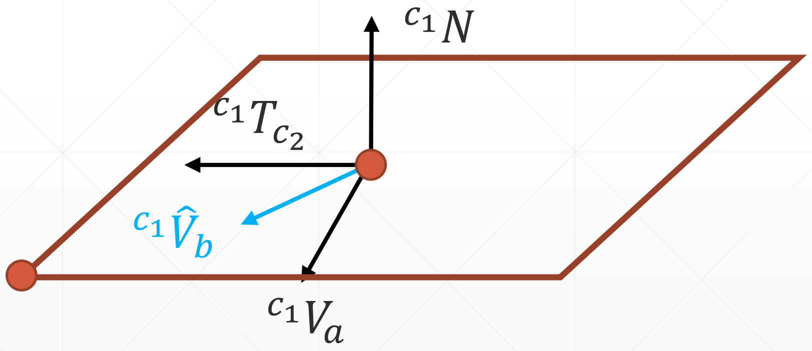
$$\begin{pmatrix} u_1 & v_1 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{f_{1,y}} \\ 0 & 0 & 1 \end{pmatrix}^T (S_T)^T \, {}^{c_1}R_{c_2} \begin{pmatrix} \frac{1}{f_{2,x}} & 0 & -\frac{u_{2,o}}{f_{2,x}} \\ 0 & \frac{1}{f_{2,y}} & -\frac{v_{2,0}}{f_{2,y}} \\ 0 & 0 & 1 \end{pmatrix}} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = 0$$



$F_{3\times3}$: Fundamental matrix

$$\begin{pmatrix} u_1 & v_1 & 1 \end{pmatrix} F_{3\times3} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = 0 \qquad \Rightarrow a\,u_2 + b\,v_2 + c = 0$$

Epipolar line constraint



Continuous Epipolar line constraint

$E.g.$

$$left\ cam: \left(f_{1,x}, f_{1,y}, u_{1,0}, v_{1,0}\right) = (365.6\,, 365.6\,, 256\,, 256\,)$$

$$^{c_1}M_r = \begin{pmatrix} 0.9848 & -0.17 & 0.03 & -121.61 \\ 0 & -0.173 & -0.984 & 179.59 \\ 0.173 & 0.969 & -0.171 & 113.82 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$right\ cam: \left(f_{2,x}, f_{2,y}, u_{2,0}, v_{2,0}\right) = (365.6\,, 365.6\,, 256\,, 256\,)$$

$$^{c_2}M_r = \begin{pmatrix} 0.9848 & 0.17 & -0.03 & -153.8 \\ 0 & -0.173 & -0.984 & 189.44 \\ -0.173 & 0.969 & -0.171 & 164.15 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Continuous Epipolar line constraint

Eg

$$ {}^{c_1}M_{c_2} = {}^{c_1}M_r \; {}^{r}M_{c_2} = {}^{c_1}M_r ({}^{c_2}M_r)^{-1} = \left(\begin{array}{c|c} {}^{c_1}R_{c_2} & {}^{c_1}T_{c_2} \\ \hline 000 & 1 \end{array}\right) $$

$$ F_{3\times 3} = \begin{pmatrix} \frac{1}{f_{1,x}} & 0 & -\frac{u_{1,0}}{f_{1,x}} \\ 0 & \frac{1}{f_{1,y}} & -\frac{v_{1,0}}{-f_{1,y}} \\ 0 \quad 0 & & 1 \end{pmatrix}^T (S_T)^T \, {}^{c_1}R_{c_2} \begin{pmatrix} \frac{1}{f_{2,x}} & 0 & -\frac{u_{2,o}}{f_{1,x}} \\ 0 & \frac{1}{f_{2,y}} & -\frac{v_{2,0}}{-f_{2,y}} \\ 0 \quad 0 & & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0.0001 & -0.0045 \\ 0.0001 & 0 & 0.1848 \\ -0.061 & -0.239 & 19.06 \end{pmatrix} $$

$$ (u_1 \quad v_1 \quad 1)\, F_{3\times 3} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = 0 $$

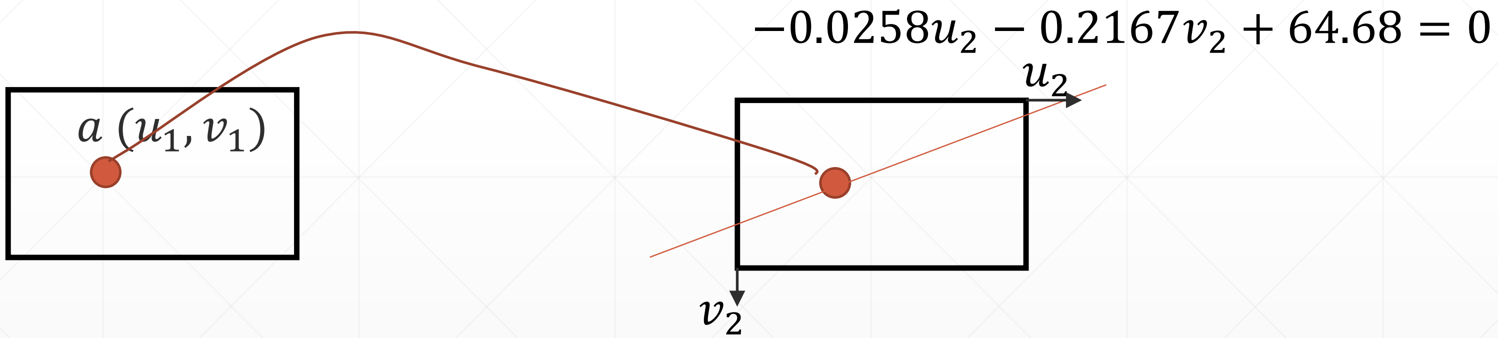
$$ if\,(u_1, v_1) = (252{,}253) \Rightarrow \; -0.0258u_2 - 0.2167v_2 + 64.68 = 0 $$

Continuous Epipolar line constraint

Eg

$$\begin{pmatrix} u_1 & v_1 & 1 \end{pmatrix} F_{3\times 3} \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = 0$$

$$if\,(u_1, v_1) = (252{,}253) \Rightarrow \; -0.0258u_2 - 0.2167v_2 + 64.68 = 0$$



Correspondence problems:

1. Issues
   i) Select candidate match
   ii) Determine the goodness of the match

2. Correspondence algorithm
   i) Correlation based
      -By matching image intensity over a window of pixels
   ii) Feature based
      -By matching a sparse sets of image features such as edges, lines, etc

3. Correlation based algorithm
   Inputs:
   a) Left camera image intensity: I_l
      Right camera image intensity: I_r
   b) Width of sub window : $2w + 1$
   c) Search region in the right image $R(P_l)$ associated with a pixel p_l in left image

Correspondence problems:

For each pixel $P_l(i,j)$ in the left image

a) for each displacement $\mathrm{d} = (d_1, d_2) \in R(P_l)$

compute cross-correlation $\mathrm{C}(\mathrm{d})$

$$\mathrm{C}(\mathrm{d}) = \sum_{k=-w}^{w} \sum_{l=-w}^{w} I_l\,(i+k, j+l)\, I_r(i+k-d_1, j+l-d_2)$$

b) select $\bar{d} = \left(\overline{d_1}, \overline{d_2}\right)$ that maximizes $\mathrm{C}(\mathrm{d})$ over $R(P_l)$

Other forms of $\mathrm{C}(\mathrm{d})$

$$\mathrm{C}(\mathrm{d}) = \sum_{k=-w}^{w} \sum_{l=-w}^{w} [I_l(i+k, j+l) - I_r(i+k-d_1, j+l-d_2)]^2$$

or

$$\mathrm{C}(\mathrm{d}) = \sum_{k=-w}^{w} \sum_{l=-w}^{w} |I_l(i+k, j+l) - I_r(i+k-d_1, j+l-d_2)|$$

4. Feature based method

a) select for a feature in an image that matches a feature in the other image

b) Typical features

-edge points

-line segments

-corners

Correspondence problems:

4. Feature based method
 e.g. Line feature descriptor
   -length l
   -orientation θ
   -mid point m
   -average intensity along line i

Similarity function:

$$S = \frac{1}{w_0\,(l_l - l_r)^2 + w_1(\theta_l - \theta_r)^2 + w_2(m_l - m_r)^2 + w_3(i_l - i_r)^2}$$

w_0, w_1, w_2, w_3: weighting factors
Select maximum value of S to find a feature.

Discrete Epiploar line constraint

-one coordinate varies within a fixed range

Assume Z from the object varies within $[Z_{min},\ Z_{max}]$

$Z_i = Z_{min} + i\ \Delta Z,\ \ i \in [0, n]$

ΔZ: accuracy

By Binocular vision

$$s_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = H_1 \begin{pmatrix} {}^rX \\ {}^rY \\ Z_i = Z_{min} + i\ \Delta Z \\ 1 \end{pmatrix}, \qquad i = 0,1,2, \qquad \ldots n$$

Also

$$s_2 \begin{pmatrix} u_2 \\ v_2 \\ 1 \end{pmatrix} = H_2 \begin{pmatrix} {}^rX \\ {}^rY \\ Z_i = Z_{min} + i\ \Delta Z \\ 1 \end{pmatrix}, \qquad i = 0,1,2, \qquad \ldots n$$

Procedures

Step1: Specify the range and accuracy of one coordinate

Step 2: Setup binocular vision equation

Discrete Epiploar line constraint

Procedures

Step 3: From

$$s_1 \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = H_1 \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ Z_i = Z_{min} + i\,\Delta Z \\ 1 \end{pmatrix}, \qquad i = 0,1,2, \qquad \ldots n$$

Compute the predicted coordinates $({}^{r}X_i\ , {}^{r}Y_i, {}^{r}Z_i)$ by $(u_1\ \ v_1)$ and given ${}^{r}Z_i$

Step 4: Given $({}^{r}X_i\ , {}^{r}Y_i, {}^{r}Z_i)$, compute $(u_{2i}\ \ v_{2i})$ from

$$s_2 \begin{pmatrix} u_{2i} \\ v_{2i} \\ 1 \end{pmatrix} = H_2 \begin{pmatrix} {}^{r}X_i \\ {}^{r}Y_i \\ {}^{r}Z_i \\ 1 \end{pmatrix}, \qquad i = 0,1,2, \qquad \ldots n$$

Step 5: Compute the intensity difference between at $(u_1\ \ v_1)$ and $(u_{2i}\ \ v_{2i})$

$\Delta I_i = I(u_1\ \ v_1) - I(u_{2i}\ \ v_{2i})$: Dissimilarity

Step 6: Repeat steps 3,4,5 for all i

Step 7: Choose the location $(u_{2j}\ \ v_{2j})$ where the dissimilarity is minimum

$$\left[(u_{2j}\ \ v_{2j}) |\ \min_i \Delta I_i\right]$$

Discrete Epiploar line constraint

$$\left[(u_{2j} \;\; v_{2j}) \mid \min_i \Delta I_i\right]$$

$(u_{2j} \;\; v_{2j})$ is the correspondence

$(u_{2j} \;\; v_{2j})$ exists on an Epipolar line

$\Rightarrow$ $({}^{r}X_j \;, {}^{r}Y_j, {}^{r}Z_j)$ is the 3D geometry of the binocular vision's inverse projective-mapping

$\Rightarrow$ No need to use $X=\begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \end{pmatrix} = (A^T A)^{-1} A^T B$

$$A = \begin{pmatrix} i_{11} - i_{31}u_1 & i_{12} - i_{32}u_1 & i_{13} - i_{33}u_1 \\ i_{21} - i_{31}v_1 & i_{22} - i_{32}v_1 & i_{23} - i_{33}v_1 \\ j_{11} - j_{31}u_2 & j_{12} - j_{32}u_2 & j_{13} - j_{33}u_2 \\ j_{21} - j_{31}v_2 & j_{22} - j_{32}v_2 & j_{23} - j_{33}v_2 \end{pmatrix}, \; B=\begin{pmatrix} u_1 - i_{14} \\ v_1 - i_{24} \\ u_2 - j_{14} \\ v_2 - j_{24} \end{pmatrix}$$

Calibration matrices

$$H_1 = \begin{pmatrix} i_{11} & i_{12} & i_{13} & i_{14} \\ i_{21} & i_{22} & i_{23} & i_{24} \\ i_{31} & i_{32} & i_{33} & 1 \end{pmatrix} \;, H_2 = \begin{pmatrix} j_{11} & j_{12} & j_{13} & j_{14} \\ j_{21} & j_{22} & j_{23} & j_{24} \\ j_{31} & j_{32} & j_{33} & 1 \end{pmatrix}$$