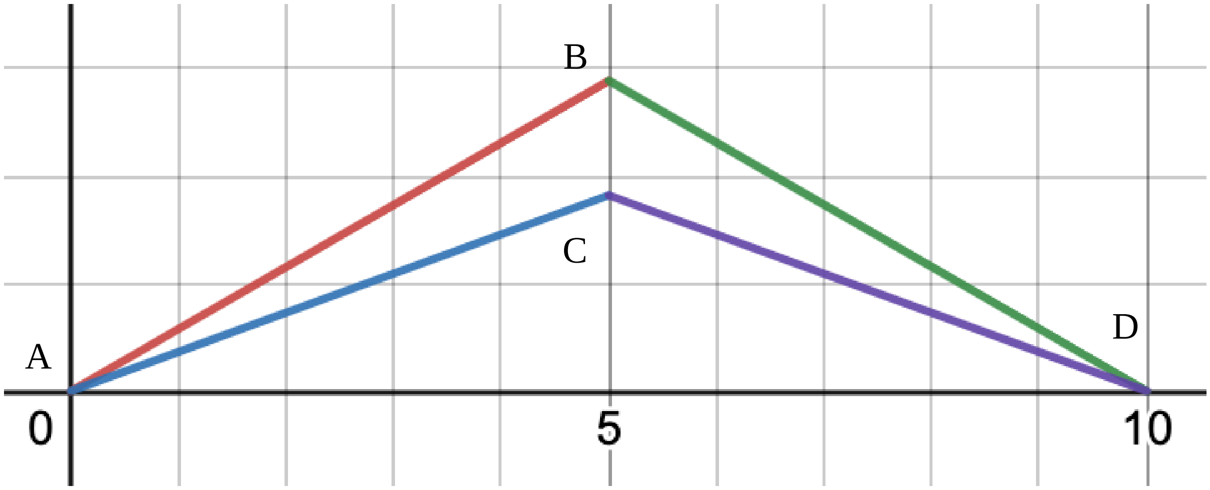
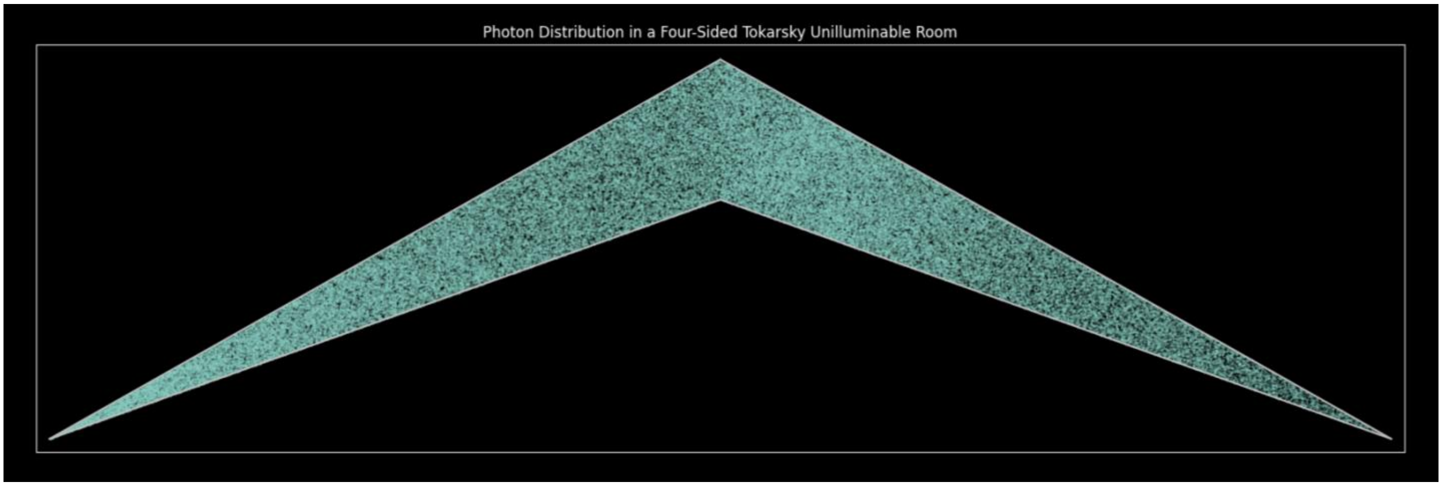
In 1995, mathematician George Tokarsky devised an "impossible pool shot", in which a billiard ball hit from one point on his constructed table will never reach the pocket at another point on the table, no matter what angle the ball is struck at and no matter how many times it is allowed to bounce off the walls. The shape of the original table is shown here as a Desmos graph, where vertex A is where the ball is placed, vertex D is where the pocket is, and the shape of the room is defined such that vertex B has angle 120° and vertices A and D have angle 10°.

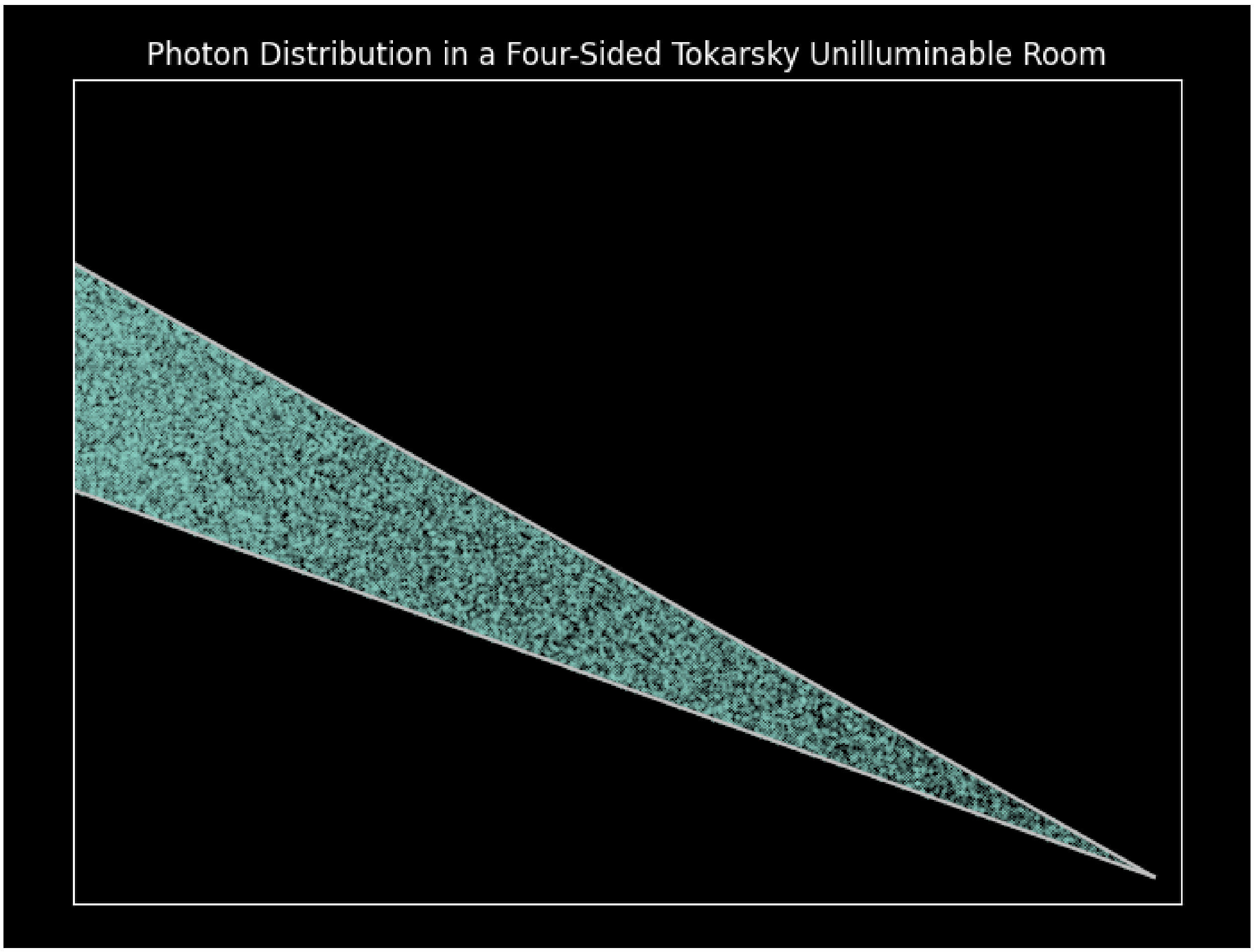



While initially conceived of as an impossible pool shot, this shape lends itself to other interesting physical interpretations. For example, imagining the walls as mirrors and placing a light source at vertex A, we now have an "unilluminable point" at vertex D. The paths light rays take from point A are rather mesmerizing, as illustrated in *ray_trace.mp4*. It appears that a photon leaving point A will spend the bulk of its time reflecting off the walls in the center of the room, with some ability to reapproach point A and progressively less time spent in the region towards point D. Since point D is theoretically unilluminable, will a large number of photons leaving point A produce a shadow gradient approaching vertex D? And what will that gradient look like?

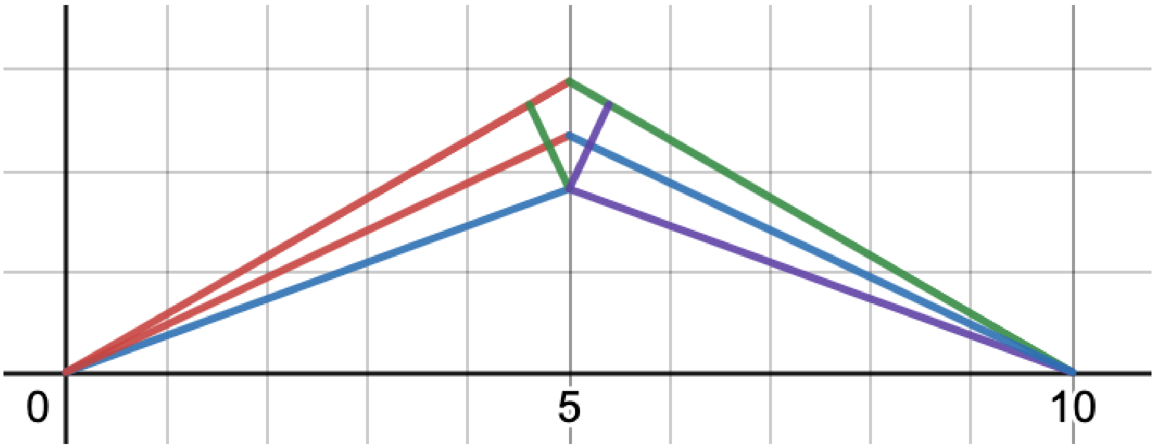
To investigate this question, I wrote a simulation of the room in Python and simulated many photons traversing the room from different starting angles to see where they ended up. An animation of what this traversal looked like for 1,000 photons is provided in *particles_long.mp4*. Once again, the results are mesmerizing, but to get an accurate distribution of the shadow that would be produced by a light source I decided to simulate 100,000 photons. Each photon's trajectory is randomly assigned as a normal vector between the slopes of walls AB and AC. Each photon is also given a velocity of -0.05 in the z-direction and placed at a random height between 0 and 10. The light source can be thought of as a vertical bar at vertex A. In this scenario, we'll be able to detect a shadow by counting the distribution of photons once they reach the floor at height z=0.

The distribution of 100,000 photons on the floor of the room is presented in *Tokarsky_room100k.png*. Already a shadow is visible near vertex D in *Tokarsky_room100k_close.png*.

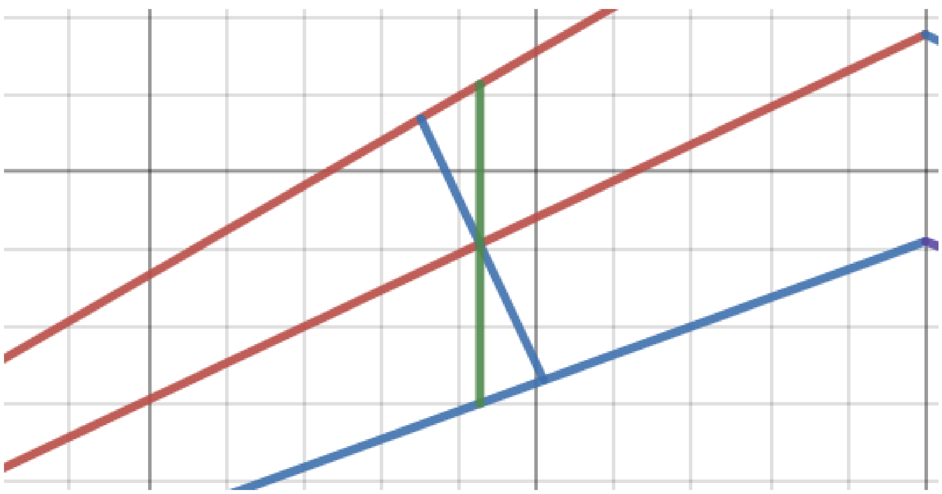


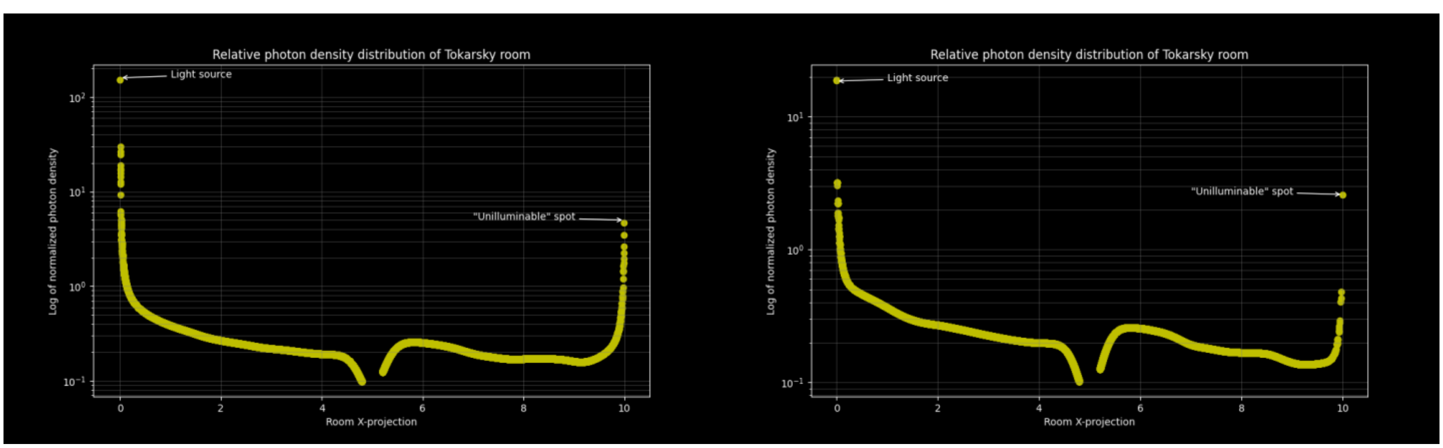

To measure this shadow and see if it is in fact significant, it will help to project these light points onto some line through the room, measure the distribution density of photons along this line, and finally normalize the distribution by dividing the density by the width of the room at that point. The projection line was chosen to be the average slope of the walls. The projection was performed for all points except those in the top “kite”, where the width of the room relative to the projection line starts to shrink and points nearest the top vertex don’t have a valid projection point (see Desmos graph of room with projection lines):

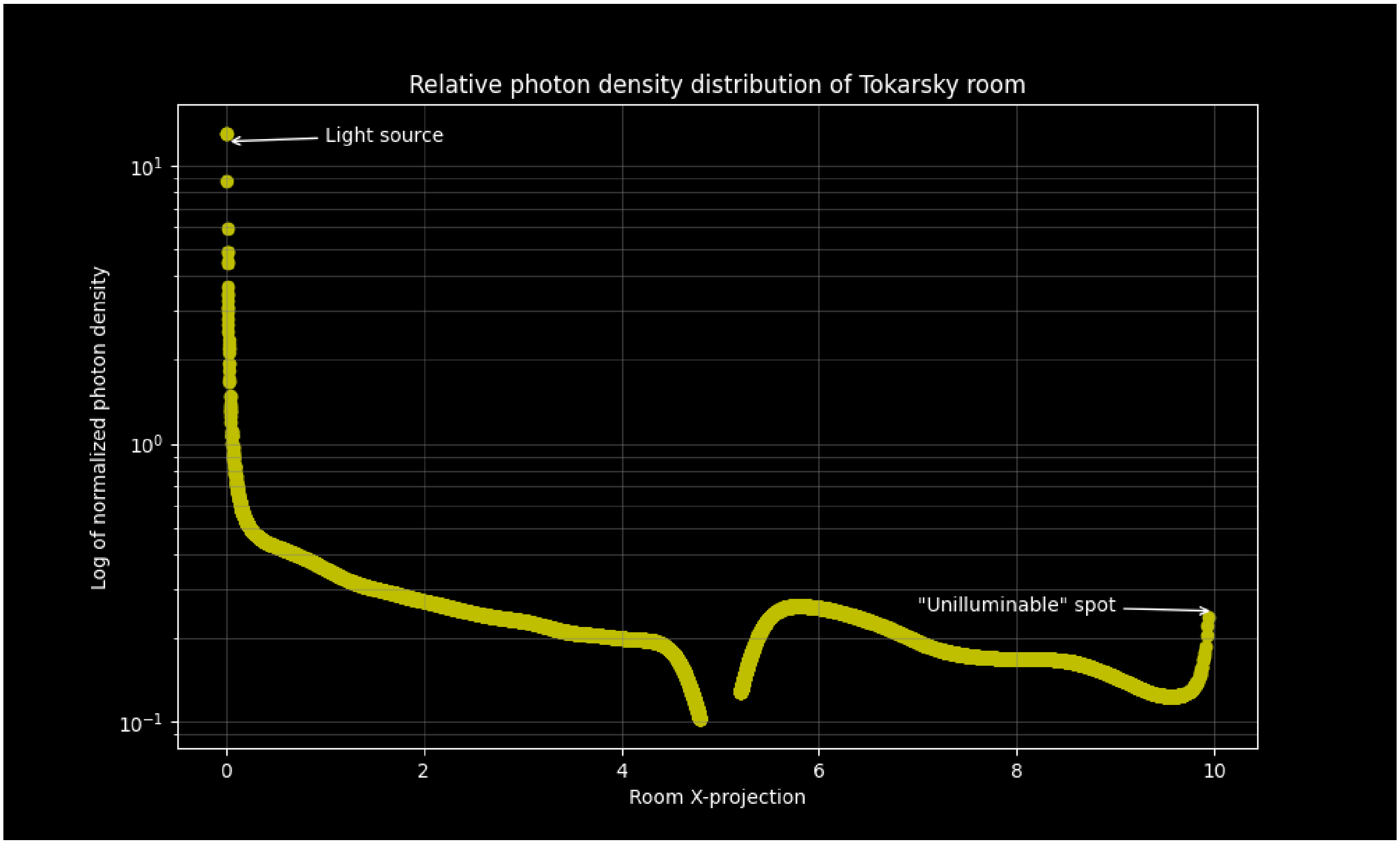



After the projection, points were referred to by their projected x-value. A Gaussian kernel density estimation was performed on the projected points to get the distribution of light as a function of the projected x-value. Finally, the room width at each x-value was obtained for normalization by solving the triangles that extend to the upper and lower walls:



The resultant photon density of the room is presented in *normalized_density.png*. Since the room width approaches zero at points A and D and the numerical nature of the photon reflections produce some inaccuracies, the photon density at point D is relatively significant. However, comparisons between photon densities at D with step sizes 0.05, 0.01 and 0.001 indicate this is merely a numerical artifact. With increasingly small step sizes, the shadow at point D becomes more pronounced.






A notable feature of the logarithmic density distribution of photons are the linear decline in the left half of the room, implying a power law decay. Whether or not this is a result of the choice to start photons at differing heights could be explored by starting all photons at the same height. Another notable feature is the relative preference for the right center of the room as opposed to the left center of the room. This can be explained by examining the *ray_trace.mp4* video, in which the left center of the room is traversed more or less parallel to the walls while the right center of the room holds a majority of the reflections. Since photons with similar angles to the example photon in *ray_trace.mp4* will spend so much time reflecting in this region, it is more likely that they will terminate there. A similar argument can be made for the relative bump at around x=8 with a different group of starting angles.

Switching from discrete collision detection to continuous collision detection will improve the simulation and eliminate artifacts in the measurement of photon density distribution. Along with implementing this in the future, I will simulate photons from a single starting height and check whether or not it has an effect on the distribution. Finally, I may examine other unilluminable room shapes for analysis.