Review of Scientific Instruments

Optimum Geometry of Saddle Shaped Coils for Generating a Uniform Magnetic Field

Cite as: Review of Scientific Instruments **41**, 122 (1970); https://doi.org/10.1063/1.1684235
Submitted: 08 August 1969 . Published Online: 11 November 2003

D. M. Ginsberg, and Melvin J. Melchner

Optimum Geometry of Saddle Shaped Coils for Generating a Uniform Magnetic Field*

D. M. GINSBERG AND MELVIN J. MELCHNER
Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801

(Received 8 August 1969; and in final form, 12 September 1969)

Saddle shaped coils for generating a field perpendicular to the axis of a cylindrical shell to which the coils are confined can be designed in a compact and easily constructed form. The central magnetic field of such a system and its second derivatives with respect to displacements from the center are given as functions of the coil dimensions, and conditions for minimizing these derivatives are discussed. A coil pair with a length-to-diameter ratio of 2 and circular arcs of 120° will have no second order central field derivatives in any direction.

SADDLE shaped coils which are confined to a thin cylindrical shell, such as near a Dewar wall, are compact and easily constructed and are convenient for generating a uniform magnetic field perpendicular to the cylinder axis.[1–3] Such a coil system (see Fig. 1) consists of two identical parts wound on the surface of a circular cylinder oriented in the z direction. Each coil carries the same current, circulating as shown to generate a field which at the origin is in the x direction. It is our purpose to derive the field uniformity which can be obtained as a function of the coil dimensions. The field anywhere in space may be calculated from a straightforward but tedious application of the Biot–Savart law. The field may then be expressed as a power series involving derivatives of the field with respect to x, y, and z evaluated at the center of the coil system (the "origin"); this power series has the advantage of providing a direct measure of the local field uniformity. The first (and all odd) derivatives vanish because of symmetry, and the second derivatives have no y or z component at the origin.

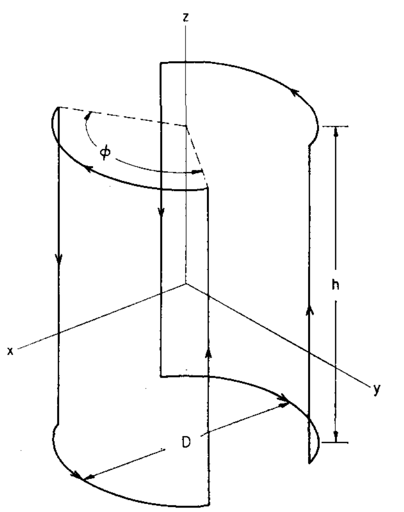

FIG. 1 The geometry of the field coils.

The central field $\mathbf{B}_0$ is given in mks units by

$$\mathbf{B}_0=(4\mathbf{i}/\pi)\mu_0 NI(h/D^2)(s^{-1/2}+s^{-3/2})\sin(\phi/2), \qquad (1)$$

where $\mathbf{i}$ is a unit vector in the x direction, $\mu_0=4\pi\times10^{-7}$ is the permeability of free space, N is the number of turns in each of the two coils, I is the current in the wire, and

$$s=1+(h/D)^2. \qquad (2)$$

Our calculations yield the following expressions for the second derivatives of the field at the origin:

$$(\partial^2\mathbf{B}/\partial x^2)_0=\mathbf{B}_0(D/2)^{-2}[-C_1\sin^2(\phi/2)+C_2]/C_0, \qquad (3)$$

$$(\partial^2\mathbf{B}/\partial y^2)_0=\mathbf{B}_0(D/2)^{-2}[C_1\sin^2(\phi/2)-C_3]/C_0, \qquad (4)$$

$$(\partial^2\mathbf{B}/\partial z^2)_0=\mathbf{B}_0(D/2)^{-2}[C_3-C_2]/C_0, \qquad (5)$$

where

$$C_0=s^2+s^3, \qquad (6)$$

$$C_1=5+3s+4s^2+8s^3, \qquad (7)$$

$$C_2=15+3s^2+6s^3, \qquad (8)$$

$$C_3=3s+3s^2+6s^3. \qquad (9)$$

The sum of the second derivatives, $\nabla^2\mathbf{B}=\nabla\nabla\cdot\mathbf{B}-\nabla\times(\nabla\times\mathbf{B})$, must vanish because in free space both $\nabla\cdot\mathbf{B}$ and $\nabla\times\mathbf{B}$ are zero; we can therefore make all three second

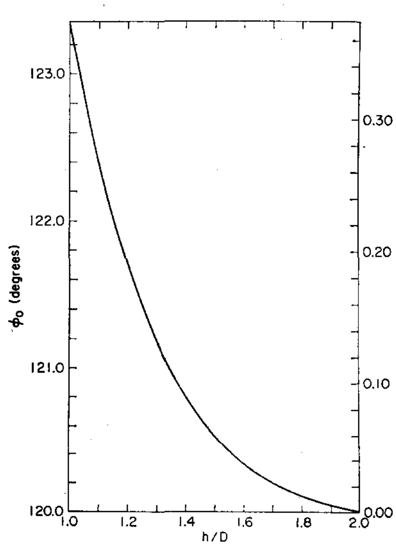

FIG. 2. The angle ϕ_0 and the field-uniformity parameter Q, defined in Eqs. (10) and (14), as functions of h/D.

* This research was supported in part by the National Science Foundation and in part by the Advanced Research Projects Agency under Grant SD-131.

[1] H. R. Hart, Jr., Ph.D. thesis, University of Illinois, 1960 (unpublished).
[2] R. J. Sarwinski, Ph.D. thesis, University of Illinois, 1966 (unpublished).
[3] W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Physics 1, 337 (1965).

derivatives vanish, although we have only two adjustable parameters, ϕ and h/D. The values required are $\phi=120°$ and $h/D=2$ (so that $s=5$). If clearance problems constrain one to a height less than this optimum, one can choose ϕ so as to minimize a weighted sum of the second derivatives. A special case[3] is that of zero second derivative with respect to x; superior, however, if the x and y dimensions of the sample are comparable, is to design for equal x and y second derivatives, since the field nonuniformity is then minimized and, in addition, is independent of angular position. The value of ϕ which accomplishes this is ϕ_0, where

$$\sin^2(\phi_0/2)=(C_2+C_3)/2C_1. \tag{10}$$

This relation is shown in Fig. 2. Evidently the optimum angle, as defined here, is near 120° for a wide range of values for h/D. If $\phi=\phi_0$, the second derivatives are

$$(\partial^2\mathbf{B}/\partial x^2)_0=\mathbf{B}_0(D/2)^{-2}Q, \tag{11}$$

$$(\partial^2\mathbf{B}/\partial y^2)_0=\mathbf{B}_0(D/2)^{-2}Q, \tag{12}$$

and

$$(\partial^2\mathbf{B}/\partial z^2)_0=-2\mathbf{B}_0(D/2)^{-2}Q, \tag{13}$$

where

$$Q=(15-3s)/(2s^2+2s^3). \tag{14}$$

Coincidentally, in the range $1\leq h/D\leq 2$, the curve showing Q as a function of h/D is almost identical in shape to that of ϕ_0 vs h/D, and therefore simply rescaling the vertical axis as shown in Fig. 2 suffices to indicate how Q depends on h/D. The equations, of course, are valid for values of h/D beyond the range covered by Fig. 2.

If the sample size is comparable to the coil diameter D, then higher derivatives of the field may be significant. However Abel *et al.*[3] found that the field of a coil pair with $h/D=1.6554$ and $\phi=120.76°$ (designed to make $\partial^2\mathbf{B}/\partial x^2=0$) varied by only 0.5% on going from the origin to the points $(x, y, z)=(0.26R, 0, 0)$, $(0, 0.33R, 0)$, or $(0, 0, 0.35R)$, where $R=D/2$ is the radius of the coil system.

Saddle shaped coils can also be used to generate a uniform magnetic field *gradient* by making the currents in the two coils equal but *opposite*; ϕ and h/D should then be selected to minimize some combination of the third order derivatives of the central magnetic field.[1-3]

THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 41, NUMBER 1 JANUARY 1970

Ultracentrifuge Data Acquisition from a Central Control System

Alfred A. Windsor, Lin C. Jensen, David A. Hoopes, and Frank T. Lindgren
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
(Received 1 August 1969)

Three analytic ultracentrifuges are controlled and useful operating measurements recorded by a central control and data acquisition system. Rotor speed, temperature, picture number of each schlieren photograph, and time of each measurement are recorded on cards by a printing summary punch machine. A FORTRAN IV computer program processes the data, yielding graphs of rotor speed and temperature vs time and the accumulated value of $\omega_{FS}^{-2}\int\omega^2(t)dt$ for the mean time of each schlieren photograph. These data permit error detection as well as more accurate schlieren analysis of lipoprotein distributions and flotation rates.

INTRODUCTION

In an earlier study[1] automatic acceleration to full speed was provided for an analytic ultracentrifuge using a modified electronic speed control system. However, during an ultracentrifuge run it is useful and frequently necessary to obtain accurate information about the acceleration phase, full speed stability, rotor temperature, and exact time when schlieren photographs are taken. The present automatic system is designed to provide all such data from one or more analytic ultracentrifuges.

When data are collected from several devices and channeled into one recorder, a method of coding and collating is required. We have chosen to use a master clock to synchronize the start of each machine and to code the data by the time at which measurements are taken. A time sequencer strobes the measurements by switching from machine to machine in a one–two–three sequence. This data collecting system is available for any one or all three ultracentrifuges, which may run on totally independent operation schedules.[2]

A digital clock (Parabam model DA24, Hawthorne, California) reading hours, minutes, and seconds on a 24 h basis controls the timing. An interface coupler (Dymec model 2526, Hewlett–Packard, Palo Alto, California) operates with an IBM 526 summary punch machine to store and record 12 input characters. The IBM

[1] A. A. Windsor, T. H. Rich, R. E. Doyle, and F. T. Lindgren, Rev. Sci. Instrum. 38, 949 (1967).

[2] J. V. Maloney, Jr., *Computers: The Physical Sciences and Medicine*, AFIPS Fall Conf. (Spartan Books, Washington, D. C., 1965) Vol. 27, Pt. 2.