CJ Hess

1. For $n = 0$

$$\frac{0^2}{(2(0)-1)(2(0)+1)} = \frac{0(0+1)}{2(2(0)+1)}$$

$$\frac{0}{-1} = \frac{0}{2}$$

$$0 = 0$$

For $n = k+1$

$$\frac{(k+1)^2}{(2(k+1)-1)(2(k+1)+1)} + \sum_{n=0}^{k} \frac{n^2}{(2n-1)(2n+1)}$$

$$\frac{(k+1)^2}{(2(k+1)-1)(2(k+1)+1)} + \frac{k(k+1)}{2(2k+1)} = \frac{(k+1)(k+1+1)}{2(2(k+1)+1)}$$

$$2(2k+1)(2k+3) \cdot \left(\frac{(k+1)^2}{(2k+1)(2k+3)} + \frac{k(k+1)}{2(2k+1)} = \frac{(k+1)(k+2)}{2(2k+3)} \right)$$

$$2(k+1)^2 + k(k+1)(2k+3) = (k+1)(k+2)(2k+1)$$

$$\boxed{\begin{gathered} 2k^3 + 7k^2 + 7k + 2 = 2k^3 + 7k^2 + 7k + 2 \\ \text{for all } k \end{gathered}}$$

2. $\prod_{i=0}^{n} \left(\frac{1}{2i+1} \cdot \frac{1}{2i+2}\right) = \frac{1}{(2n+2)!}$

for $n = 0$:

$$\frac{1}{2(0)+1} \cdot \frac{1}{2(0)+2} = \frac{1}{(2(0)+2)!}$$

$$\frac{1}{1} \cdot \frac{1}{2} = \frac{1}{2!}$$

$$\frac{1}{2} = \frac{1}{2}$$

for $(n = 1)$:

$$\frac{1}{2(1)+1} \cdot \frac{1}{2(1)+2} = \frac{1}{(2(1)+2)!}$$

$$\frac{1}{2} \cdot \left(\frac{1}{3} \cdot \frac{1}{4}\right) = \frac{1}{4!}$$

$$\frac{1}{2} \cdot \frac{1}{12} = \frac{1}{24}$$

$$\frac{1}{24} = \frac{1}{24}$$

3. $2^n < (n+2)!$ for $n \geq 0$

$n = 0$

$2^0 < (0+2)!$

$1 < 2!$

$1 < 2$

$n = k+1$

$2^{k+1} < (k+3)!$

$2(2^k) < (k+3)!$

for all $k \geq 0$

4. C_0, C_1, C_2 defined by $C_0 = 5$ $C_k = (C_{k-1})^2$

for $k = 1$

$C_1 = (5)^2$

for $k = 2$

$C_2 = ((5)^2)^2$

for $k = k+1$

$(C_k)^2$

for n

$C_n = 5^{2^n}$

due to the ratio between each step being some change of 5^2