Discrete Math Homework 5

1a. $\forall$ real string s_1 and s_2, if $G(s_1) = G(s_2)$ then $s_1 = s_2$

Counterexample:

s_1 = 'bab' s_2 = 'bba'

$G(s_1) = G(s_2)$

$2 = 2$

So $G(s_1) = G(s_2)$ but $s_1 \neq s_2$

so G is not one-to-one

1b. $G: R \to R$ is the function defined by the rule $G(s)$ = is the number of bs in a string of nonnegative length

$y \leftarrow R$ $\exists s$ in R that $G(s) = y$

Since the length of the string can be infinitely positive and can contain any positive number of bs, $G(s)$ is onto.

2. Let $f, g, h \; \mathbb{Z} \to \mathbb{Z}$: $f(n) = 2n + 5$

$g(n) = 3n - 7$

$h(n) = 4n^2 - 3$

a. $g(f(n)) = 3(2n+5) - 7$

$= 6n + 15 - 7$

$= 6n + 8$

$h(g(n)) = 4((3n-7)^2) - 3$

$= 4(9n^2 - 42n + 49) - 3$

$= 36n^2 - 168n + 216 - 3$

$= 36n^2 - 168n + 213$

b. Suppose we have n_1 and n_2 such that

$g \circ f(n_1) = g \circ f(n_2)$

$6n_1 + 8 = 6n_2 + 8$

$-8 \qquad -8$

$\frac{6n_1}{6} = \frac{6n_2}{6}$

$n_1 = n_2$

$g \circ f$ is one-to-one

$0 \leftarrow \mathbb{Z}$

$6n + 8 = 0$

$6n = -8$

$n = -\frac{8}{6}$

Since $-\frac{8}{6}$ is not an integer, there exists no integer n such that $g \circ f(n) = 0$

$g \circ f(n)$ is not onto.

2b. Counterexample

$$h \circ g(n) = 21$$

$$21 = 36n^2 - 168n + 213$$

$$0 = 36n^2 - 168n + 192$$

Quad. Formula!

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{168 \pm \sqrt{-168^2 - 4(36)(192)}}{72}$$

$$= \frac{168 \pm \sqrt{576}}{72}$$

$$= \frac{168 \pm 24}{72}$$

$$n = \frac{168 + 24}{72} \text{ or } \frac{168 - 24}{72}$$

$$n = 5/3, 2$$

since $n_1 \neq n_2$, $h \circ g(n)$ is not one-to-one

26. $0 \leftarrow \mathbb{Z}$

$hog(n) = 0$

$36n^2 - 168n + 213 = 0$

Quad Formula

$$\frac{168 \pm \sqrt{-168^2 - 4(36)(213)}}{72}$$

$$\frac{168 \pm \sqrt{-2448}}{72}$$

$$\frac{168 \pm 4i\sqrt{153}}{72}$$

Since there is no real integer such that $hog(n) = 0$, hog is not onto.

2c. $(g \circ f)^{-1}(\{-11, -8, -3, 0, 7, 17, 28, 34, 40\})$

$g \circ f(n) = -11$
$6n + 8 = -11$
$6n = -19$
$n = -19/6$

$g \circ f(n) = -8$
$6n + 8 = -8$
$6n = -16$
$n = -16/6 = -8/3$

$g \circ f(n) = -3$
$6n + 8 = -3$
$6n = -11$
$n = -11/6$

$g \circ f(n) = 0$
$6n + 8 = 0$
$6n = -8$
$n = -8/6 = -4/3$

$g \circ f(n) = 7$
$6n + 8 = 7$
$6n = -1$
$n = -1/6$

$g \circ f(n) = 17$
$6n + 8 = 17$
$6n = 9$
$n = 3/2$

$g \circ f(n) = 28$
$6n + 8 = 28$
$6n = 20$
$n = 10/3$

$g \circ f(n) = 34$
$6n + 8 = 34$
$6n = 26$
$n = 13/3$

$g \circ f(n) = 40$
$6n + 8 = 40$
$6n = 32$
$n = 16/3$

$$\boxed{(g \circ f)^{-1}(\{-11, -8, -3, 0, 7, 17, 28, 34, 40\}) = \{-19/6, -8/3, -11/6, -4/3, -1/6, 3/2, 10/3, 13/3, 16/3\}}$$

3. $xSy \iff |x| = |y|$

- Relation S is reflexive, because $xSx \iff |x| = |x|$ which is true $\forall x \in S$.
- Relation S is symmetric, because $\forall x, y \in S$, if $|x| = |y|$ then $|y| = |x|$.
- Relation S is transitive, because $\forall x, y, z \in S$ xSy means $|x| = |y|$, ySz means $|y| = |z|$, and xSz means xSz means $|x| = |z|$
  if $|x| = |y|$ and $|y| = |z|$, then $|x| = |z|$.

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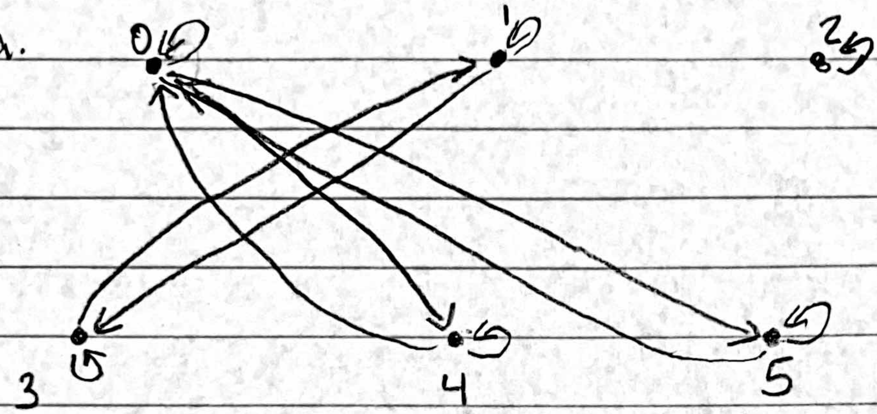
a.



b.

$$\begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 1 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

c. R is reflexive, because $\forall x$ in R xRx is true. R is transitive, because $\forall xy$ in R if xRy is true then yRx is true. R is not transitive, because $4R0$ and $0R5$ are true, but $4R5$ is false.