CJ Hess

1 a:

i. True
ii. True
iii. False

b:

$L(1010) = 4$
Range of $L = [0, \infty)$
Codomain of $L = [0, \infty)$
Inverse image of 1 = "0"

c: $5\sum_{k=0}^{n+1} (2k^2 - 3k) + 2\sum_{k=1}^{n+2} (k^2 + 5k + 1)$

$\sum_{k=0}^{n+1} (10k^2 - 15k) + 2\sum_{k=0}^{n+1} ((k-1)^2 + 5(k-1) + 1)$

$\sum_{k=0}^{n+1} (10k^2 - 15k) + \sum_{k=0}^{n+1} (2k^2 + 6k - 6)$

$$\boxed{\sum_{k=0}^{n+1} (12k^2 - 9k - 6)}$$

1 d. $3\sum_{k=0}^{n+1} (-1)^n (7)^{n+1}$

$(n+1) = 3(-1)^{n+1} (7)^{n+2}$

$= \boxed{3(-1)^{n+1} (7)^{n+2}}$

e. a = 1932, b = 612

a = a/b = 3 b = 1932 % 612 = 96

a = 612 b = 96

a = a/b = 6 b = 612 % 96 = 36

a = 96 b = 36

a = a/b = 2 b = 96 % 36 = 24

a = 36 b = 24

a = a/b = 1 b = 36 % 24 = 12

a = 24 b = 12

a = a/b = 2 b = 0

$\boxed{\gcd(1932, 612) = 2}$

2 a. as $i \to \infty$, we approach $(2, 2)$

at $i = 1$, we get $(1, 3)$

$$\bigcup_{i=1}^{\infty} A_i = \boxed{[1, 3]}$$

$$\bigcap_{i=1}^{\infty} A_i = \boxed{(-\infty, 1) \cup (3, \infty)}$$

b

i. $(A \cap C) \times A^c$

$\{a, 2\} \times A^c$

$\{a, 2\} \times \{c, 1\}$

$$\boxed{\{\langle a, c\rangle, \langle a, 1\rangle, \langle 2, c\rangle, \langle 2, 1\rangle\}}$$

ii. $P((B-A) \times C)$

$P(\{1\} \times C)$

$P(\{\langle 1, 2\rangle, \langle 1, 3\rangle\})$

$$\boxed{P = \{\emptyset, \{\langle 1,2\rangle\}, \{\langle 1,3\rangle\}, \{\langle 1,2\rangle, \langle 1,3\rangle\}\}}$$

3 Step 1:

$$P(n) = \frac{n+1}{3(2n+5)} \quad n \geq 0$$

Step 2:

$$P(0) = \frac{0+1}{3(2(0)+5)} \quad \frac{1}{(2(0)+3)(2(0)+5)}$$

$$= \frac{1}{15} = \frac{1}{15}$$

Step 3:

$$P(k+1) = \frac{k+2}{3(6k+27)} \quad \frac{1}{(2(k)+3)(2(k)+5)}$$

$$P(k) = \frac{k+1}{6k+15} = \frac{1}{4k^2+16k+15}$$

Step 4:

Since we have proven both the basis and inductive step, the theorem is true

$$\frac{6k^2+21k+15}{6k+15} \quad \frac{k+1}{k+15}$$

4 a. $\forall a,b,c,r$ if $\frac{7r^3}{2a} + \frac{3c}{b} - 5$ is irrational

a, b, c are rational, then r is irrational

Negation:

$\exists a,b,c,r$ if $\frac{7r^3}{2a} + \frac{3c}{b} - 5$ is not irrational

then r is not irrational.

b. $\frac{7r^3}{2a} + \frac{3c}{b} - 5 = \frac{x}{y}$ where x and y are integers $(y \neq 0)$

$$\frac{7r^3}{2a} = \frac{x}{y} + 5 - \frac{3c}{b}$$

$$7r^3 = 2a\left(\frac{x}{y} + 5 - \frac{3c}{b}\right)$$

$$r^3 = \frac{2a}{7}\left(\frac{x}{y} + 5 - \frac{3c}{b}\right)$$

r is not both even and odd

thus contradiction.

5 $P(n) = 1 + 4n < 2^n$

$P(5) = 1 + 4(5) < 2^5$

$= 21 < 32$

true

$P(k) = 1 + 4k < 2^k$

$P(k+1) = 4k + 5 < 2^{k+1}$

$\boxed{8k + 10 < 2(2^k)}$

for all $n \geq 5$, $1 + 4n < 2^n$ is true