CHAPTER SIX

Transmission Line Transformers

6.1 INTRODUCTION

An RF transceiver often requires impedance transformation, power splitting, or transformation from a balanced to an unbalanced (balun) transmission line. Such circuits appropriate to the RF range are described in this chapter. The subject matter of Chapter 3 was impedance transformation. This subject is taken up here again, but now with more careful attention given to the special problems and solutions required for RF designs. The discrete-element designs described previously can be used in RF designs with the understanding that element values will change as frequency changes. The alternative to discrete-element circuits are transmission line circuits. The classical microwave quarter-wavelength transformer can be used up to hundreds of gigahertz in the appropriate transmission line medium. However, at 1 GHz, a three-section quarter-wavelength transformer would be a little less than a meter long! The solution lies in finding a transformation structure that may not work at 100 GHz but will be practical at 1 GHz.

The conventional transformer consists of two windings on a high-permeability iron core. The flux, ϕ, is induced onto the core by the primary winding. By Faraday's law, the secondary voltage is proportional to $d\phi/dt$. For low-loss materials, the primary and secondary voltages will be in phase. Ideal transformers have perfect coupling and no losses. The primary-to-secondary voltage ratio is equal to the turns ratio, n, between the primary and secondary windings, namely $V_p/V_s = n$. The ratio of the primary-to-secondary current ratio is $I_p/I_s = 1/n$. This implies conservation of power, $V_pI_p = V_sI_s$. As a consequence, the impedance seen by the generator or primary side in terms of the load impedance is

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$$Z_G = n^2 Z_L \tag{6.1}$$

When the secondary side of the ideal transformer is an open circuit, the input impedance of the transformer on the primary side is infinity.

In a physical transformer the ratio of the leakage inductances on primary and secondary sides is $L_p/L_s = n$. For the ideal transformer, L_p and L_s approach ∞, but their ratio remains finite at n. The physical transformer has an associated mutual inductance, $M = k\sqrt{L_p/L_s}$, where k is the coupling coefficient. The leakage inductance together with the interwire capacitances limits the high-frequency response. The transmission line transformer avoids these frequency limitations.

6.2 IDEAL TRANSMISSION LINE TRANSFORMERS

It was found in Chapter 2 that inductive coils always come with stray capacitance. It was this capacitance that restricted the frequency range for a standard coupled-coil transformer. The transmission line transformer can be thought of as simply tipping the coupled-coil transformer on its side. The coil inductance and stray capacitance now form the components for an artificial transmission line whose characteristic impedance is

$$Z_0 = \sqrt{\frac{L}{C}} \tag{6.2}$$

The artificial transmission line can be used, in principle, up to very high frequencies because the shunt capacitance forms part of the transmission line characteristic impedance. The transmission line transformer can be made from a variety of forms of transmission lines such as two parallel lines, a twisted pair of lines, a coaxial cable, or a pair of wires on a ferrite core. The transmission line transformer can be defined as having the following characteristics:

1. The transmission line transformer is made up of interconnected lines whose characteristic impedance is a function of such mechanical things as wire diameter, wire spacing, and insulation dielectric constant.
2. The transmission lines are designed to suppress even-mode currents and allow only odd-mode currents to flow (Fig. 6.1).

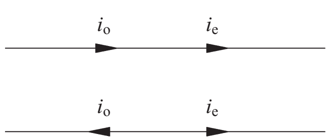

FIGURE 6.1 Two-wire transmission line showing odd- and even-mode currents.

3. The transmission lines carry their own "ground" so that transmission lines relative to true ground are unintentional.
4. All transmission lines are of equal length and typically $<\lambda/8$.
5. The transmission lines are connected at their ends only.
6. Two different transmission lines are not coupled together either by capacitance or inductance.
7. For a short transmission line, the voltage difference between the terminals at the input port is the same as the voltage difference at the output port.

Some explanation of these points is needed to clarify the characteristics of the transmission line transformer. In property 2, for a standard transmission line, the current going to the right in one conductor must be equal to the current going to the left in the other in order to preserve current continuity (Fig. 6.1). Since only odd-mode currents are allowed, the external magnetic fields are negligible. The net current driving the magnetic field outside of the transmission line is low. The third point is implied by the second. The transmission line is isolated from other lines as well as the ground. The equality of the odd-mode currents in the two lines of the transmission line as well as the equivalence of the voltages across each end of the transmission line is dependent on the transmission line being electrically short in length. The analysis of transmission line transformers will be based on the given assumptions above.

As an example, consider the transmission line transformer consisting of one transmission line with two conductors connected as shown in Fig. 6.2. The transformation ratio will be found for this connection. Assume first that v_1 is the voltage across R_G and i_1 is the current leaving the generator resistance:

1. Current i_1 passes through the upper conductor of the transmission line.
2. The odd-mode current i_1 flows in the opposite direction in the lower conductor of the transmission line.
3. The sum of the two transmission line currents at the output node is $2i_1$.

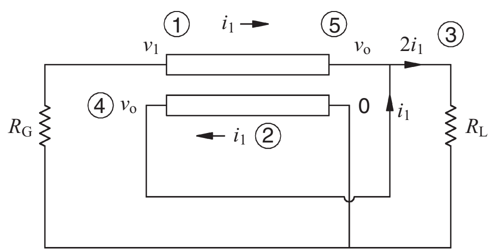

FIGURE 6.2 Analysis steps for transmission line transformer.

4. The voltage at the output node is assumed to be v_o. Consequently, the voltage at the left side of the lower conductor in the transmission line is v_o above ground.
5. On the left-hand side, the voltage difference between the two conductors is $v_1 - v_o$. This is the same voltage difference on the right-hand side. Consequently,

$$v_o - 0 = v_1 - v_o$$
$$v_o = v_1/2$$

If $R_G = v_1/i_1$, then

$$R_L = \frac{v_o}{2i_1} = \frac{v_1/2}{2i_1} = \frac{R_G}{4} \tag{6.3}$$

This 4:1 circuit steps down the impedance level by a factor of 4.

A physical connection for this transformer is shown in Fig. 6.3, where the transmission line is represented as a pair of lines. In this diagram the nodes in the physical representation are matched to the corresponding nodes of the schematic representation. The transmission line is bent around to make the *B–C* distance a short length. The transmission line, shown here as a two-wire line, can take a variety of forms such as a coupled line around a ferromagnetic core, flexible microstrip line, or coaxial line. If the transformer is rotated about a vertical axis at the center, the circuit shown in Fig. 6.4 results. Obviously, this results in a 1:4 transformer where $R_L = 4R_G$. Similar analysis to that given above verifies this result. In addition multiple two-wire transmission line trans-

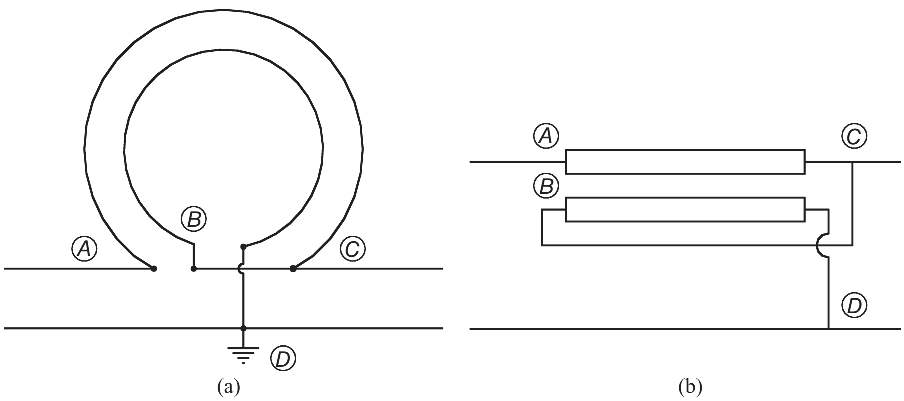

FIGURE 6.3 Physical two-wire transmission line transformer and equivalent formal representation.

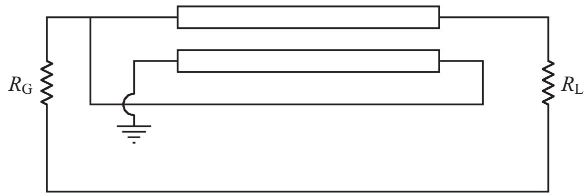

FIGURE 6.4 Alternate transmission line transformer connection.

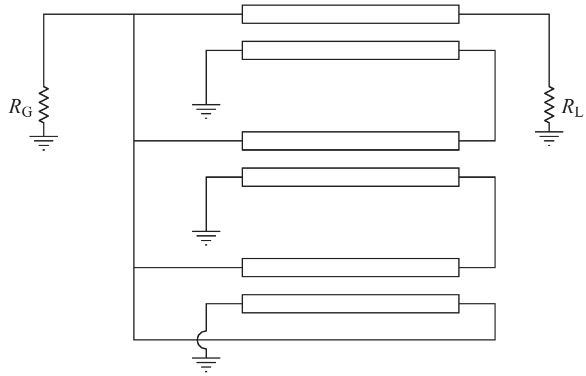

FIGURE 6.5 A 16:1 transmission line transformer.

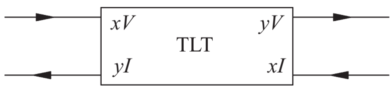

FIGURE 6.6 Symbol for general transmission line transformer.

formers may be tied together to achieve a variety of different transformation ratios. An example of three sections connected together is shown in Fig. 6.5. In this circuit the current from the generator splits into four currents going into the transmission lines. Because of the equivalence of the odd-mode currents in each line, these four currents are all equal. The voltages on the load side of each line pair build up from ground to 4× the input voltage. As a result, for match to occur, $R_L = 16R_G$.

The voltages and currents for a transmission line transformer (TLT) having a wide variety of different interconnections and numbers of transmission lines can be represented by the simple diagram in Fig. 6.6, where x and y are integers. The impedance ratios, $R_G = (x/y)^2 R_L$, range from 1:1 for a 1-transmission line circuit to 1:25 for a 4-transmission line circuit with a total of 16 different transformation ratios [1]. A variety of transmission line transformer circuits are found in [1] and [2].

6.3 TRANSMISSION LINE TRANSFORMER SYNTHESIS

All the transmission lines in the transmission line transformer shown in Fig. 6.5 have their left-hand sides near the generator connected in parallel and all their right-hand sides near the load connected in series. In this particular circuit, there are three transmission lines, and analysis shows that $V_{in}:V_{out} = 1:4$, and $R_G:R_L = 1:16$. The number of transmission lines, m, is the order of the transformer, so that when all the transmission lines on the generator side are connected in shunt and on the load side in series, the voltage ratio is $V_{in}:V_{out} = 1:(m+1)$. Synthesis of impedance transformations of 1:4, 1:9, 1:16, 1:25, and so on are all obvious extensions of the transformer shown in Fig. 6.5. To obtain a voltage ratio that is not of the form $1:(m+1)$ there is a simple synthesis technique [3]. The voltage ratio is $V_{in}:V_{out} = H:L$, where H is the high value and L the low value. This ratio is decomposed into an $V_{in}:V_{out} = H - L:L$. If now $H - L < L$, this procedure is repeated where now $H' = L$ and $L' = H - L$. This ratio is now $V_{out}:V_{in} = H':L'$, which in turn can be decomposed into $H' - L':L'$. These steps are repeated until a 1:1 ratio is achieved, all along keeping track of which ratio is being done, $V_{in}:V_{out}$ or $V_{out}:V_{in}$. The allowed voltage ratios, upon being squared, give the impedance ratios as shown in Table 6.1.

An example given in [3] illustrates the procedure. If an impedance ratio of $R_G:R_L = 9:25$ is desired, the corresponding voltage ratio is $V_{in}:V_{out} = 3:5$:

Step 1 $H:L = V_{out}:V_{in} = 5:3 \rightarrow (5-3):3 = 2:3$
Step 2 $H:L = V_{in}:V_{out} = 3:2 \rightarrow (3-2):2 = 1:2$
Step 3 $H:L = V_{out}:V_{in} = 2:1 \rightarrow (2-1):1 = 1:1$

Now working backward from step 3, a $V_{in}:V_{out} = 1:2$ transmission line transformer is made by connecting two transmission lines in shunt on the input side and in series on the output side (Fig. 6.7*a*). From step 2, the V_{out} is already 2, so another transmission line is attached to the first pair in shunt on the output side and series on the input side (Fig. 6.7*b*). Finally from step 1, $V_{in} = 3$ already,

TABLE 6.1 Voltage Ratios for Transmission Line Transformers

Number of Lines	1	2	3	4
	1:1	2:3	3:4	4:5
	1:2	1:2	3:5	5:7
	—	1:3	2:5	5:8
	—	—	1:4	4:7
	—	—	—	3:7
	—	—	—	3:8
	—	—	—	2:7
	—	—	—	1:5

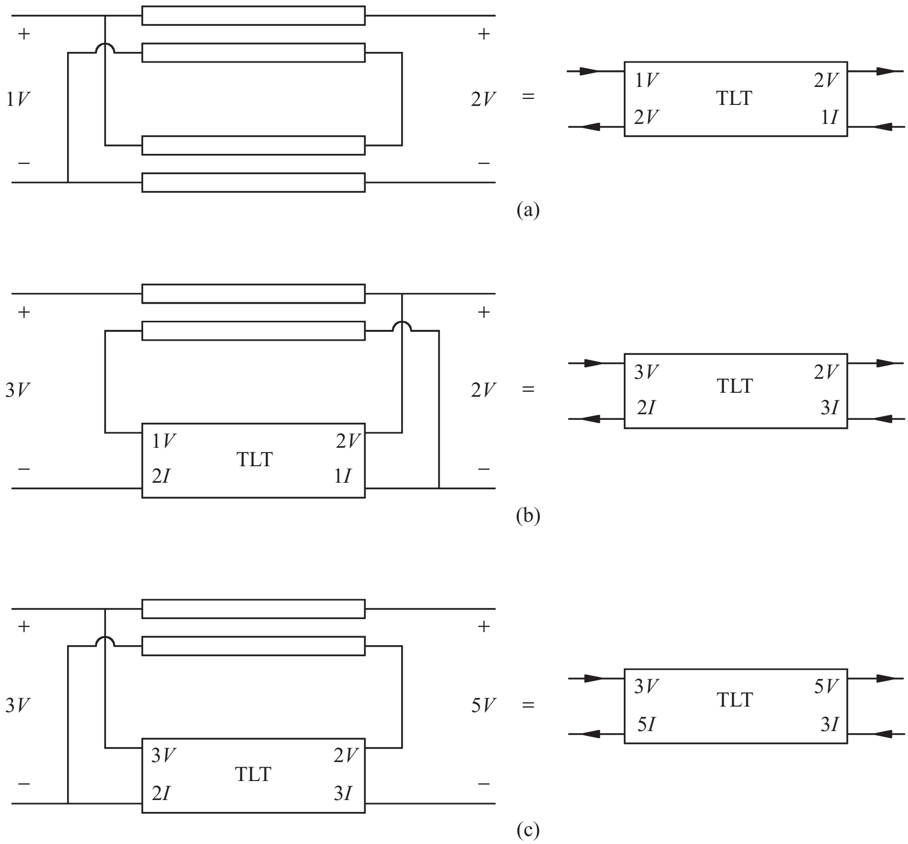

FIGURE 6.7 Step-by-step procedure for synthesis for desired impedance ratio.

so the input is connected in shunt with another added transmission line and the outputs connected in series (Fig. 6.7*c*). The final design has $V_{in}:V_{out} = 3:5$ as desired.

6.4 ELECTRICALLY LONG TRANSMISSION LINE TRANSFORMERS

One of the assumptions given in the previous section was that the electrical length of the transmission lines was short. Because of this the voltages and currents at each end of an individual line could be said to be equal. However, as the line becomes electrically longer (or the frequency increases), this assumption ceases to be accurate. It is the point of this section to provide a means of determining the amount of error in this assumption. Individual design goals would dictate whether a full frequency-domain analysis is needed.

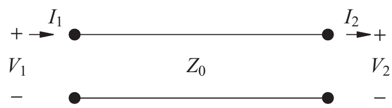

FIGURE 6.8 Electrically long transmission line.

As pointed out in Chapter 4, the total voltage and current on a transmission line are each expressed as a combination of the forward and backward terms (Fig. 6.8). In this case let V_2 and I_2 represent the voltage and current at the load end, where V^+ and V^- are the forward- and backward-traveling voltage waves:

$$V_2 = V^+ + V^- \tag{6.4}$$

$$I_2 = \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \tag{6.5}$$

Assuming that the transmission line is lossless, the voltage and current waves at the input side, 1, given in terms of their values at port 2 are modified by the phase associated with the electrical length of the line:

$$V_1 = V^+ e^{j\theta} + V^- e^{-j\theta} \tag{6.6}$$

$$I_1 = \frac{V^+}{Z_0} e^{j\theta} - \frac{V^-}{Z_0} e^{-j\theta} \tag{6.7}$$

The sign associated with the phase angle, $+\theta$, for V^+ is used because the reference is at port 2 while a positive phase is associated with traveling from left to right. The Euler formula is used in converting the exponentials to sines and cosines. The voltage at the input, V_1, is found in terms of V_2 and I_2 with the help of Eqs. (6.4) and (6.5):

$$V_1 = V_2 \cos\theta + jZ_0 I_2 \sin\theta \tag{6.8}$$

Similarly, I_1 can be expressed in terms of the voltage and current at port 2:

$$I_1 = I_2 \cos\theta + j\frac{V_2}{Z_0} \sin\theta \tag{6.9}$$

The 1:4 transmission line transformer shown in Fig. 6.4 is now reconsidered in Fig. 6.9 to determine its frequency response. The generator voltage can be expressed in terms of the transmission line voltages and currents:

$$V_G = (I_1 + I_2) R_G + V_1 \tag{6.10}$$

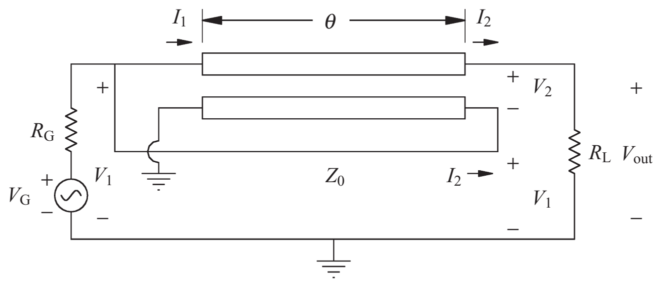

FIGURE 6.9 Electrically long 1:4 transmission line transformer.

The nontransmission line connections are electrically short. Therefore, the output voltage across R_L is $V_o = V_1 + V_2$, and

$$V_G = (I_1 + I_2)R_G + I_2 R_L - V_2 \tag{6.11}$$

In Eqs. (6.11), (6.8), and (6.9), V_1 is replaced by $I_2R_L - V_2$ to give three equations with three unknowns I_1, I_2, and V_2:

$$V_G = I_1 R_G + I_2(R_G + R_L) - V_2 \tag{6.12}$$

$$0 = 0 + I_2(jZ_0 \sin\theta - R_L) + V_2(1 + \cos\theta) \tag{6.13}$$

$$0 = -I_1 + I_2 \cos\theta + j\frac{V_2}{Z_0}\sin\theta \tag{6.14}$$

The determinate of this set of equations is

$$\Delta = -2R_G(1 + \cos\theta) - R_L \cos\theta + j\sin\theta\left(\frac{-R_G R_L}{Z_0} - Z_0\right) \tag{6.15}$$

and the current I_2 is

$$I_2 = \frac{-V_G(1 + \cos\theta)}{\Delta} \tag{6.16}$$

Consequently, the power delivered to the load from the source voltage is

$$\begin{aligned} P_o &= \tfrac{1}{2}\left|I^2\right|^2 R_L \\ &= \frac{1}{2}\frac{|V_G|^2(1 + \cos\theta)^2 R_L}{[2R_G(1 + \cos\theta) + R_L(\cos\theta)]^2 + \left[\left(R_G R_L + Z_0^2\right)/Z_0\right]^2 \sin^2\theta} \end{aligned} \tag{6.17}$$

Now the particular value of R_L that guarantees maximum power transfer into the load is found by maximizing Eq. (6.17). Let D represent the denominator in Eq. (6.17):

$$\frac{dP_o}{dR_L}=0=\frac{1}{2}|V_G|^2\frac{(1+\cos\theta)^2}{D}$$
$$\times\left(1-\frac{R_L}{D}\{2[2R_G(1+\cos\theta)+R_L\cos\theta]\cos\theta+[\cdots]\sin^2\theta\}\right) \quad (6.18)$$

In the low-frequency limit where $\theta\to 0$, the coefficient of $\sin^2\theta$ in Eq. (6.18) will be multiplied by zero. Furthermore, $D(\theta=0)=(4R_G+R_L)^2$ so that Eq. (6.18) requires that $R_L=4R_G$. The optimum characteristic impedance is found by maximizing P_o with respect to Z_0, while this time keeping the line length $\neq 0$. The result is not surprising, as it is the geometric mean between the generator and load resistance:

$$Z_0=\sqrt{R_L/R_G}=2R_G \quad (6.19)$$

From Eq. (6.17) the output power when $Z_0=2R_G$ and $R_L=4R_G$ is

$$P_o=\frac{1}{2}\frac{|V_G|^2(1+\cos\theta)^2}{R_G(1+3\cos\theta)^2+4R_G\sin^2\theta} \quad (6.20)$$

This reduces to the usual form for the available power when $\theta\to 0$.

More complicated transmission line transformers might benefit from using SPICE to analyze the circuit. The analysis above gives a clue to how the values of Z_0 and the relative values of R_G and R_L might be chosen with the help of a low-frequency analysis.

As an example, consider the circuit in Fig. 6.9 again where $R_G=50\,\Omega$ so that $R_L=200\,\Omega$, $Z_0=\sqrt{50\times 200}=100\,\Omega$, the length of the transformer is 4 cm, and the frequency is 1.0 GHz. The return loss (= 20 log of the reflection coefficient) in Fig. 6.10 shows that in principle a good match is obtained even at 1 GHz.

The SPICE net list used to analyze this circuit makes use of the conversion of voltages to S parameters:

```
Analysis of a circuit for S11 and S21
*
* R01 and R02 are input and output resistance levels.
* RL is the load resistance.  The load may be supplemented
* with additional elements.
* Lines beginning with ** may be used for PSPICE instead
**.PARAM R01=50, R02=50.   RLOAD=50.    IN1=-1/R01
**.PARAM R01=50, R02=200.  RLOAD=200.   IIN=-1/R01
**.FUNC N(R01,R02) SQRT(R02/R01)
**R01   1   0   {R01}
r01     1   0   50
```

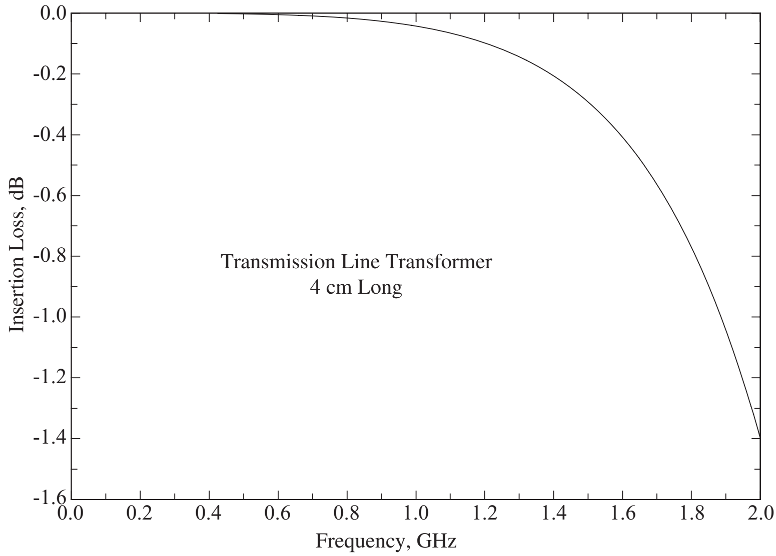

FIGURE 6.10 Return loss for frequency-dependent transmission line transformer of Fig. 6.9.

```
vin      10   11    ac   1
**GI1    1    0     VALUE={-V(10,11)/R01}
*gi1     1    0     10     11     "-1/R01"
gi1      1    0     10     11     -.02
e11      10   0     1      0     2
r11      11   0     1
xcircuit      1     2      tltckt
**RL     2    0     {RLOAD}
rl       2    0     200
**E21    21   0     VALUE={V(2)*2/N(R01,R02)}
*    n = SQRT(R02/R01)
*e21     21   0    2   0   "2/n"
e21      21   0    2   0   1
r21      21   0    1
*
.subckt  tltckt  1   4
* Input side
* 4  cm = 0.1333 wavelength  at  1  GHz
TLT4    1    0    4   1    Z0=100  F=1GHZ   NL=.1333
* Output side
.ends    tltckt
* Code  for  S11 and S21
*.AC DEC  "num"  "f1"    "f2"
.ac  lin  301  .1meg    2ghz
**.PROBE V(11)  V(21)
.end
```

6.5 BALUNS

A balun (balanced–unbalanced) is a circuit that transforms a balanced transmission line to an unbalanced one. An example of a balanced line is the two-wire transmission line. An unbalanced line is one where one of the lines is grounded, such as in coaxial line or microstrip. One situation where the balun plays an important role is in feeding a dipole antenna with a coaxial line where the antenna is balanced and the coaxial line is unbalanced. One simple structure is shown in Fig. 6.11 where the difference between the inputs of the antenna is forced to be 180° by addition of a half wavelength line between them. At radio frequencies, a more practical way to perform this same function is to use a transmission line transformer as shown in the example of the 1:1 balun in Fig. 6.12*a*. There is no specified ground on the right-hand side of this circuit, but since the voltage difference on the input side is V, the voltage across the load must also be V. For the dipole application, where a $+V$ is needed on one side and $-V$ on the other side, one of the output sides can be grounded as indicated in Fig. 6.12*b*. The (R_G:R_L = 1:4) balun in Fig. 6.13 shows that impedance matching and changing to a balanced line can be accomplished with a balun. Analysis of this circuit may be aided by assuming some voltage, V_x, at the bottom side of R_L. When the voltage at the top side of R_L is found, it also contains V_x. The difference between the bottom and top sides of R_L removes the V_x.

6.6 DIVIDERS AND COMBINERS

Transmission lines can be used to design power dividers and power combiners. These are particularly important in design of high-power solid-state RF amplifiers where the input can be split between several amplifiers or where the outputs of several amplifiers may be effectively combined into one load. A very simple two-way power divider is shown in Fig. 6.14. In this circuit $R_L = 2R_G$ and the transmission line characteristic impedance should be $Z_0 = \sqrt{2}R_G$. The current in R_n ordinarily would be 0 because of the equal voltages on either

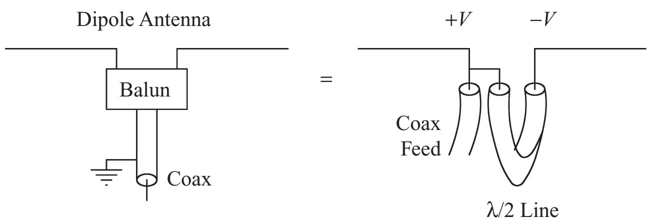

FIGURE 6.11 Balun example used for dipole antenna.

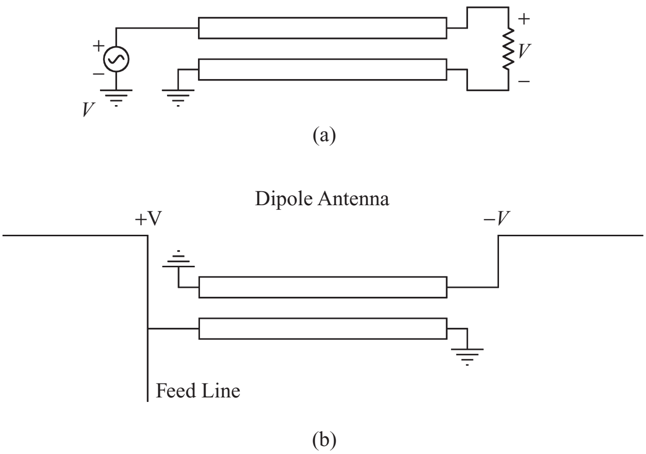

FIGURE 6.12 (*a*) Transmission line transformer implementation of a (1:1) balun, and (*b*) grounding one side gives a +V and −V to two sides of dipole antenna.

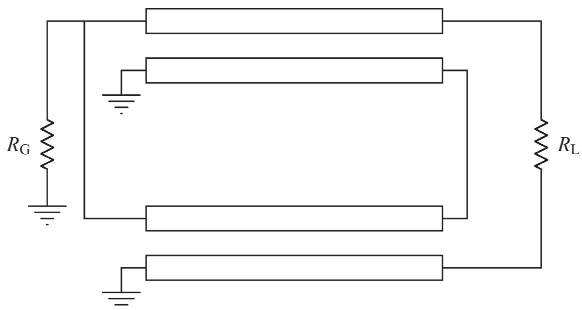

FIGURE 6.13 Balun with R_G:R_L = 1:4 impedance ratio.

side of that resistance. Under unbalanced load conditions, R_n can absorb some of the unbalanced power and thus protect the load. The two loads are both $2R_G$. The input voltage is V_1 on the top conductor, and the voltage on the bottom conductor is V_x (Fig. 6.14). On the right-hand side of the transmission line, the bottom conductor is V_1 and so the top conductor must be $2V_1 - V_x$ to ensure that both sides of the transmission line have the same voltage across the terminals, that is, $V_1 - V_x$. Since the current flowing through the top load resistor and the bottom load resistor must be the same, the voltage on either side of R_n is the same. Consequently, $2V_1 - V_x = V_x$ or $V_x = V_1$, so the voltage-to-current ratio at the load is

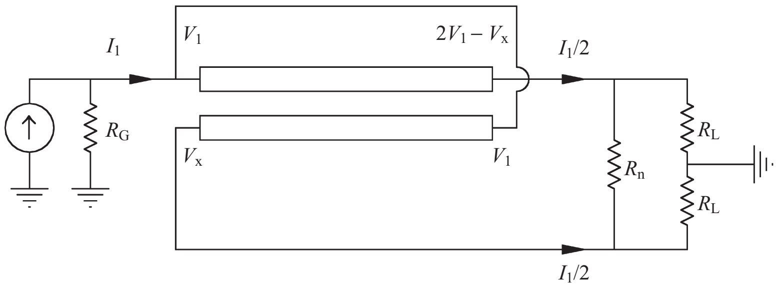

FIGURE 6.14 Two-way power divider.

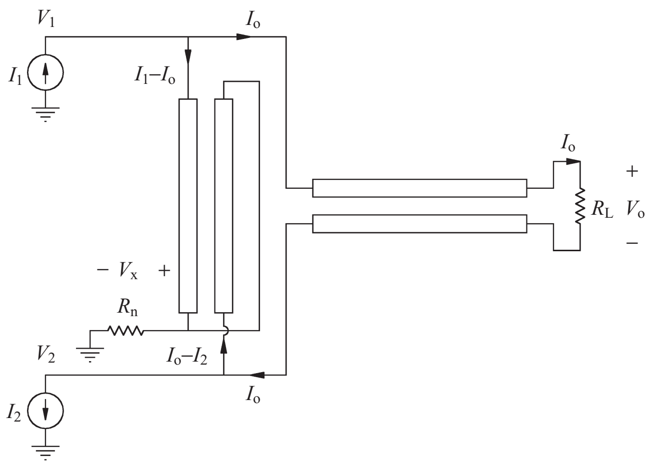

FIGURE 6.15 Two-way 180° power combiner.

$$R_L = \frac{V_1}{I_1/2} = 2R_G \tag{6.21}$$

A two-way 180° power combiner shown in Fig. 6.15 makes use of a hybrid coupler and a balun. The resistor, R_n, is used to dissipate power when the two inputs are not exactly equal amplitude or exactly 180° out of phase so that matched loading for the two sources is maintained. For example, consider when $I_1 = I_2$ as shown in Fig. 6.15 so that I_1 is entering the circuit and I_2 is leaving the circuit. The current flowing through the load, R_L, is I_o. The current flowing into the hybrid transmission line from the top is $I_1 - I_o$ while the current flowing into the bottom is $I_o - I_2$. The odd-mode current in the transmission line forces

$$I_1 - I_o = I_o - I_2$$

or

$$I_o = I_1 \tag{6.22}$$

All the current goes through the balun and no current flows through the hybrid. The current through R_n is therefore 0, leading to $V_x = 0$. The voltage difference between the two ends of the transmission lines of the hybrid is the same and implies that

$$V_1 - V_x = V_x - V_2$$

or

$$V_1 = -V_2 \tag{6.23}$$

and

$$V_o = V_1 - V_2 = 2V_1 \tag{6.24}$$

The matching load resistance is then

$$\frac{V_o}{I_o} = R_L = \frac{2V_2}{I_1} = 2R_G \tag{6.25}$$

When I_1 and I_2 are both entering the circuit so that $I_1 = -I_2$, and $V_1 = V_2$, then voltages across the top and bottom of the transmission line in the hybrid circuit of Fig. 6.15 are

$$V_1 - V_x = V_x - V_2$$

or

$$V_x = V_1 \tag{6.26}$$

The voltage across the load is $V_o = 0$ and $I_o = 0$. The current in the hybrid transmission line is I_1, so the current flowing through R_n is $2I_1$:

$$R_n = \frac{V_x}{2I_1} = \frac{V_1}{2I_1} = \frac{R_G}{2} \tag{6.27}$$

The choices for R_L and R_n assure impedance matching for an arbitrary phase relationship between I_1 and I_2. Optimum performance would be expected if the characteristic impedances of the transmission lines were

$$Z_{0\text{-balun}} = \sqrt{2}R_G \tag{6.28}$$

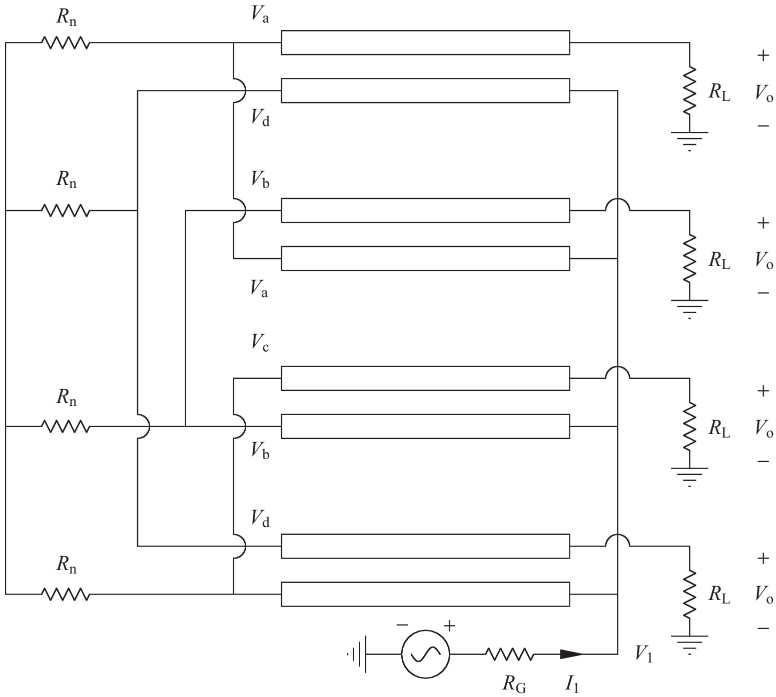

FIGURE 6.16 Four-way power divider.

$$Z_{0-\text{hybrid}} = R_G / \sqrt{2} \tag{6.29}$$

The four-way power divider illustrated in Fig. 6.16 has some similarities with the Wilkinson power divider used at microwave frequencies. In the Wilkinson divider, matching impedances between the input and output is done by choosing the quarter-wavelength transmission lines to have a characteristic impedance $Z_0 = \sqrt{N}R_G$ where N is the power division ratio, and $R_n = R_G$. In the present circuit, impedance matching is done using an impedance transformer at the voltage source (not shown in Fig. 6.16). If it is desired that all the output loads and voltages be equal to one another, then it follows that the currents in the R_n resistors is 0. This can be shown easily. The voltage difference between the conductors on the right-hand side in each of the transmission lines is $V_o - V_1$. Then for the left-hand side,

$$V_o - V_1 = V_a - V_d = V_b - V_a = V_c - V_b = V_d - V_c \tag{6.30}$$

Combining the second and third expressions, then the third and fourth expressions, and so on leads to the following:

$$2V_a = V_b + V_d \tag{6.31}$$

$$2V_b = V_c + V_a \tag{6.32}$$

$$2V_c = V_b + V_d \tag{6.33}$$

Equations (6.31) and (6.33) clearly show that $V_a = V_c$ and Eq. (6.32) shows $V_b = V_a$ and finally $V_d = V_a$. This means there is no current flowing in the R_n resistors and that on the right-hand side, $V_o = V_1$. The current entering each transmission line must then be $I_1/4$ where I_1 is the input current from the source. The load currents are also $I_1/4$ so the impedance transformation at the input requires $R_G = R_L/4$.

6.7 THE 90° COUPLER

The 90° coupler is commonly used to do power division and combining. This is a four-port lossless circuit in which power entering one port will divide between two output ports. The two output signals are 90° out of phase with one another. The fourth port is isolated from the input. The typical branch line or rat race coupler used at microwave frequencies use quarter-wavelength transmission lines. Even techniques such as capacitive loading or folding of transmission lines would still produce a cumbersome design in the lower RF range. A compact design using lumped capacitances and coupled inductors is given in [4, 5]. The coupled inductors are essentially an iron core transformer turned on its side. The four terminals become four ports when they are all referenced with respect to a ground plane. An alternative design that does not require coupled inductors is shown in Fig. 6.17 [6].

The circuit is excited with a voltage wave amplitude of ½ for the odd mode and ½ for the even mode. The superposition of inputs will give an input amplitude of 1 at port 1 and zero at port 4. The outputs at ports 2 and 3 are b_2 and b_3. The symmetry across a central horizontal line enables use of this odd- and even-mode analysis [7]. When ports 1 and 4 are excited in the odd mode,

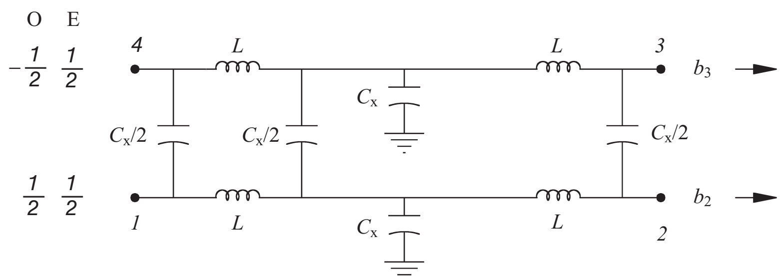

FIGURE 6.17 Four-port 90° lumped-element coupler design with odd- and even-mode excitation.

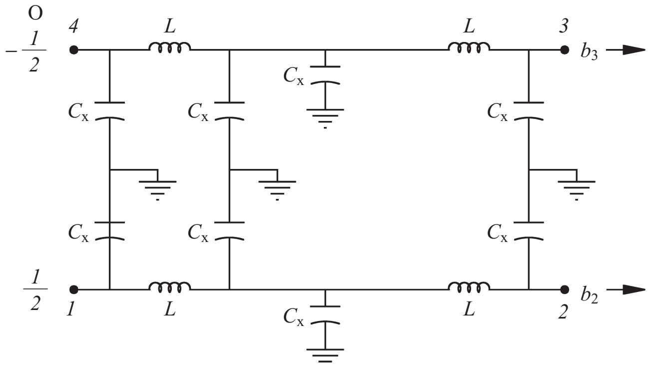

FIGURE 6.18 Half circuit that results from odd-mode excitation.

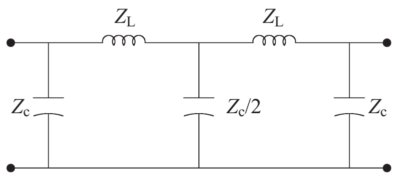

FIGURE 6.19 Odd-mode two-port half circuit.

voltages on the horizontal centerline are zero, so the voltages and currents in the circuit would not be affected if these points were all grounded. When ports 1 and 4 are excited in the even mode, no currents flow across the central line of symmetry, and these points can be open circuited without changing any of the internal voltages or currents. The odd-mode circuit is shown in Fig. 6.18.

The two halves of the circuit are each two-port circuits that can be analyzed separately, one of which is shown in Fig. 6.19 where $Z_L = sL$ and $Z_C = 1/sC_x$. The analysis process proceeds by (1) determine the $ABCD$ parameters of the circuit, (2) find the reflection and transmission coefficients of the four-port circuit, (3) repeat the process for the even-mode excitation, (4) specify that $b_1 = 0$, thereby enforcing a condition for impedance matching, (5) specify that $b_4 = 0$ to enforce isolation of the fourth port, (6) determine the values of Z_L and Z_C, and (7) show that b_2 and b_3 have equal amplitudes and a 90° phase difference.

For the circuit in Fig. 6.19, the $ABCD$ parameters are determined for the odd-mode circuit by standard circuit analysis:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{2Z_L^2}{Z_C^2} + \frac{4Z_L}{Z_C} + 1 \tag{6.34}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 2Z_L\left(\frac{Z_L}{Z_C} + 1\right) \tag{6.35}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{2}{Z_C}\left(\frac{Z_L}{Z_C}+2\right)\left(\frac{Z_L}{Z_C}+1\right) \tag{6.36}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{2Z_L^2}{Z_C^2}+\frac{4Z_L}{Z_C}+1 \tag{6.37}$$

The impedance, Z_0, is the impedance level to which the coupler is attached.

The reflection and transmission coefficients for the two-port circuit may be found from Table D.1 in Appendix D as $S_{11} = \Gamma$ and $S_{21} = T$. It should be noted that $AD - BC = 1$ for a linear reciprocal network. For the odd mode:

$$\Gamma_o = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \tag{6.38}$$

$$T_o = \frac{2}{A + B/Z_0 + CZ_0 + D} \tag{6.39}$$

It will be shown later that if $b_1 = b_4 = 0$, then $\Gamma_o = \Gamma_e = 0$. The latter is the even-mode reflection coefficient that is yet to be found. Since $A = D$ from Eqs. (6.34) and (6.37), then for Γ_o to be zero:

$$\frac{1}{Z_0^2} = \frac{C}{B} \tag{6.40}$$

The impedances in Fig. 6.19 are really reactances, which are $Z_L = jX_L$ and $Z_C = -jX_C$. Filling in these values for C/B gives

$$\left(\frac{X_L}{Z_0}\right)^2 = \frac{X_L}{X_C}\left(2-\frac{X_L}{X_C}\right) \tag{6.41}$$

Equation (6.41) was obtained by requiring a match at the input port so that $\Gamma_o = 0$. Since $A = D$ and $B/Z_0 = CZ_0$, the transmission coefficient is

$$T_o = \frac{1}{A + B/Z_0} \tag{6.42}$$

The denominator is complex, so after multiplying numerator and denominator by its complex conjugate and substituting the requirement for match, Eq. (6.41), the resulting real denominator can be shown to be equal to 1 and

$$T_o = \left(1 - 2\frac{X_L^2}{Z_0^2}\right) - j2\frac{X_L}{Z_0}\left(1-\frac{X_L}{X_C}\right) \tag{6.43}$$

This process for finding the odd-mode transmission coefficient for Fig. 6.19 must now be repeated for the even-mode circuit in Fig. 6.20. The even-mode

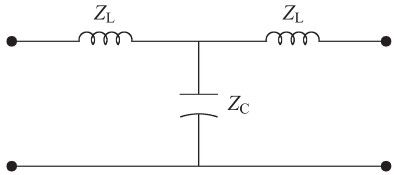

FIGURE 6.20 Even-mode two-port half-circuit.

circuit is found by putting an open circuit at the line of symmetry rather than a short circuit to ground. The *ABCD* parameters are found again by circuit analysis:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_L}{Z_C} + 1 \tag{6.44}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_L\left(\frac{Z_L}{Z_C} + 2\right) \tag{6.45}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_C} \tag{6.46}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{Z_L}{Z_C} + 1 \tag{6.47}$$

The requirement for match at port 1 and isolation at port 4 requires $B/Z_0 = CZ_0$ just as required for the odd mode. For the even-mode circuit:

$$\frac{Z_0^2}{X_C^2} = \frac{X_L}{X_C}\left(2 - \frac{X_L}{X_C}\right) \tag{6.48}$$

Then

$$T_e = \frac{1}{A + CZ_0} \tag{6.49}$$

$$T_e = 1 - \frac{X_L}{X_C} - j\frac{Z_0}{X_C} \tag{6.50}$$

Equation (6.50) for T_e is found by a similar process used to find T_o. When the coupler in Fig. 6.17 is excited as shown, the four outputs are

$$b_1 = \tfrac{1}{2}(\Gamma_e + \Gamma_o) = 0 \quad \text{matched} \tag{6.51}$$

$$b_2 = \tfrac{1}{2}(T_e + T_o) \tag{6.52}$$

$$b_3 = \tfrac{1}{2}(T_e - T_o) \tag{6.53}$$

$$b_4 = \tfrac{1}{2}(\Gamma_e - \Gamma_o) = 0 \quad \text{isolation} \tag{6.54}$$

The only way for b_1 and b_4 to be zero is for $\Gamma_o = \Gamma_e = 0$. The $\Gamma_o = 0$ requirement gave Eq. (6.41) and the $\Gamma_e = 0$ requirement gave Eq. (6.48). The variables X_L and X_C can be normalized with respect to Z_0 so that $\bar{X}_L = X_L/Z_0$ and $\bar{X}_C = X_C/Z_0$, From Eq. (6.41),

$$\bar{X}_L = \frac{2\bar{X}_C}{\bar{X}_C^2 + 1} \quad \text{odd mode} \tag{6.55}$$

and from Eq. (6.48)

$$\bar{X}_C = \frac{\bar{X}_L^2 + 1}{2\bar{X}_L} \quad \text{even mode} \tag{6.56}$$

Simultaneous solution gives

$$0 = \left(\bar{X}_L^2 - 1\right)\left(\bar{X}_L^2 + 3\right) \tag{6.57}$$

The only physically meaningful root is $\bar{X}_L = 1$, which implies $\bar{X}_C = 1$. From Eqs. (6.52) and (6.53),

$$b_{2,3} = \frac{1}{2}\left\{\left[1 - \frac{X_L}{X_C} - j\frac{Z_0}{X_C}\right] \pm \left[\left(1 - \frac{2X_L^2}{Z_0^2}\right) - j\frac{2X_L}{Z_0}\left(1 - \frac{X_L}{X_C}\right)\right]\right\} \tag{6.58}$$

This is evaluated for the known values of X_L and X_C giving:

$$b_2 = \frac{\sqrt{2}}{2}\angle -135° \tag{6.59}$$

$$b_3 = \frac{\sqrt{2}}{2}\angle -45° \tag{6.60}$$

Thus, the outputs are each 3 dB down from the input and are in phase quadrature.

This somewhat tedious process has yielded some fruit. It has shown how a compact 90° coupler might be designed for radio frequencies too low for quarter-wavelength transmission lines. It has also demonstrated how symmetry may be used for analysis of four-port networks.

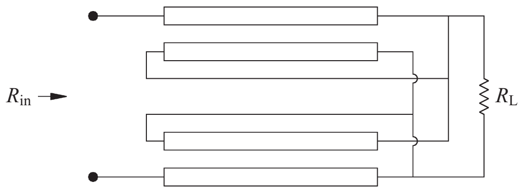

FIGURE 6.21 Transmission line transformer for Problem 6.1.

PROBLEMS

6.1. Indicate the direction of the currents in the transmission line transformer shown in Fig. 6.21. Determine the value of R_{in} in terms of R_L.

6.2. Design a transmission line transformer that matches a 250-Ω generator impedance to a 10-Ω load impedance. What characteristic impedance would you use for the transmission lines? Verify that your design gives the desired result.

6.3. For the 4:1 transformer shown in Fig. 6.2, find the output power, $P_o = |I_2|^2 R_L/2$, where the frequency dependence of the transmission lines is used. You will have three equations in the three unknowns I_1, I_2, and V_2. The final answer is similar to Eq. (6.17).

6.4. Design a transmission line transformer that matches a 200-Ω load to a 50-Ω source impedance. The transmission lines are to be 4 cm long, but the transmission line characteristic impedance can be chosen to give an acceptable match by not deviating from 50 Ω by more than 25 Ω to at least 2.5 GHz. Using SPICE, plot the return loss at the input side as a function of frequency. What is the return loss at 1 GHz?

6.5. Repeat Problem 6.4 for a transmission line transformer that matches 800 Ω to 50 Ω. The SPICE analysis should again show the return loss versus frequency. For this circuit, what is the return loss at 1 GHz?

6.6. Synthesize a transmission line transformer using the technique described in Section 6.3 to give a resistance ratio of $R_G:R_L = 25:16$. You may use the TLT symbol given in Fig. 6.6 as long as each symbol is defined by a known two-conductor transmission line.

6.7. Derive Eq. (6.50).

REFERENCES

1. P. L. D. Abrie, *The Design of Impedance-Matching Networks for Radio-Frequency and Microwave Amplifiers*, Dedham, MA: Artech, 1985.

2. H. L. Krauss, C. W. Bostian, and F. H. Raab, *Solid State Radio Engineering*, New York: Wiley, pp. 371–382, 1980.
3. E. Rotholz, "Transmission Line Transformers," *IEEE Trans. Microwave Theory Tech.*, **MTT-29**, pp. 327–331, April 1981.
4. G. F. Avitable, A. Cidronali, C. Salvador, and M. Speciale, "A Compact MMIC 90° Coupler for ISM Applications," *1997 MTT-S Digest*, Piscataway, NJ: IEEE Press, pp. 281–284, 1997.
5. R. W. Vogel, "Analysis and Design of Lumped- and Lumped-Distributed-Element Directional Couplers for MIC and MMIC Applications," *IEEE Trans. Microwave Theory Tech.*, **40**, pp. 253–262., 1992.
6. P. Vizmuller, *RF Design Guide: Systems, Circuits, and Equations*, Boston: Artech, p. 100, 1995.
7. J. Reed and G. J. Wheeler, "A Method of Analysis of Symmetrical Four-Port Networks," *IRE Trans. Microwave Theory Tech.*, **MTT-4**, pp. 246–252, 1956.