Brute Force Convex Hull

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Short description of the brute force convex hull algorithm implementation.

Summary of Algorithm:

For each pt (x_i, y_i):
    For each second pt (x_j, y_j):
        For each third pt (x_k, y_k):
            If $(x_k - x_i) * (y_j - y_i) - (y_k - y_i) * (x_j - x_i) >= 0$: **(1)**
                Then (x_k, y_k) is on same side of line as other pts
        If all third pts (x_k, y_k) for k = 1 to n are on same side
            Then allPtsOneSide = true
            If allPtsOneSide == true:
                Add line segment with pts (x_i, y_i) and (x_j, y_j) to the convex hull

Runtime: Three nested for loops from 1 to n imply $O(n^3)$

Explanation: For each pt given, we check the segment it makes with all other points. For each segment, we check if all points are on the same side of the line.

The key equation is $(x_k - x_i) * (y_j - y_i) - (y_k - y_i) * (x_j - x_i)$. **(1)**

We can FOIL this and get: $x_k y_j - x_k y_i - x_i y_j - y_k x_j + y_k x_i + y_i x_j$ **(2)**

Derivation: If we have two points (x_i, y_i) and (x_j, y_j), we have a line with the formula $ax + by = c$ where $a = y_j - y_i$ and $b = x_i - x_j$ and $c = x_i y_j - y_i x_j$.
Let (x_k, y_k) be the third point.
We can substitute the third point and a,b,c into $ax + by - c$ as follows:

$S = (y_j - y_i)x_k + (x_i - x_j)y_k - (x_i y_j - y_i x_j)$

$S = x_k y_j - x_k y_i + x_i y_j - x_j y_k - x_i y_j + y_i x_j$ which is equivalent to **(2)**.