CS23 Assignment Three

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1 Is $(1, 2, 3, 4) = (1, 2, 4, 3)$?

No. *Unlike sets, the order of elements in a tuple matters.*

2 Which of the following are equivalent:

$\{a, b, c\}, \{\{a, b\}, c\}, (a, b, c), (a, (b, c)), (b, c, a),$
$\{\{a, b, c\}\}, \{b, c, a\}, \{\}, \{\{\}\}$

$\{a, b, c\}$ **is equivalent to** $\{b, c, a\}$**.** *All other tuples and sets are inequivalent due to differences in strcture or order of elements.*

3 Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$ and define a relation R from A to B as follows: For every $(x, y) \in A \times B$, $(x, y) \in R$ means that $\frac{y}{x}$ is an integer.

a. Is $4R6$? Is $4R8$? Is $(3, 8) \in R$? Is $(2, 10) \in R$?

$\frac{6}{4} = 1.5$ *is not an integer.* **Therefore,** $4R6$ **is not true.** $\frac{8}{4} = 2$ *is an integer.* **Therefore,** $4R8$ **is true.** $\frac{8}{3}$ *is not an integer.* **Therefore,** $(3, 8) \in R$ **is not true.** $\frac{10}{2} = 5$ *is an integer.* **Therefore,** $(2, 10) \in R$ **is true.**

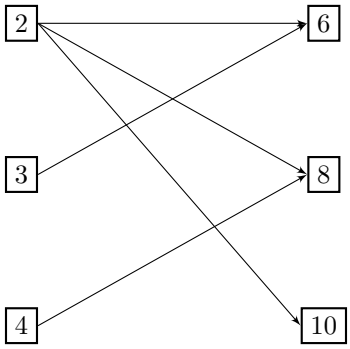
b. Write R as a set of ordered pairs.

$R = \{(2, 6), (2, 8), (2, 10), (3, 6), (4, 8)\}$.

c. Write the domain and co-domain of R.

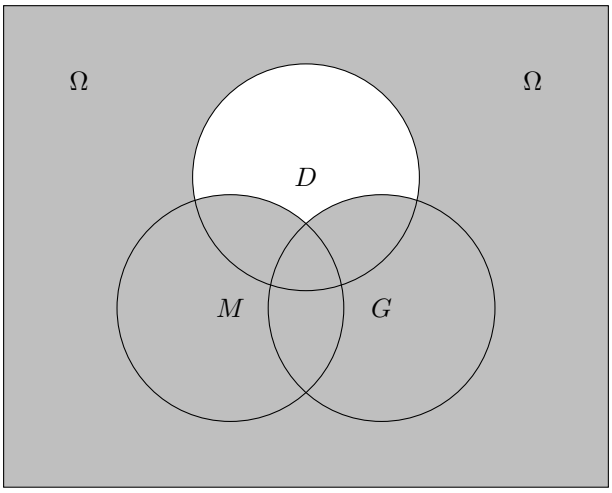
The domain of R *is* A *and the co-domain is* B.

d. Draw an arrow diagram for R.

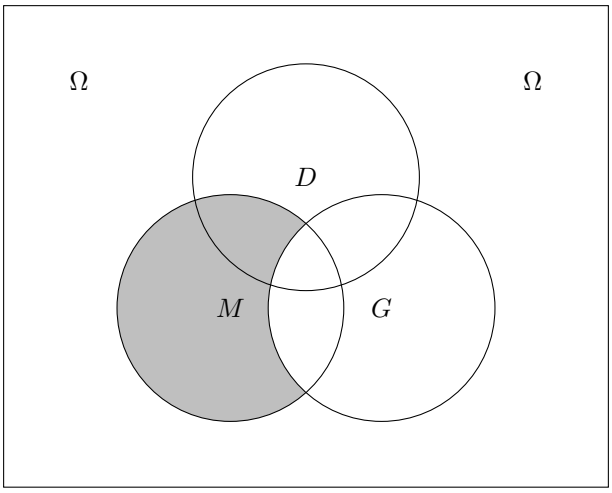


4 Given that Ω = all students in university, D = day students, M = mathematics majors, and G = graduate students, draw a venn diagram for this situation and copy it once for each of the given problems. Shade the following sets.

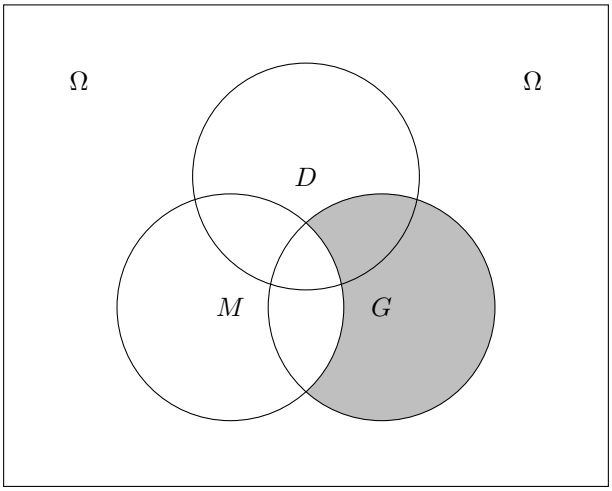
a. Evening and online students:



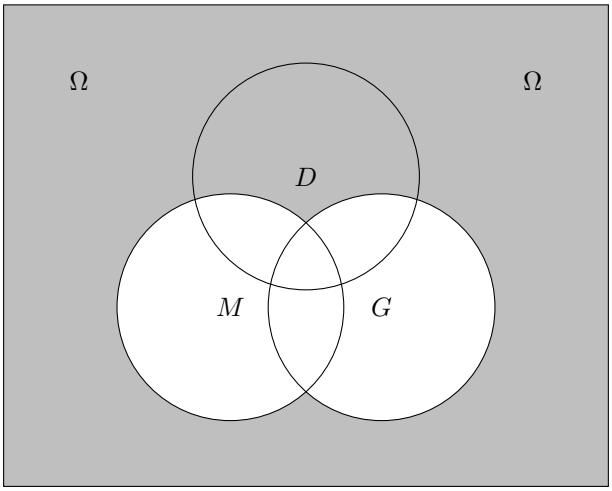
b. Undergraduate mathematics majors:



c. Non-math graduate students:



d. Non-math undergraduate students:



5 Two faces of a six sided dice are painted red, two are painted green and two are painted blue. The dice is rolled three times and the colors are recorded.

a. List the 27 possible outcomes in the format RGB (1st, 2nd, 3rd roll).

RRR, RRG, RRB, RGR, RGG, RGB, RBR, RBG, RBB, GRR, GRG, GRB, GGR, GGG, GGB, GBR, GBG, GBB, BRR, BRG, BRB, BGR, BGG, BGB, BBR, BBG, BBB.

b. Consider the event where all three rolls produce a different color. One outcome in the event is RGB. List all outcomes in the event and its probability.

RGB, GRB, GBR, BGR, RBG, BRG. $\frac{6}{27}$ **gives us a** 22.2% **probability.**

c. Consider the event where all two of the three rolls produce the same color. One outcome in the event is RRB. List all outcomes in the event and its probability.

RRG, RRB, RGR, RGG, RBR, RBB, GRR, GRG, GGR, GGB, GBG, GBB, BRR, BRB, BGG, BGB, BRR, BBG. $\frac{18}{27}$ **gives us a** 67.7% **probability.**

6 You have two hats. The first hat contains 2 orange marbles and 7 red marbles; the second hat contains 4 orange marbles and 3 red marbles. Bob selects a marble by first picking one of the two hats at random. He then selects one of the marbles in this hat at random. If Bob has selected a red marble, what is the probability that he selected a marble from the first hat?

To solve this problem, we can use Bayes' Theorem:

Function	Meaning	Probability
$Pr(H_1)$	First hat is picked	1/2
$Pr(H_2)$	Second hat is picked	1/2
$Pr(R\|H_1)$	Red marble in first hat	7/9
$Pr(R\|H_2)$	Red marble in second hat	3/7
$Pr(O\|H_1)$	Orange marble in first hat	2/9
$Pr(O\|H_2)$	Orange marble in second hat	4/7

$$Pr(H_1|R) = \frac{Pr(R|H_1) \cdot Pr(H_1)}{Pr(R)}$$

Solve for $Pr(R)$ using the total probability theorem:

$$Pr(R) = Pr(R|H_1) \cdot Pr(H_1) + Pr(R|H_2) \cdot Pr(H_2)$$

$$= \frac{7}{9} \cdot \frac{1}{2} + \frac{3}{7} \cdot \frac{1}{2} = \frac{7}{18} + \frac{3}{14} = \frac{38}{63}$$

Finish by plugging in known values:

$$Pr(H_1|R) = \frac{(7/9)(1/2)}{(38/63)} \approx 64.47\%$$

7 Suppose that the words "Act now" occur in 250 out of 2500 messages known to be spam and in 5 of 1000 messages known not to be spam. Estimate the probability that an incoming message containing the words "Act now" is spam, assuming that it is equally likely that an incoming message is spam or not spam. If our threshold for rejecting a message as spam is 0.9, will we reject this incoming message?

To solve this problem, we can use Bayes' Theorem:

Function	Meaning	Probability
$Pr(I)$	Informative (real) text	1/2
$Pr(S)$	Spam (useless) text	1/2
$Pr(A\|S)$	"Act now" in spam text	1/10
$Pr((\sim A)\|S)$	"Act now" NOT in spam text	9/10
$Pr(A\|I)$	"Act now" in real text	1/200
$Pr((\sim A)\|I)$	"Act now" NOT in real text	199/200
$Pr(R)$	Spam probabilirt rejection threshold	0.9

$$Pr(S|A) = \frac{Pr(A|S) \cdot Pr(S)}{Pr(A)}$$

Solve for $Pr(A)$ using the total probability theorem:

$$Pr(A) = Pr(A|I) \cdot Pr(I) + Pr(A|S) \cdot Pr(S)$$

$$= \frac{1}{200} \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{400} + \frac{1}{20} = \frac{21}{400}$$

Finish by plugging in known values:

$$Pr(S|A) = \frac{(1/10)(1/2)}{(21/400)} = \frac{400}{20 \cdot 21} \approx 95.24\% > Pr(R) = 90\%.$$

Since $Pr(S|A) > Pr(R)$, the text would be rejected by the system.