

CS23 Assignment Five

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April 16, 2024

1 If 12 people shake hands with each other, how many handshakes take place.

In order to solve this problem, we are looking for how many ways we can create groups of 2 people from the 12. This leads us to the combinations formula: nCr , where n is the total number of people and r represents a single handshake between 2 people. See below:

$$nCr = C(n, r) = \frac{n!}{r!(n-r)!} \rightarrow C(12, 2) = \frac{12!}{2!(12-2)!} = 66.$$

Thus, 66 handshakes take place.

2 In a group of six people, is it possible for everyone to be friends with exactly two other people in the group? How about three other people? Four?

While drawing out graphs is an option, an easier and more straightforward approach involves counting the number of degrees between each node. The sum of degrees must be always be even because it takes two nodes to form an edge. Let's apply this logic to the question. If each person is represented by a node, and each friendship by an edge, then the number of degrees on each node is the number of friends any given person has. Thus, we can evaluate the boolean of the following statement see if each case is possible:

True or false: For n nodes, each with r degrees, is $\frac{n \cdot r}{2} \in \mathbb{Z}$?

Number of People	Number of Friends	Statement	True or False
6	2	$(6 \cdot 2)/2 = 6$	True
6	3	$(6 \cdot 3)/2 = 9$	True
6	4	$(6 \cdot 4)/2 = 12$	True

Thus, we can see that each combination of number of friends is possible.