

CS23 Assignment Five

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1 If 12 people shake hands with each other, how many handshakes take place.

In order to solve this problem, we are looking for how many ways we can create groups of 2 people from the 12. This leads us to the combinations formula: nCr , where n is the total number of people and r represents a single handshake between 2 people. See below:

$$nCr = C(n, r) = \frac{n!}{r!(n-r)!} \rightarrow C(12, 2) = \frac{12!}{2!(12-2)!} = 66.$$

Thus, 66 handshakes take place.

2 In a group of six people, is it possible for everyone to be friends with exactly two other people in the group? How about three other people? Four?

While drawing out graphs is an option, an easier and more straightforward approach involves counting the number of degrees between each node. The sum of degrees must be always be even because it takes two nodes to form an edge. Let's apply this logic to the question. If each person is represented by a node, and each friendship by an edge, then the number of degrees on each node is the number of friends any given person has. Thus, we can evaluate the boolean of the following statement see if each case is possible:

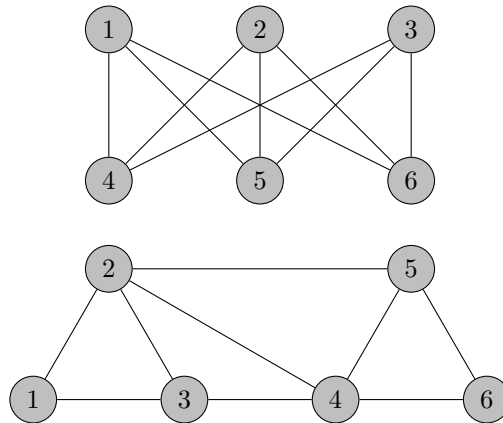
True or false: For n nodes, each with r degrees, is $\frac{n \cdot r}{2} \in \mathbb{Z}$?

Number of People	Number of Friends	Statement	True or False
6	2	$(6 \cdot 2)/2 = 6$	True
6	3	$(6 \cdot 3)/2 = 9$	True
6	4	$(6 \cdot 4)/2 = 12$	True

Thus, we can see that each combination of number of friends is possible.

3 Is it possible for two graphs with the same number of nodes and edges to not be isomorphic? What if the degrees of the nodes in the two graphs are the same? Give example graphs or explain why.

Take a look at the graphs below. They each have 6 nodes and 9 edges, yet are not isomorphic. While the sum of all node degrees in both graphs is 18, the bipartite graph's nodes are evenly distributed, whereas its opposition's are not. Therefore, if the node degrees differ, non-isomorphic graphs are clearly possible.



But what if all the nodes have the same degree? In that case, every graph with the same number of nodes and edges would be isomorphic. (Note: this logic requires the graph to be nondirectional. If the connections between nodes indicate or carry direction, then you could still find ways to create two or more non-isomorphic graphs.)

4 What is the largest number of edges possible in a graph with 10 vertices? What is the largest number of edges possible in a bipartite graph with 10 vertices? What is the largest number of edges possible in a tree with 10 vertices?

The first portion of this question is nearly identical in logic to Problem 1. The maximum number of edges possible in a graph with 10 vertices would be one where each vertex connects to each and every other vertex; Every person shakes hands with each and every other person. Thus:

$$nC_r = C(n, r) = \frac{n!}{r!(n-r)!} \rightarrow C(10, 2) = \frac{10!}{2!(10-2)!} = 45 \text{ edges.}$$

What about in a bipartite graph? In a bipartite graph, edges are formed between the top and bottom vertices. Thus, we can count the node degrees on either row, sum them up, and that will give us our edges. If V is our number of vertices, then we can form, at most, $V/2$ connections per node, and still have a bipartite graph. if we sum up this max per every node in either (but not both) row(s), we would sum the node degrees $V/2$ times. Thus, our equation becomes $(V/2)^2$. (Note: This calculation assumes that V is an even number.)

$$\left(\frac{V}{2}\right)^2 \rightarrow \left(\frac{10}{2}\right)^2 = 25 \text{ edges.}$$

What about in a tree graph? One of characterizing traits of tree graphs is that they do not contain cycles. In a tree graph, there will always be $(V - 1)$ edges, where V is the number of vertices. Therefore:

$$(V - 1) \rightarrow (10 - 1) = 9 \text{ edges.}$$