Math 4329 Mock Midterm 03

Name: ____

Student ID: ____

Number	Max Possible	Points
1	20	
2	30	
3	15	
4	20	
5	15	
Total	100	

1. (20 pts) (**Quadrature Rules**)

(a) Find c_1 and c_2 in the following quadrature formula:

$$\int_{-1}^{1} f(x)dx \approx c_1 f(-1) \ + \ c_2 f(1)$$

so that is exact for all polynomials of the largest degree possible. What is the degree of precision for this formula ?

(b) i. Approximate $\int_{-1}^{1} x^8 \, dx$ using the two point Gaussian quadrature rule with nodes $\pm 3^{-1/2}$ and weights 1.

ii. Calculate the exact integral $\int_{-1}^{1} x^8 \, dx$ and compare the error between the true value and the approximation obtained in 1(b)i.

2. (30 pts) Gaussian Elimination

(a) Use Gaussian elimination with back substitution to solve the system:

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 1 \\ 2x_1 + 6x_2 + 8x_3 &= 3 \\ 6x_1 + 8x_2 + 18x_3 &= 5 \end{aligned}$$

Please specify the multipliers m_{21}, m_{31} and m_{32}.

(b) Use the multipliers from the previous part (b) to form the LU factorization of the coefficient matrix of the linear system.

3. (15 pts) Consider the following table:

x	$f(x)$
0.3	1.5
0.4	2
0.5	5
0.6	7
0.7	12

where $x_{i+1} = x_i + h, \quad i = 0, 1, ..., 3..$

(a) Approximate $f'(0.5)$ using $D_h^+ f(0.5)$ and $h = 0.1$.

(b) Compute $D_h^{(2)} f(0.5)$ using the Central Difference Formula and step size $h = 0.2$.
<u>Note:</u> You may use the following formula for Central Difference Formula:

$$D_h^{(2)} f(x_1) = \frac{D_h^+ f(x_1) - D_h^- f(x_1)}{h}$$

(c) Compare the answer from (b) with the following approximation :

$$D_h^{(2)} f(x_1) = \frac{f(x_2) - 2f(x_1) + f(x_0)}{h^2},$$

with $x_1 = 0.5, \ h = 0.2$.

4. (20 pts) Consider the Jacobi and Gauss Seidel methods applied to solve the following system:

$$\begin{aligned} 3x_1 - x_2 &= -4, \\ 2x_1 + 5x_2 &= 2. \end{aligned}$$

Compute $\mathbf{x}_J^{(k)}$, $\mathbf{x}_{GS}^{(k)}$ for $k = 1, 2$ with initial guess $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}$.

Do we have convergence ?

5. (15 pts) State whether the following statement is true or false: Consider the following linear system:

$$\begin{aligned} x + y &= 0 \\ x + \frac{801}{800} y &= 1. \end{aligned}$$

The solution computed using Gaussian Elimination on a computer with three digits of significance is $x = -800, \ y = 800$.