Math 4329 **Homework 04**

Due on December 28th 2017

I. Consider the linear system written as

$$Ax = b. \tag{1}$$

The system is specified by A and b, with

$A = (a_{ij})_{i,j=1}^{n}$ being a $n \times n$ matrix, n is a positive integer with entries a_{ij} specified as

$$a_{ij} = \max\{i, j\},\ 1 \le i, j \le n, \quad \text{and}$$

$b = (b_i)_{i=1}^{n}$ with $b_i = 1\ \forall i = 1, \cdots, n.$

For $n = 2, 5, 10, 20$ please perform the following tasks:

1. Use the MATLAB function `inv(A)` to directly compute the inverse of `A` and thus the solution $\mathbf{x}_\mathbf{D}^{(n)}$ of the above linear system (1).
2. Use the MATLAB program `GEpivot.m` to solve the linear system (1). Call the computed solution $\mathbf{x}_\mathbf{P}^{(n)}$.
3. Modify the MATLAB program `GEpivot.m` so that no pivoting is used. Then use this modified program to solve the same linear system (1) by using Gaussian elimination without pivoting. Call the solution $\mathbf{x_{NP}}^{(n)}$.

   Please turn in the $\|\mathbf{x_D}^{(n)} - \mathbf{x_P}^{(n)}\|_\infty$, $\|\mathbf{x_D}^{(n)} - \mathbf{x_{NP}}^{(n)}\|_\infty$ comparing the solutions $\mathbf{x_P}^{(n)}$ and $\mathbf{x_{NP}}^{(n)}$ to the true value $\mathbf{x_D}^{(n)}$ for $n = 2, 5, 10, 20$. The output should look like

```
*******************************************************************
 n             ||xD - xP||                 ||xD - xNP||
*******************************************************************
 2
 5
10
20
*******************************************************************
```

   You can compute the $||\cdot||_\infty$ by using the MATLAB function `norm(xD-xP, Inf)` to compute the error for $\|\mathbf{x_D}^{(n)} - \mathbf{x_P}^{(n)}\|_\infty$ etc.

II. Consider the linear system given below:

$$x_1 + x_2 + 20x_3 = 5 \tag{2}$$

$$5x_1 - 3x_2 + x_3 = 2 \tag{3}$$

$$2x_1 - 4x_2 + x_3 = 1 \tag{4}$$

The above system (2)–(4) can be expressed as $Ax = b$.

1. Compute the inverse of A by hand to obtain the exact solution to (2)–(4).
2. Please set up the Gauss Jacobi and Gauss Seidel Methods for the system (2)–(4).
3. Will Gauss Jacobi and Gauss Seidel Methods converge for any choice of initial guess x_1^0, x_2^0, x_3^0?
4. Interchange the rows of the system of linear equations (2)–(4) to obtain a system with a strictly diagonally dominant coefficient matrix A. **Please explicitly mention the rows that you have interchanged.**
5. Apply one step of the Gauss-Seidel method to approximate the solution to two significant digits. Assume an initial approximation of $x_1 = x_2 = x_3 = 0$.