Exam 2

Graded

Student

Hasin Raihan

Total Points

93.84 / 100 pts

Question 1

(no title) **2** / 2 pts

✔ **– 0 pts** Correct

Question 2

(no title) **2** / 2 pts

✔ **– 0 pts** Correct

Question 3

(no title) **2** / 2 pts

✔ **– 0 pts** Correct

Question 4

(no title) **2** / 2 pts

✔ **– 0 pts** Correct

Question 5

(no title) **2** / 2 pts

✔ **– 0 pts** Correct

Question 6

(no title) **2** / 2 pts

✔ **– 0 pts** Correct

Question 7

(no title) **3** / 4 pts

✔ **– 1 pt** did not use unit vector, or small mistake

Question 8

(no title) **3** / 4 pts

✔ **– 1 pt** small mistake

2 it's e^z, not e^x

Question 9

(no title) 2.67 / 4 pts

✔ – 1.33 pts Erroneous limits of integration

Question 10

(no title) 2.67 / 4 pts

✔ – 1.33 pts Erroneous limits of integration

Question 11

(no title) 4 / 4 pts

✔ – 0 pts Correct

Question 12

(no title) 4 / 4 pts

12.1 (no title) 2 / 2 pts

✔ – 0 pts Correct

12.2 (no title) 2 / 2 pts

✔ – 0 pts Correct

Question 13

(no title) 20.5 / 22 pts

13.1 (no title) 9.5 / 11 pts

Integral Set Up

✔ – 1.5 pts Erroneous limits of integration

1

Integral Evaluation

✔ – 0 pts Correct

13.2 (no title) 11 / 11 pts

Integral Set Up

✔ – 0 pts Correct

Integral Evaluation

✔ – 0 pts Correct

Question 14

(no title) **21** / 21 pts

✔ **+ 21 pts** Correct

+ 4 pts Correct first partial derivatives

+ 4 pts Correct second partial derivatives and D

+ 5 pts Find critical points

+ 8 pts Correctly classify critical points and find local extreme values

− 2 pts Did not write the max/min *values* but only the points (or did not write the saddle points but only its values)

− 3 pts one misclassification or missing classification

+ 0 pts miswrite saddle point

+ 2 pts partial credit for critical points/ partial derivatives

− 2 pts computational mistake

+ 4 pts partial credit for classification

+ 0 pts no work toward solution

+ 2 pts minimal partial credit on classifying the critical points

− 1 pt notational mistake or small mistake

Question 15

(no title) **21** / 21 pts

✔ **− 0 pts** Correct

MA-UY 2114 BCD2 EXAM 2 NOVEMBER 13, 2023

Print Name: Hasin Raihan Section: C

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Directions: You have **110 minutes** to answer the following questions. ***You must show all your work where stated*** as neatly and clearly as possible and indicate the final answer clearly. No calculator may be used. Cell phones and other electronic devices may **NOT** be used during the exam.

Problem	Points
1 – 6	2 pts each
7 –12	4 pts each
13	22
14	21
15	21
Total	100

Answer the following questions. No partial credit will be given for the multiple choice or fill in the blank. You do not need to show your work for questions 1–6. Be sure to fill in your bubbles completely.

? 1. Which of the following is the domain of $f(x,y) = \ln(x - y - 1)$?

- ○ All $(x,y) \in \mathbb{R}^2$ such that $y = x - 1$
- ○ All $(x,y) \in \mathbb{R}^2$ such that $y \neq x - 1$
- ○ All $(x,y) \in \mathbb{R}^2$ such that $y > x - 1$
- ● All $(x,y) \in \mathbb{R}^2$ such that $y < x - 1$
- ○ None of the above.

$x - y - 1 \leq 0$

$\cancel{\ln}(0)$

$x - y - 1 \leq 0$

$x - y \leq 1$

$x - 1 \leq y \rightarrow x - 1 > y$

2. If $xy^2z + z^2e^x = 1$, then $\partial z/\partial y$ is

- ○ $-\dfrac{y^2z + z^2e^x}{xy^2 + 2ze^x}$
- ○ $\dfrac{y^2z + z^2e^x}{xy^2 + 2ze^x}$
- ● $-\dfrac{2xyz}{xy^2 + 2ze^x}$
- ○ $\dfrac{2xyz}{xy^2 + 2ze^x}$
- ○ None of the above.

$xy^2z + z^2e^x - 1 = 0$

$\dfrac{-F_y}{F_z} = -\dfrac{2yxz}{xy^2 + 2ze^x}$

3. Let $f(x,y,z) = xe^y + yz^4\sin(x)$. Then f_{xxz} is

- ● $-4yz^3\sin(x)$
- ○ e^y
- ○ 0
- ○ xe^y
- ○ None of the above.

$f_x = e^y + yz^4\cos(x)$

$f_{xx} = -yz^4\sin(x)$

$f_{xxz} = -4yz^3\sin(x)$

4. Find the differential of the function $z = y\sin(e^x)$.

○ $dz = ye^x\cos(e^x) + \sin(e^x)$

○ $dz = ye^x\cos(e^x)\sin(e^x)\,dxdy$

● $dz = ye^x\cos(e^x)\,dx + \sin(e^x)\,dy$

○ $dz = y\sin(e^x)\,dxdy$

○ None of the above.

$dz = f_x\,dx + f_y\,dy$

$= y\cos(e^x)e^x\,dx + \sin(e^x)\,dy$

5. Use the chain rule to find $\partial z/\partial s$ when $z = e^r\cos\theta$, $r = st$, $\theta = \sqrt{s^2+t^2}$.

○ $e^r t\sqrt{s^2+t^2}\cos\theta$

○ $e^r s\cos\theta - e^r t\sin\theta\,(s^2+t^2)^{-1/2}$

● $e^r t\cos\theta - e^r s\sin\theta\,(s^2+t^2)^{-1/2}$

○ 0

○ None of the above.

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta}\cdot\frac{\partial \theta}{\partial s}$

$e^r\cos\theta\, t \quad - \quad e^r\sin\theta$

$(s^2+t^2)^{1/2} \to \frac{1}{2}(s^2+t^2)^{-1/2}\cdot 2s$

6. What is the maximum rate of change of $f(x,y) = 2x + e^{-y^2}$ at the point $(1,0)$?

● 2

○ $\langle 1, 0\rangle$

○ 1

○ $\left\langle 1, -2ye^{-y^2}\right\rangle$

○ None of the above.

$\|\nabla f\|$

$\nabla f = \langle 2, e^{-y^2}(-2y)\rangle$

$\nabla f(1,0) = \langle 2, 0\rangle$

$\|\nabla f\| = \sqrt{2^2+0^2} = \sqrt{4} = \underline{2}$

For the following questions, fill in the provided box.

7. Find the directional derivative of $f(x,y) = x^2y - xy^2$ at $P(1,0)$ in the direction of $\mathbf{i} - \mathbf{j}$. Write your answer in the box provided.

$\langle 1, -1 \rangle$

$\nabla f(x,y) = \langle 2xy - y^2, x^2 - 2xy \rangle$

$\nabla f(1,0) = \langle 2(1)(0) - 0^2, 1^2 - 2(1)(0) \rangle$

$= \langle 0, 1 \rangle$

$\boxed{-1}$

$\langle 0, 1 \rangle \cdot \langle 1, -1 \rangle$

$1 \cdot 0 + 1(-1)$

$0 + (-1) = -1$

8. Find the equation of the tangent plane to $x^2 + y = z + e^z$ at the point $(1,0,0)$. Write your answer in the box provided.

$0 = f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0)$

$0 = x^2 + y - z - e^x$

$f_x = 2x - e^x = 2 - e$

$f_y = 1$

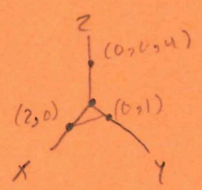
$f_z = -1$

$0 = 2 - e(x-1) + y - z$

$z = y + (2-e)(x-1)$

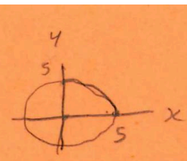
$\boxed{z = y + (2-e)(x-1)}$

9. Set up a double integral to find the volume of the solid below the plane $x + 2y + z = 4$ and above the triangular region in the xy-plane with vertices $(0,0), (2,0), (0,1)$. Write your answer in the box provided. You must write all of your limits of integration out and your differentials for credit.



$\boxed{\int_0^1 \int_0^2 (4 - x - 2y)\, dx\, dy}$

$z = 4 - x - 2y$



10. Convert the following integral to polar coordinates. Write your answer in the box.

$$\int_0^5 \int_0^{\sqrt{25-x^2}} (x^2+y^2)\, dy dx$$

$x = r\cos\theta$
$y = r\sin\theta$
$x^2+y^2=r^2$

$\tan\frac{y}{x} = \theta$

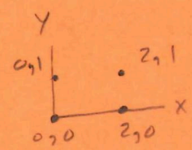
$x^2+y^2 = 25$
$y = \sqrt{25-x^2}$

$dV = r\,dr\,d\theta$

$\int_{x=0}^{x=5} \int_{y=0}^{y=\sqrt{25-x^2}} (x^2+y^2)\,dy\,dx$

$$\boxed{\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{25-r^2\cos^2\theta}} r^3\, dr\, d\theta}$$

11. The density of a rectangular lamina with vertices $(0,0), (2,0), (2,1)$ and $(0,1)$ is given to be $\rho(x,y) = xy$. Set up an integral that gives the mass of the lamina. Write your answer in the box provided. You must write all of your limits of integration out and your differentials for credit.



Mass = $\boxed{\int_0^1 \int_0^2 (xy)\, dx\, dy}$

12. Write the equation $z^2 = 4 - x^2 - y^2$ in cylindrical and spherical coordinates. Simplify your answer.

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$z^2 = 4 - r^2\cos^2\theta - r^2\sin^2\theta$
$4 - r^2(\cos^2\theta + \sin^2\theta)$
$z^2 = 4 - r^2(1)$

Cylindrical: $\boxed{z^2 = 4 - r^2}$

Spherical: $\boxed{\rho^2 = 4}$

$x = \rho\sin\phi\cos\theta$
$y = \rho\sin\phi\sin\theta$
$z = \rho\cos\phi$

$\rho^2\cos^2\phi = 4 - \rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta$
$= 4 - \rho^2\sin^2\phi\,(\cos^2\theta + \sin^2\theta)$
$\rho^2\cos^2\phi = 4 - \rho^2\sin^2\phi$
$\rho^2\cos^2\phi + \rho^2\sin^2\phi = 4$

$\rho^2 = 4$

For the following questions, you must show all of your work where stated. Any answer given without valid supporting work will not receive any credit.

13. (a) Use cylindrical coordinates to evaluate $\iiint_E z\,dV$, where E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

$z = z$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2=r^2$

$z = r^2$

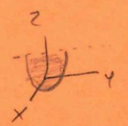
$4 = x^2+y^2$

$0 \le z \le 4$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$$\int_0^4 \int_0^{2\pi} \int_0^2 r^2 \, r\,dr\,d\theta\,dz$$



$$= \int_0^4 \int_0^{2\pi} \int_0^2 r^3\,dr\,d\theta\,dz$$

$dV = r\,dr\,d\theta$

$$\int_0^4 \int_0^{2\pi} \left(\frac{r^4}{4} \Big|_{r=0}^{r=2} \right) d\theta\,dz$$

$$\int_0^4 \left(4\theta \Big|_{\theta=0}^{\theta=2\pi} \right) dz$$

$$\int_0^4 8\pi\,dz \to 8\pi z \Big|_{z=0}^{z=4} = 8\pi(4)$$

32π

Answer $\boxed{32\pi}$

(b) Use spherical coordinates to evaluate $\iiint_E (x^2+y^2+z^2)^{3/2}\,dV$ where E is the region above $z = \sqrt{x^2+y^2}$ and below $x^2+y^2+z^2 = 1$.

$z = \rho\cos\phi$

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$x^2+y^2+z^2=\rho^2$

$\rho\cos\phi = \rho\sin\phi$

$1 = \tan\phi$

$\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

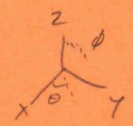
$\rho^2 = 1$

45 $\frac{\pi}{4}$

$$\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^1 (\rho^2)^{3/2} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

$\downarrow$

$\rho^3 \cdot \rho^2$



$$\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^1 \rho^5 \sin\phi\,d\rho\,d\phi\,d\theta$$

$$\frac{\rho^6}{6}\sin\phi \Big|_{\rho=0}^{\rho=1} \to \int_0^{\frac{\pi}{4}} \frac{\sin\phi}{6}\,d\phi\,d\theta$$

$$\frac{1}{6}(-\cos\phi)\Big|_{\phi=0}^{\phi=\frac{\pi}{4}}$$

$$-\frac{\sqrt{2}}{2} - (-1)$$

$$\frac{1}{6}\left(-\frac{\sqrt{2}}{2}+1\right)$$

Answer $\boxed{\dfrac{-\sqrt{2}+2}{12}(2\pi)}$

$$\int_0^{2\pi} \frac{-\sqrt{2}+2}{12}\,d\theta \to \frac{-\sqrt{2}+2}{12}\,\theta \Big|_{\theta=0}^{\theta=2\pi}$$

$$\frac{-\sqrt{2}+2}{12}$$

14. Find the local maximum and minimum values and saddle points of the function given below. Show all of your work and justify all of your answers. If an answer does not exist, write DNE in the box provided.

$$f(x, y) = x^3 + y^3 - 3x^2 - 3y^2 - 9y$$

Find critical points

$f_x = 3x^2 - 6x$

$f_y = 3y^2 - 6y - 9$

$f_{xx} = 6x - 6$

$f_{yy} = 6y - 6$

$f_{xy} = 0$

$-1-3(1)+9$

$-4+9$

$8+\cancel{27}-12-\cancel{27}-9(3)$

$-4-27$

$f(0,-1) = 5$

$f(2,3) = -31$

$f_x = 0 \rightarrow 3x^2-6x=0$

$f_y = 0 \rightarrow 3y^2-6y-9=0$

$3(y^2-2y-3)=0$ $\quad 3(x^2-2x)=0$

$y^2-2y-3=0$ $\quad x^2-2x=0$

$(y-3)(y+1)=0$ $\quad x^2=2x$

$y=3, y=-1$ $\quad x=0$, $x=2$

points $= (0,3)$

$= (0,-1)$

$= (2,3)$

$= (2,-1)$

$D > 0 \quad f_{xx} < 0 \quad$ max

$D > 0 \quad f_{xx} > 0 \quad$ min

$D < 0 \quad$ saddle

$D = f_{xx}f_{yy} - f_{xy}^2$

$(0,3)$?

$D = (-6)(12) - 0^2 < 0$ saddle

$(0,-1)$?

$D = (-6)(-12) > 0$

$f_{xx} < 0$

$(2,3)$?

$D = (6)(12) - 0^2 > 0$

$f_{xx} > 0$

$(2,-1)$

$D = (6)(-12) - 0^2 < 0$

saddle

Local maximum value(s): $(0,-1) \rightarrow$ value $= 5$

Local minimum value(s): $(2,3) \rightarrow$ value $= -31$

Saddle point(s): $(0,3), (2,-1)$

15. Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y) = 1 + e^{-xy}$ subject to the constraint $4x^2 + y^2 = 1$

$\nabla f = \lambda \nabla g$

$\langle -ye^{-xy}, -xe^{-xy} \rangle = \langle \lambda(8x), \lambda(2y) \rangle$

$-ye^{-xy} = \lambda 8x \longrightarrow \lambda = \dfrac{-ye^{-xy}}{8x}$

$-xe^{-xy} = \lambda 2y \qquad \lambda = \dfrac{-xe^{-xy}}{2y}$

$\dfrac{-y\cancel{e^{-xy}}}{8x} = \dfrac{-x\cancel{e^{-xy}}}{2y}$

$-2y^2 = -8x^2$

$8x^2 = 2y^2$

$4x^2 = y^2$

constraint

$4x^2 + y^2 = 1$

$4x^2 + 4x^2 = 1$

$8x^2 = 1 \to x^2 = \frac{1}{8}$

$x = \pm \frac{1}{\sqrt{8}}$

$4(\frac{1}{8}) = \frac{1}{2} = y^2$

$y = \pm \frac{1}{\sqrt{2}}$

Critical points:

$f(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}) = 1 + e^{-\frac{1}{4}}$

$f(-\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{2}}) = 1 + e^{\frac{1}{4}}$

$f(\frac{1}{\sqrt{8}}, -\frac{1}{\sqrt{2}}) = 1 + e^{1/4}$

$f(-\frac{1}{\sqrt{8}}, -\frac{1}{\sqrt{2}}) = 1 + e^{-1/4}$

Maximum value: $\boxed{1 + e^{1/4}}$

Minimum value: $\boxed{1 + e^{-1/4}}$