## MA-UY 2114 Formula Sheet

Here are some potentially useful formulas for the exam. Note that you may not need to use them, they are here so you do not need to memorize them. You must know the context in which each is applied.

$$\bullet \ \operatorname{comp}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|}$$

$$\bullet \operatorname{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \mathbf{a}$$

• 
$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c}$$

• 
$$\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1, \qquad 0 \le t \le 1$$

• 
$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

$$\bullet \ \mathbf{T} = \frac{\mathbf{r}'}{\|\mathbf{r}'\|}$$

$$\bullet \ \ \mathbf{N} = \frac{\mathbf{T}'}{\|\mathbf{T}'\|}$$

• 
$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \left\|\frac{d\mathbf{T}}{ds}\right\| = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

• 
$$\mathbf{a} = v'\mathbf{T} + \kappa v^2\mathbf{N} = a_T\mathbf{T} + a_N\mathbf{N}$$

• 
$$\int_C f(x,y)dx = \int_a^b f(x(t),y(t))x'(t)dt$$

• 
$$D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$$

• 
$$D = D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

• 
$$A(S) = \iint_D \sqrt{1 + z_x^2 + z_y^2} dA$$

• 
$$A(S) = \iint_D \|\mathbf{r}_u \times \mathbf{r}_v\| dA$$

• 
$$\int_{C} f(x,y)ds = \int_{a}^{b} f(x(t),y(t)) \sqrt{\left(\frac{dx}{dt}\right)^{2} + \left(\frac{dy}{dt}\right)^{2}} dt$$

• 
$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$$

• 
$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n} \ dS$$

• 
$$\iint_{S} \mathbf{F} \cdot d\mathbf{S} = \iint_{D} (-Pg_x - Qg_y + R) dA$$

• 
$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

• 
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt = \int_C \mathbf{F} \cdot \mathbf{T} ds = \int_C P dx + Q dy + R dz$$

• 
$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

$$\bullet \ \frac{\partial(x,y)}{\partial(u,v)} = x_u y_v - x_v y_u$$

• 
$$\iint_{S} f(x, y, z) dS = \iint_{D} f(\mathbf{r}(u, v)) \|\mathbf{r}_{u} \times \mathbf{r}_{v}\| dA$$

• curl 
$$\mathbf{F} = (R_y - Q_z)\mathbf{i} + (P_z - R_x)\mathbf{j} + (Q_x - P_y)\mathbf{k}$$

- Stokes' Theorem Integral:  $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$
- Divergence Theorem Integral:  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F} dV$