Example Solution

GLMS - Exponential Families and Link Functions Stat 495

The text presents exponential families via a formula relating any two densities in the family via a renormalized exponential tilt. There are other ways to recognize that a density is in an exponential family.

Suppose X is a random variable with density given by: $f(x \mid \theta) = a(\theta)b(x)\exp[c(\theta)d(x)]$, where a() and c() are functions only of the parameter theta, and b() and d() are functions of x (the data). If X's density can be written in this form, then the density is in an exponential family.

For example, for the Bernoulli distribution, some re-writing enables us to see that:

$$f(x \mid p) = p^x(1-p)^{1-x} = (1-p)\left(\frac{p}{1-p}\right)^x = (1-p)\exp\left[x\log\left(\frac{p}{1-p}\right)\right].$$

Here, $a(p) = 1-p, b(x) = 1, c(p) = \log\left[\frac{p}{1-p}\right], d(x) = x$. Hint: $r = \exp(\log(r))$.

The link function relates the parameters of the distribution to a linear function of X. The link function is c(). What is the link function for the Bernoulli distribution? Where have you seen this before?

$$c(\pi) = \log\left(\frac{\pi}{1-\pi}\right) = \text{logit}(\pi)$$

Logistic Regression

Verify that the Poisson density can be written in this form.

Poisson $\quad$ Poi(μ) $\quad$ $\frac{e^{-\mu}\mu^x}{x!}$

$$f(x \mid \mu) = \frac{e^{-\mu}\mu^x}{x!} = e^{-\mu}\frac{1}{x!}e^{x\log\mu}$$

$a(\mu) = e^{-\mu}$ $\quad$ $c(\mu) = \log\mu$

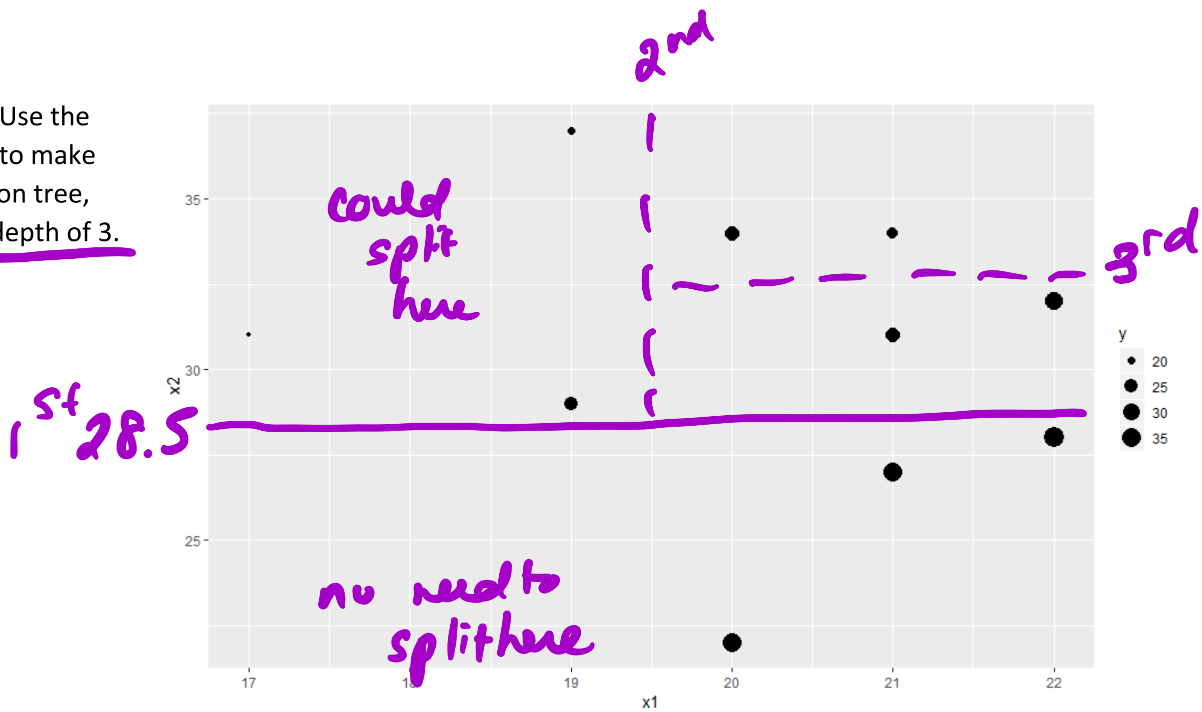
$b(x) = \frac{1}{x!}$ $\quad$ $d(x) = x$

What is the link function for the Poisson distribution? Your textbook calls these the natural parameters (the lambdas).

$$c(\mu) = \log\mu$$

Regression trees. Use the data set provided to make your own regression tree, with a maximum depth of 3.

x1	x2	y
17	31	19
19	37	20
19	29	25
20	22	34
20	34	27
21	34	22
21	31	26
21	27	35
22	28	37
22	32	33



(You can aim for this to be a "good" tree, but don't try to maximize any particular criteria through computation. Guestimate / eye-ball the cutoffs and write down what your tree would be.)