Chapter 15 Large-Scale Hypothesis Testing and False-Discovery Rates

<u>Notation</u>

$n =$

$N =$

$\alpha =$

FDR $=$

$p_{(i)} =$

$\mathcal{H}_{0(i)} =$

What does large-scale refer to?

15.1 - 15.2 We outline 4 procedures related to adjusting for multiple testing covered in these sections.

1. No adjustment. Reject $\mathcal{H}_{0(i)}$ if $p_{(i)} \le \alpha$.
2. Bonferroni. Reject $\mathcal{H}_{0(i)}$ if $p_{(i)} \le \frac{\alpha}{N}$.
3. Holm's procedure.
   a. Order p-values from smallest to largest
   $$p_{(1)} \le p_{(2)} \le \ldots \le p_{(N)}$$
   b. Let i_0 be the smallest index i such that $p_{(i)} > \frac{\alpha}{(N-i+1)}$.
   c. Reject $\mathcal{H}_{0(i)}$ for $i < i_0$.
4. Benjamini - Hochberg FDR procedure.
   a. Order p-values as in Holm's.
   b. Define i_{max} to be the largest index for which $p_{(i)} \le \frac{i}{N} \cdot q$.
   $q = 0.1$ is typical practice.
   c. Reject $\mathcal{H}_{0(i)}$ for $i \le i_{max}$. Call results "interesting" rather than "significant."

Be careful with the text's "acceptance" language.

Now we compare / contrast these methods.

FWER stands for...

Procedure #1 does NOT control FWER.

Procedures #2 and #3 control FWER @ level ___, but the difference btw them is...

The False discovery proportion is $\frac{\#\text{ true null hyp. rejected}}{\#\text{ total rejections}}$. This is Fdp.

For a decision rule D, how is FDR(D) related to Fdp(D)?

Procedure #4 controls FDR @ level q, ie. $FDR(D) \leq q$.

Example

In a setting where $n = 20$ and $N = 100$, with $\alpha = .05$ and $q = 0.1$, the smallest 15 p-values were:

0.00005 0.00016 0.00196 0.00214 0.00694 0.00963 0.01256 0.01657
0.02804 0.04022 0.04024 0.04345 0.05524 0.05822 0.06142

Apply the four procedures. How many times do you reject H_0 for each?

15.3

The theoretical development in this section uses cdfs and Bayes rule.

Let z_0 be a threshold and z_i be the test statistic for the i^{th} case.

$Fdr(z_0) = P(\text{case } i \text{ is null} \mid z_i \geq z_0)$

$Fdr(z_0)$ is the ______ false-discovery rate, as contrasted

with FDR which is ____________.

Can obtain an empirical Bayesian estimate of $Fdr(z_0)$, $\widehat{Fdr}(z_0)$.

Concludes FDR control relates to Bayes posterior probability of nullness.

$\widehat{Fdr}(z_0)$ rejects $\mathcal{H}_{0(i)}$ when...

(general, no formula)

15.4

$Fdr(z_0)$ is based on ____ areas. This is not desirable from a Bayesian perspective. Instead, we can define

$fdr(z_0) = P(\text{case } i \text{ is null} \mid z_i = z_0)$ as the ______ false-discovery rate.

We can get reasonable empirical Bayes estimates of fdr.

How are $Fdr(z_0)$ and $fdr(z_0)$ related?

15.5

Using the empirical null dist. means the significance of an outlying case is judged relative to the dispersion of the majority, not a theoretical ideal.

What are some reasons to doubt the theoretical null in large-scale situations?

1.

3.

2.

4.

Should you always expect to need to adjust the theoretical null?

15.6

List four "big" take-away messages from the chapter summary.

1.

2.

3.

4.