School of Electronics and Computer Science
University of Southampton

COMP3206/6229 (2015/16): Machine Learning Assignment 20%

Issue	Thursday, 19 November 2015
Deadline	Friday, 11 December 2015 (12:00 Noon)

- This is an assessment and you are expected to work independently.
- Spend no more than 20 hours on this assignment. Parts of it are designed to build on previous short tasks you have done. Please revise those exercises.

Neural Network Approximation

Artificial neural networks are good function approximators. In the case of pattern recognition, they are used as estimators of posterior probabilities of class membership.
Consider a two-class pattern classification problem in two dimensions, in which each class is Guassian distributed with distinct means and covariance matrices $\mathcal{N}(\boldsymbol{m}_j, \boldsymbol{C}_j)$. Take

$$\boldsymbol{m}_1 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \quad \boldsymbol{C}_1 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad \boldsymbol{m}_2 = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{C}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- Compute the posterior probability on a regular grid in the input space and plot the decision boundary for which the posterior probability satisfies $P[\omega_1 \,|\, \boldsymbol{x}] = 0.5$. Show 100 samples drawn from each of the classes superposed on the same graph. Draw the posterior probability as a three dimensional graph.
- Using the data sampled from each of the distributions, train a feedforward neural network using `MATLAB`'s `Neural Networks` toolbox. The following are commands you are likely to use.

```
[net] = feedforwardnet(20)
[net] = train(net, X, y)
[output] = net(x)
```

  Evaluate the network output on a regular grid, and plot the decision contour. How well does it compare with the Bayes' optimal boundary? Compare the approximation at two different sizes of the network.

Time Series Prediction

Time series prediction is an important application of machine learning techniques. The prediction problem is formulated as a regression problem. Given a series, denoted by $s(n), n = 1, ..., N$, we wish to predict the n^{th} sample $s(n)$ from p samples that occurred in the past ($p << N$): $[s(n-1)\ s(n-2)\ ...\ s(n-p)]$.

Chaotic Time Series

The Mackey-Glass model [1] is a popular chaotic time series. It is obtained by integrating the nonlinear differential equation,

$$\frac{dx}{dt} = \frac{a\, x(t-\tau)}{1 + x(t-\tau)^{10}} - b\, x(t),$$

which, for certain parameter values gives sustained oscillatory signals.

- Obtain the functions `mgfunc.m`, `mackeyglass_rk4.m` and `mackeyglass_eq` from `MathWork`'s contributed software site `http://www.mathworks.co.uk/matlabcentral/fileexchange/24390-mackey-glass-time-series-generator`, and run `mgfunc` to generate a time series of 2000 samples.
- Use the firt $N = 1500$ samples to train a prediction model and the remaining 500 as test data. With $p = 20$, construct the design matrix and output of a regression problem. Your input matrix will have $N - p + 1$ rows and p columns, with each row being a time shifted version of the previous one.
- Estimate a linear predictor from the training data and check how well it does *one step ahead* prediction on the test data.
- Train a feedforward neural network and evaluate how well it performs on one step ahead prediction.
- Use the trained neural network in a free running mode, feeding back predicted outputs feeding back into the input and check if sustained oscillations are possible [2][1].

Financial Time Series

Obtain daily `FTSE100` data for the past five years from a financial data provider *e.g.* `finance.yahoo.com`. Formulate a neural network predictor that predicts tomorrows FTSE index value from 20 past trading days. Use market `Close` prices. Was there an opportunity to make money using your knowledge of neural networks? Does the ability to predict stock index improve if you were to use past values of `Volume Traded` information as additional input?

Report

Present your work in a report of no more than six pages.

Marking Scheme

In each part, 3/5 marks will be awarded for completing the work correctly. 5/5 may be gained for *something extra* in the form of insightful discussion of the problem and its solution.

Neural network approximation	5
Mackey-Glass Time Series	5
Financial Time Series	5
Clarity of presentation	5

References

[1] M. C. Mackey and L. Glass, "Oscillation and chaos in physiological control systems," *Science*, vol. 197, no. 4300, pp. 287–289, 1977.

[2] E. A. Wan, "Modeling nonlinear dynamics with neural networks: Examples in time series prediction," in *Proceedings of the Fifth Workshop on Neural Networks*, pp. 327–232, 1993.

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[1] A copy of [2] is available in the course notes page as `Wan1993.pdf`