**Hello Everyone,**

**I hope you all are fine and doing well.**

# MABAC (Multi-Attributive Border Approximation Area Comparison) Models

MABAC（多屬性邊界近似區域比較）模型

# Theoretical Framework (MABAC Model)

理論架構（MABAC模型）

The Fuzzy MABAC (Multi-Attributive Border Approximation Area Comparison) model incorporates fuzzy logic into the traditional MABAC approach. This is particularly useful when dealing with uncertainty or imprecision in decision-making, allowing for a more flexible evaluation of alternatives.

模糊 MABAC（多屬性邊界近似區域比較）模型將模糊邏輯融入傳統的 MABAC 方法中。這在處理決策中的不確定性或不精確性時特別有用，可以更靈活地評估替代方案。

## Fuzzy MABAC Model Steps

**Decision Matrix Construction:** Create a matrix where rows represent alternatives and columns represent criteria, using fuzzy numbers.

**Fuzzy Normalization:** Normalize the fuzzy decision matrix to bring all criteria to a common scale.

## 模糊 MABAC 模型步驟

決策矩陣構造：使用模糊數建立一個矩陣，其中行代表備選方案，列代表標準。

模糊標準化：標準化模糊決策矩陣，使所有標準達到共同的尺度。

**Weight Assignment:** Assign weights to each criterion, also using fuzzy numbers.

**Fuzzy Border Approximation Area Calculation:** Calculate performance scores for each alternative using the weighted normalized values.

**Ranking of Alternatives:** Rank the alternatives based on their calculated performance scores.

權重分配：為每個標準分配權重，也使用模糊數。
模糊邊界近似面積計算：使用加權歸一化值計算每個備選方案的效能分數。
備選方案排名：根據計算的效能分數對備選方案進行排名。

# Mathematical Framework (MABAC Model)

數學框架（MABAC 模型）

The MABAC (Multi-Attributive Border Approximation Area Comparison) model is a multi-criteria decision-making method used to evaluate and rank alternatives based on multiple attributes. Below is the mathematical expression of the MABAC model followed by a detailed example.

MABAC（多屬性邊界近似區域比較）模型是一種多標準決策方法，用於基於多個屬性對備選方案進行評估和排序。以下是 MABAC 模型的數學表達式，並附有詳細的範例。

# Mathematical Expressions of MABAC

**1. Decision Matrix Construction**: Construct a decision matrix $A$ where $A = [a_{ij}]$:

- $i = 1,2, \ldots, m$ (alternatives)
- $j = 1,2, \ldots, n$ (criteria)
- Each element $a_{ij}$ represents the performance of alternative $i$ on criterion $j$.

MABAC 的數學表達式

1. 決策矩陣建構：建構決策矩陣$A$，其中$A = [a_{ij}]$：
   - $i = 1,2, \ldots, m$（替代方案）
   - $j = 1,2, \ldots, n$（標準）
   - 每個元素$a_{ij}$代表替代方案$i$在標準$j$上的表現。

**2. Normalization**: Normalize the decision matrix to make the criteria comparable. This can be done using min-max normalization:

歸一化：將決策矩陣進行歸一化，使標準具有可比較性。這可以使用最小-最大歸一化來完成：

$$a'_{ij} = \frac{a_{ij} - \min(a_{*j})}{max(a_{*j}) - min(a_{*j})} \; for\ benefit\ criteria$$

$$a'_{ij} = \frac{max(a_{*j}) - a_{ij}}{max(a_{*j}) - min(a_{*j})} \; for\ cost\ criteria$$

**3. Weight Assignment**: Assign weights $w_j$ to each criterion such that:

權重分配：為每個標準分配權重 $w_j$，以便：

$$\sum_{j=1}^{n} w_j = 1$$

**4. Border Approximation Area Calculation**: Compute the performance score for each alternative:

邊界近似面積計算：計算每個備選方案的效能得分：

$$B_i = \sum_{j=1}^{n} w_j a'_{ij}$$

**5. Ranking of Alternatives**: Rank the alternatives based on their scores $B_i$.

備選方案排名：根據得分$B_i$對備選方案進行排名。

# Example

例子

Let's evaluate three projects (A, B, C) based on two criteria: **Cost** (lower is better) and **Expected Profit** (higher is better).

## Step 1: Decision Matrix

讓我們根據兩個標準來評估三個項目（A、B、C）：成本（越低越好）和預期利潤（越高越好）。

第 1 步：決策矩陣

| Project | Cost (in thousands) | Profit (in thousands) |
|---|---|---|
| A | 100 | 150 |
| B | 80 | 120 |
| C | 120 | 200 |

## Step 2: Normalize the Decision Matrix

步驟 2：標準化決策矩陣

**Criteria**:

標準：

- Cost is a cost criterion (lower is better).

成本是一個成本標準（越低越好）。

- Profit is a benefit criterion (higher is better).

利潤是一個效益標準（越高越好）。

**Normalize Cost**:

標準化成本：

$$a'_{A1} = \frac{120 - 100}{120 - 80} = \frac{20}{40} = 0.5$$

$$a'_{B1} = \frac{120 - 80}{120 - 80} = \frac{40}{40} = 1$$

$$a'_{C1} = \frac{120 - 120}{120 - 80} = \frac{0}{40} = 0$$

利潤標準化

**Normalize Profit:**

$$a'_{A2} = \frac{150-120}{200-120} = \frac{30}{80} = 0.375$$

$$a'_{B2} = \frac{120-120}{200-120} = 0$$

$$a'_{C2} = \frac{200-120}{200-120} = \frac{80}{80} = 1$$

## Step 3: Weight Assignment

Assume the following weights:

- Weight for Cost ($w_1$) = 0.6
- Weight for Profit ($w_2$) = 0.4

## 第 3 步：權重分配

假設以下權重：

- 成本權重 ($w_1$) = 0.6
- 利潤權重 ($w_2$) = 0.4

## Step 4: Calculate Border Approximation Area

- **Calculate for Project *A*:**

$$B_A = w_1 . a'_{A1} + w_2 . a'_{A2}$$
$$B_A = 0.6 \cdot 0.5 + 0.4 \cdot 0.375 = 0.3 + 0.15 = 0.45$$

- **Calculate for Project *B*:**

$$B_B = w_1 . a'_{B1} + w_2 . a'_{B2}$$
$$B_B = 0.6 \cdot 1.0 + 0.4 \cdot 0 = 0.6 + 0 = 0.6$$

- **Calculate for Project *C*:**

$$B_C = w_1 . a'_{C1} + w_2 . a'_{C2}$$
$$B_C = 0.6 \cdot 0 + 0.4 \cdot 1 = 0 + 0.4 = 0.4$$

## Step 5: Ranking of Alternatives

Now, we rank the projects based on their $B_i$ values:

- **Project B**: $B_B$=0.6
- **Project A**: $B_A$=0.45
- **Project C**: $B_C$=0.4

## 第 5 步：備選方案排名

現在，我們根據項目的 $B_i$值對項目進行排名：

- 項目B：$B_B$=0.6
- 項目A：$B_A$=0.45
- 項目C：$B_C$=0.4

## Conclusion

Based on the MABAC model, **Project B** is identified as the best option, followed by **Project A**, while **Project C** is the least favorable. This example illustrates the step-by-step application of the MABAC technique for multi-criteria decision-making.