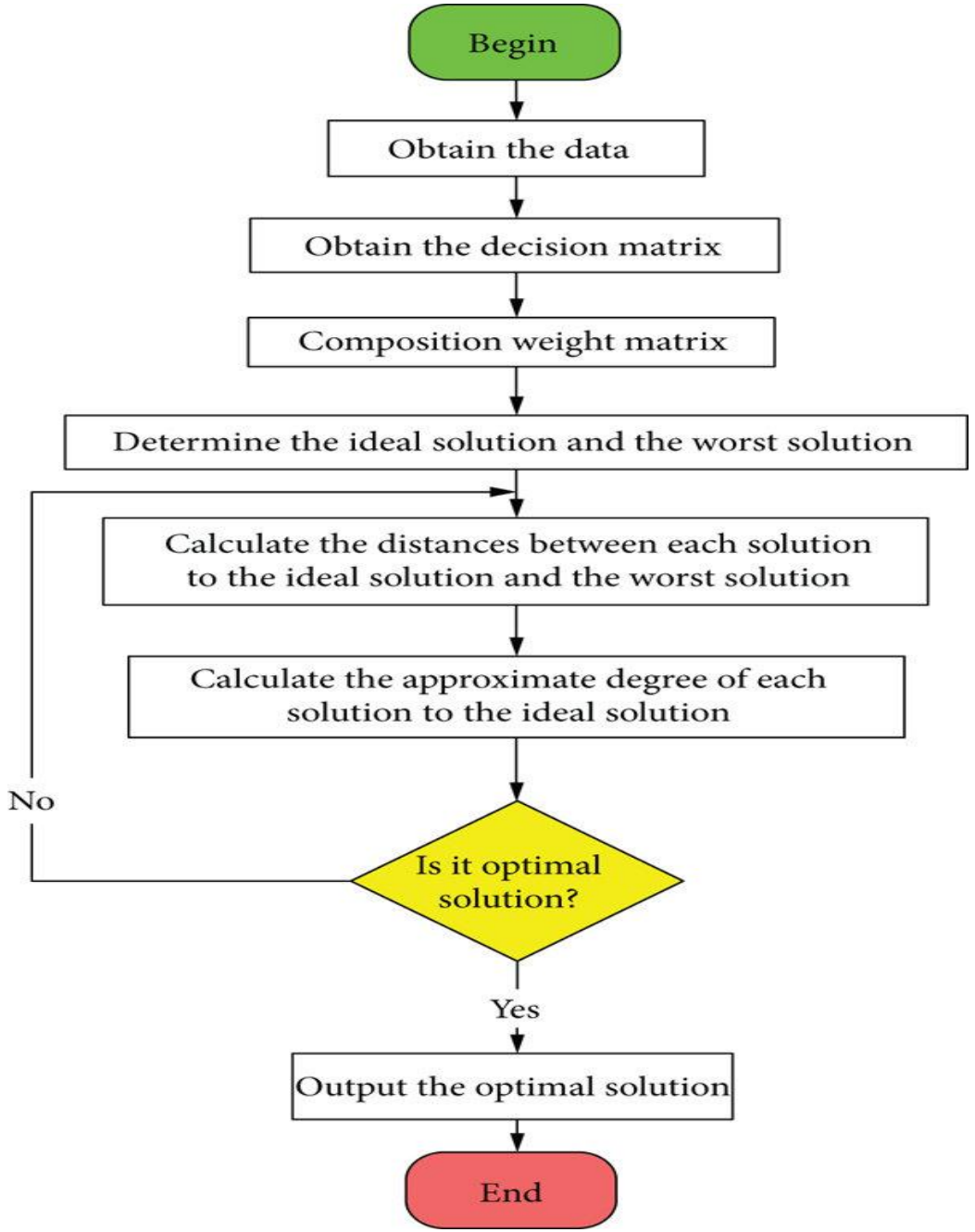
Hello Everyone,

I hope you all are fine and doing well.

TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) Method

TOPSIS（透過與理想解決方案的相似性進行優先排序的技術）方法



Geometrical form of TOPSIS Method

TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is a multi-criteria decision-making method. It helps to rank alternatives based on their distance from an ideal solution. Here are the mathematical steps involved in the TOPSIS technique, along with an example.

TOPSIS（透過與理想解決方案的相似性進行偏好排序的技術）是一種多標準決策方法。它有助於根據與理想解決方案的距離對備選方案進行排名。以下是 TOPSIS 技術中涉及的數學步驟以及一個範例。

Steps of TOPSIS

Construct the Decision Matrix: Create a decision matrix D where rows represent alternatives and columns represent criteria.

TOPSIS的步驟

建立決策矩陣：建立決策矩陣D，其中行代表替代方案，列代表標準。

Normalize the Decision Matrix: Normalize the decision matrix to ensure that different criteria can be compared. The normalized value r_{ij} is calculated using:

$$r_{ij} = \frac{x_{ij}}{\left(\sum_{k=1}^{m} x_{kj}\right)^{\frac{1}{2}}}$$

where x_{ij} is the original value, m is the number of alternatives, and j is the criterion.

標準化決策矩陣：標準化決策矩陣以確保可以比較不同的標準。標準化值r_{ij}的計算方法為：

$$r_{ij} = \frac{x_{ij}}{\left(\sum_{k=1}^{m} x_{kj}\right)^{\frac{1}{2}}}$$

其中x_{ij}是原始值，m是替代方案的數量，j是標準。

Weight the Normalized Matrix: Multiply the normalized values by the weights w_j assigned to each criterion:

$$v_{ij} = w_j . r_{ij}$$

對歸一化矩陣進行加權：將歸一化值乘以分配給每個標準的權重 w_j：

$$v_{ij} = w_j . r_{ij}$$

Determine the Ideal and Negative-Ideal Solutions: Identify the ideal (best) and negative-ideal (worst) solutions:

1) Ideal solution A^+: The best value for each criterion.

2) Negative-ideal solution A^-: The worst value for each criterion.

確定理想和負理想解決方案：確定理想（最佳）和負理想（最差）解決方案：

1）理想解A^+：每個標準的最佳值。

2）負理想解A^-：每個標準的最差值。

Calculate the Separation Measures: Calculate the distance of each alternative from the ideal and negative-ideal solutions:

計算分離度量：計算每個備選方案與理想解和負理想解的距離：

$$S_i^+ = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2 \right)^{\frac{1}{2}}$$

$$S_i^- = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2 \right)^{\frac{1}{2}}$$

Calculate the Relative Closeness to the Ideal Solution: Compute the relative closeness of each alternative to the ideal solution:

計算與理想解決方案的相對接近度：計算每個替代方案與理想解決方案的相對接近度：

$$C_i^* = \frac{S_i^-}{S_i^+ + S_i^-}$$

Rank the Alternatives: Rank the alternatives based on C_i^*. The higher the value, the better the alternative.

對替代方案進行排名：根據C_i^*對替代方案進行排名。數值越高，替代方案越好。

Example

Let's consider a simple example with three alternatives and three criteria.

Step 1: Decision Matrix

$$D = \begin{bmatrix} 7 & 9 & 6 \\ 8 & 6 & 7 \\ 9 & 7 & 8 \end{bmatrix}$$

where rows are alternatives A_1, A_2, A_3 and columns are criteria C_1, C_2, C_3.

Step 2: Normalize the Decision Matrix

$$r_{ij} = \frac{x_{ij}}{\left(x_{1j}^2 + x_{2j}^2 + x_{3j}^2\right)^{\frac{1}{2}}}$$

Calculating for each element, we get:

$$D = \begin{bmatrix} 0.424 & 0.547 & 0.408 \\ 0.485 & 0.365 & 0.48 \\ 0.545 & 0.457 & 0.552 \end{bmatrix}$$

Step 3: Weight the Normalized Matrix: Assume weights $w = [0.5,0.3,0.2]$:

$$V = R.w = \begin{bmatrix} 0.424 * 0.5 & 0.547 * 0.3 & 0.408 * 0.2 \\ 0.485 * 0.5 & 0.365 * 0.3 & 0.48 * 0.2 \\ 0.545 * 0.5 & 0.457 * 0.3 & 0.552 * 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.212 & 0.164 & 0.082 \\ 0.243 & 0.11 & 0.096 \\ 0.273 & 0.137 & 0.11 \end{bmatrix}$$

Step 4: Determine Ideal and Negative-Ideal Solutions

1) Ideal solution $A^{+} = (0.273, 0.164, 0.110)$.

2) Negative-ideal solution $A^{-} = (0.212, 0.110, 0.082)$.

Step 5: Calculate Separation Measures

$$S_i^+ = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2 \right)^{\frac{1}{2}}$$

$$S_i^- = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2 \right)^{\frac{1}{2}}$$

Calculate for each alternative:

1) S_1^+ and S_1^-.

2) S_2^+ and S_2^-.

3) S_3^+ and S_3^-.

Step 6: Relative Closeness

$$C_i^* = \frac{S_i^-}{S_i^+ + S_i^-}$$

Step 7: Rank the Alternatives Rank alternatives based on C_i^*.

Conclusion

By following these steps, you can apply the TOPSIS technique to rank various alternatives based on multiple criteria effectively. The process is systematic and can be applied in various fields, including business, engineering, and environmental studies.