Question 4

(a)

Let $P = \{(s_1, f_1), (s_2, f_2), ..., (s_n, f_n)\}$. Let $d = (s_d, f_d) \in P$ such that f_d is minimal. Then, there exists a set $S \subset P$ where $d \in S$ and for each $(s_i, f_i) \in S$, $f_i < s_{i+1}$, where $|S|$ is maximal.

(b)

If $d \in S$ then we are done, so suppose it is not. We want to construct a set $S\prime \subset P$ so that each $(s_i\prime, f_i\prime) \in S\prime$, $f_i\prime < s_{i+1}\prime$ and $d \in S\prime$, where $|S\prime|$ is maximal. Suppose we order the pairs in S by their end time, ie, each $f_i < f_{i+1}$, then the first element, say $v = (s_v, f_v) \in S$ has the lowest f, so for each $(s_i, f_i) \in S$, $f_v < f_i$ and $f_v < s_i$. But, we know that $f_d < f_v$, since f_d must be the lowest f in P. So, if we let $S\prime = \{S \backslash v\} \cup d$, then for each $(s_i\prime, f_i\prime) \in S\prime$, we have $f_d < f_i\prime$ and $f_d < s_i\prime$. So, then reordering $S\prime$ to be sorted by s, we get that for each $(s_i\prime, f_i\prime) \in S\prime$, $f_i\prime < s_{i+1}\prime$. Next, we can see that $|S\prime| = |S| - 1 + 1 = |S|$, so $|S\prime|$ is maximal.