Question 1

Proof:

(1) Let P(n) be the predicate "when the input to mySum() is n, this function terminates and returns 2n(n+1)(2n+1)." We will prove P(n) for all $n \geq 1$.

(2) Base Case: Assume the input to mySum() is 1. Then the condition on line 3 evaluates to true, so line 4 is executed and the function returns 12. Note that,

$$2n(n+1)(2n+1) = 2(1)(1+1)(2(1)+1) = 2(2)(3) = 12$$

So, P(1) is true.

(3) Induction Step: Let $n \geq 1$ be arbitrary and assume P(n). We will prove P(n+1), which states

$$2(n+1)(n+1+1)(2(n+1)+1)$$

(4) Suppose mySum() is called with input n+1. Then the parameter k has value $n+1 > 1$ when line 3 is reached, so the if-condition on line 3 evaluates to false. Thus line 4 is skipped and line 6 will be executed, so the parameter k still has value n+1 when line 6 is reached.

(5) In line 6, mySum() is called with input n+1-1 = n. By the inductive hypothesis, this function terminates and returns the value 2n(n+1)(2n+1). Thus, the value returned on line 6 is

$$2n(n+1)(2n+1) + 12(n+1)(n+1)$$

$$= (n+1)(2n(2n+1) + 12(n+1))$$

$$= (n+1)(4n^2 + 2n + 12n + 12)$$

$$= (n+1)(4n^2 + 8n + 6n + 12)$$

$$= (n+1)(4n(n+2) + 6(n+2))$$

$$= (n+1)(n+2)(4n+6)$$

$$= 2(n+1)(n+2)(2n+3)$$

$$= 2(n+1)(n+1+1)(2(n+1)+1)$$

Proving P(n+1).

(6) So, by induction, P(n) is true for all $n \geq 1$. Hence the given function is fully correct.