Question 1

Assume that the concerts in P are sorted by start time.

(a)

The items in a solution S are concerts/pairs (s_i, f_i) from P to attend, where s represents the concerts' start time, and f represent the concerts' end time. Note that $s_i < f_i$ must be true.

A feasible solution S is a set of concerts/pairs, where $S \subset P = \{(s_1, f_1)(s_2, f_2), ..., (s_n, f_n)\}$. Where for any $(s_i, f_i) \in S$, $f_i < s_{i+1}$. That is, the previous concert must end before the next concert starts, ie, no concerts overlaps.

An optimal solution is a feasible solution S, where $|S|$ is maximal. That is, S is the set of concerts to attend to ensure the most concerts are attended.

(b)

Given a solution $S \subset P = \{(s_1, f_1)(s_2, f_2), ..., (s_n, f_n)\}$, let's remove an arbitrary concert $C = (s_c, f_c) \in S$ and the concerts that overlap with it from the problem. So let $S\prime = S \backslash C$ and we are done since no concerts in S overlap with C. Then, let O be the set of concerts from P that overlap with C. Specifically, for all concerts say $J = (s_j, f_j) \in P$, if $s_j < s_c$ and $f_j > s_c$, OR, if $s_j > s_c$ and $s_j < f_c$, then add concert J to O. Then, let $P\prime = P \backslash \{C \cup O\}$. The resulting subproblem is to find a set $S\prime \subset P\prime$ so that for all $(s_i\prime, f_i\prime) \in S\prime$, $f_i\prime < s_{i+1}\prime$, ie, no overlaps, where $|S\prime|$ is maximal.