

Q2

Proof:

(1) Let $M = 1$, $n_0 = 2$, and $n > n_0$ be arbitrary.

(2) By (1), $n > 2$, so $\log_2 n > 1 \implies \frac{1}{\log_2 n} < 1$.

(3) By (1), $n > 2$, so $n > 1$.

(4) Now,

$$1 \geq M, \quad \text{by (1)}$$

$$\implies (\log_2 n)^2(2 - 1) \geq M(\log_2 n)^2$$

$$\implies (\log_2 n)^2(2 - \frac{1}{\log_2 n}) \geq M(\log_2 n)^2, \quad \text{by (2)}$$

$$\implies 2(\log_2 n)^2 - \log_2 n \geq M(\log_2 n)^2$$

$$\implies 2(\log_2 n)^3 - \log_2 n \geq M(\log_2 n)^2, \quad \text{by (3)}$$

$$\implies 2(\log_2 n)^3 - \log_2 n + 2024 \geq M(\log_2 n)^2,$$

as needed.

(4) Therefore,

$$\exists M > 0 \quad \exists n_0 > 0 \quad \forall n > n_0 \quad (2(\log_2 n)^3 - \log_2 n + 2024 \geq M(\log_2 n)^2).$$

Thus, $2(\log_2 n)^3 - \log_2 n + 2024 \in \Omega((\log_2 n)^2)$.