COMP 2080 Summer 2024 – Assignment 3

This assignment is due by 11:59pm on **Monday May 27**.

Your assignment will be submitted in Crowdmark and *not* in UM Learn. A Crowdmark invitation will be sent to your U of M email.

Proofs will be graded on both correctness and presentation. Clearly explain all of your steps.

The total number of points is 15.

Global Instructions

For this assignment, you may **only** use basic algebra, arithmetic, and the definitions of $O()$, $\Omega()$, $\Theta()$, $o()$, and $\omega()$. Do **not** use limits, statements about the hierarchy of functions, or any other properties of asymptotic notation.

Questions

1. [**3 marks**] Prove that $n! \notin O(n(4^n))$.

2. [**3 marks**] Prove that $2(\log_2(n))^3 - \log_2(n) + 2024 \in \Omega\left((\log_2(n))^2\right)$.

3. [**3 marks**] Prove that $\log_2(4^n + 2) \in \Theta(n)$.

4. [**3 marks**] First, we define the following notation: for any even number m,

$$m!! = m \cdot (m-2) \cdot (m-4) \cdot \dots \cdot (4) \cdot (2).$$

   Prove that $4^n \in o((2n)!!)$.

5. [**3 marks**] Let f and g be arbitrary functions that map $\mathbb{Z}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$. Prove that, if $g \in \omega(f)$, then $f + g \in \omega(f)$. Hints:

   - Begin by clearly stating what you are assuming and what you need to prove.
   - If you find yourself writing something like "Let $M = 3M$" in your proof, then you are using two different M's and they should have different names. You can use subscripts (M_1, n_1, M_2, n_2, …) or primes (M', n'_0, M'', n''_0, …) to distinguish betweeen your variables.