Question 2

Assume that the concerts in P are sorted by start time.

(a)

Let $S \subset P = \{(s_1, f_1)(s_2, f_2), ..., (s_n, f_n)\}$. Choose an arbitrary concert $C = (s_c, f_c) \in S$ and the concerts that overlap with it. Let O be the set of concerts from P that overlap with C. Specifically, for all concerts say $J = (s_j, f_j) \in P$, if $s_j < s_c$ and $f_j > s_c$, OR, if $s_j > s_c$ and $s_j < f_c$, then add concert J to O. Then, let $P\prime = P \backslash \{C \cup O\}$. Let $S\prime = S \backslash C$ and we are done since no concerts in S overlap with C, ie, $O \notin S\prime$ already. Then $S\prime$ is the set of concerts from $P\prime$ so that for all $(s_i\prime, f_i\prime) \in S\prime$, $f_i\prime < s_{i+1}\prime$, ie, no overlaps, where $|S\prime|$ is maximal.

(b)

We will show that $|S\prime|$ is maximal. Assume, in hopes of contradiction, that $|S\prime|$ is not maximal, that is, there exists a set of pairs $T\prime \subset P\prime$, so that for every $(s_i\prime, f_i\prime) \in T\prime$, $f_i\prime < s_{i+1}\prime$, where $|T\prime| > |S\prime|$. Let $T = T\prime \cup C$, so $|T| = |T\prime| + 1$.

First, we will show that T is feasible for P. Since $P\prime = P \backslash \{C \cup O\}$, $P\prime$ contains no pairs that overlap with C, so since $T\prime \subset P\prime$, it follows that $T\prime$ contains no pairs that overlap with C either. Therfore, when unionizing C with $T\prime$ to form T, there will be no overlapping, ie, for every $(s_i, f_i) \in T$, $f_i < s_{i+1}$. Thus, T is feasible for P.

Now, $|T| = |T\prime| + 1 > |S\prime| + 1 = |S|$, contradicting the fact that $|S|$ is maximal. Thus, $|S\prime|$ must be maximal.