$$G = (V, E)$$

$$v \in V$$

$$f_0:V\to\mathbb{Z}_2$$

 $\forall v \in V, d(v) = 0$

 $\forall v \in V, d(v) = 1$

$$A \in \mathbb{Z}_2^{|V| \times |V|}$$

$$A_{i,j} = \begin{cases} 1 & \text{if } i = j \text{ or } (v_i, v_j) \in E, \\ 0 & \text{otherwise} \end{cases}$$

$$x \in \mathbb{Z}_2^{|V|}$$

$$v_i \in S$$

$$f_0 \in \mathbb{Z}_2^{|V|}$$

$$Ax = f_0 + d,$$

$$d \in \mathbb{Z}_2^{|V|}$$

$$Ax = f_0 + d$$

$$rank(A) = |V|$$

$$f_0 + d \in \operatorname{im}(A)$$

$$f_0 + d \notin \operatorname{im}(A)$$

$$x = x_0 + \sum_{i=1}^k \alpha_i v_i$$

 v_1,\ldots,v_k

$$\alpha_i \in \mathbb{Z}_2$$

$$\alpha_1, \dots, \alpha_k \in \{0, 1\}$$

$$\alpha = (\alpha_1, \dots, \alpha_k) \in \mathbb{Z}_2^k$$