Introduction to the Theory of Computation 2020 — Midterm 2

Solutions (may be updated)

Problem 1 (15 pts). Consider $\Sigma = \{0, 1\}$ and the language

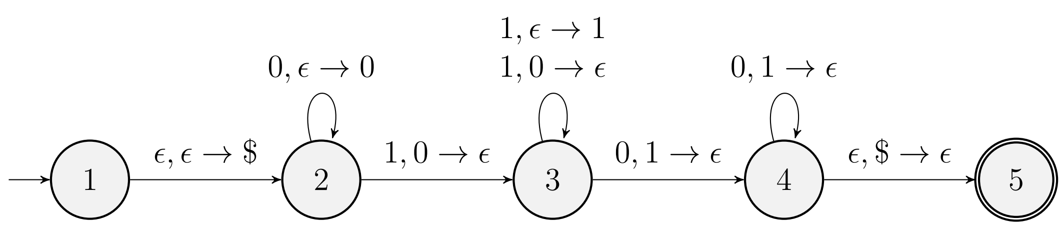
$$A = \{0^m 1^{m+n} 0^n \mid m, n \geq 1\}.$$

(a) (10 pts) Construct a PDA P_1 with ≤ 5 states to recognize A. Please check (b) and you are required to ensure P_1 satisfies the three conditions and no modification is needed.

(b) (5 pts) Convert your PDA P_1 to a CFG by using the procedure in Lemma 2.27 of the textbook. For simplicity, you only need to write each $A_{pq} \to aA_{rs}b$ rule. The rules $A_{pq} \to A_{pr}A_{rq}$ and $A_{pp} \to \epsilon$ are not needed. In order to prepare for $A_{pq} \to aA_{rs}b$ rules, please give table(s) for each stack alphabet u pushed/popped, similar to what we had in slides.

Solution.

(a) See the following diagram.



We see that the three conditions are satisfied:

(a) Single accept state

(b) By using \$, the stack is emptied before accepting

(c) Every transition either pushes or pops, but not both

(b) See the following tables.

- $u = 1$:

p	r	s	q	a	b
3	3	3	4	1	0
3	3	4	4	1	0

- $u = \$$:

p	r	s	q	a	b
1	2	4	5	ϵ	ϵ

- $u = 0$:

p	r	s	q	a	b
2	2	2	3	0	1
2	2	3	3	0	1

$$A_{23} \to 0A_{22}1$$
$$A_{23} \to 0A_{23}1$$
$$A_{34} \to 1A_{33}0$$
$$A_{34} \to 1A_{34}0$$
$$A_{15} \to A_{24}$$

Problem 2 (25 pts). Consider $\Sigma = \{0, 1\}$ and the language

$$B = \{0^m 1^n \mid m \geq n \geq 0\}.$$

(a) (10 pts) Construct a PDA P_2 with ≤ 2 states to recognize B. The stack alphabet is restricted to be

$$\Gamma = \{0\}.$$

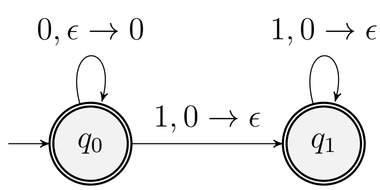
Draw the state diagram and give the formal definition of your PDA P_2. Hint: See (c), so you may want to have a diagram as deterministic as possible. Transitions like $\epsilon, \epsilon \to \epsilon$ may make (c) difficult.

(b) (10 pts) Prove that a PDA with only one state and with $\Gamma = \{0\}$ cannot recognize B. Your proof must be clearly written.

(c) (5 pts) Construct a DPDA with ≤ 3 states (including the q_r state that was introduced in examples in our slides) for B and show the table of δ.

Solution.

(a) See the following diagram.



$P_2 = (Q, \Sigma, \Gamma, \delta, q_0, F)$, where $Q = \{q_0, q_1\}, \Gamma = \{0\}, F = Q$, and δ is shown below.

Input	0		1		ϵ	
Stack	0	ϵ	0	ϵ	0	ϵ
q_0		$\{(q_0, 0)\}$	$\{(q_1, \epsilon)\}$			
q_1			$\{(q_1, \epsilon)\}$			

Table 1: δ. Blank entries signify $\emptyset$.

(b) Suppose P_2 has only one state q. To accept ϵ, q must be an accept state. Then every rule loops from q to itself.

One reasoning:

- Consider the string 1, which should be rejected. Consider the four kinds of transitions when a 1 is read.

$$1, \epsilon \to \epsilon$$
$$1, \epsilon \to 0$$
$$1, 0 \to \epsilon$$
$$1, 0 \to 0$$

  There cannot be $1, \epsilon \to \epsilon$ or $1, \epsilon \to 0$; otherwise 1 would be accepted.
- Consider the string 01, which should be accepted. To read 1, with the properties from the previous paragraph, a rule must be $1, 0 \to \epsilon$ or $1, 0 \to 0$. Thus before reading 1, a 0 must be pushed to the stack. That is, we need a rule either $0, \epsilon \to 0$ or $\epsilon, \epsilon \to 0$. However, the rule $\epsilon, \epsilon \to 0$ along with either $1, 0 \to \epsilon$ or $1, 0 \to 0$ would accept 1, so the rule $0, \epsilon \to 0$ is needed.
- We conclude that the diagram must have at least

$$0, \epsilon \to 0, \quad 1, 0 \to \epsilon$$

  or

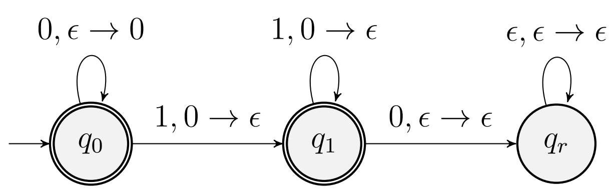
$$0, \epsilon \to 0, \quad 1, 0 \to 0$$

  The former accepts 0101 and the latter accepts 011, but neither should be accepted. Thus it is impossible to have only one state with $\Gamma = \{0\}$.

Alternative reasoning:

- To reject 1, we have $\delta(q, 1, \epsilon) = \emptyset$. This implies $\delta(q, 1, 0) \neq \emptyset$; otherwise 1 can never be processed, but strings such as 01 are in the language B. We then know that when we read a 1 there need to be at least a 0 on the stack.
- Consider the initial empty stack, when there is nothing to pop. We cannot have both $\delta(q, 0, \epsilon) = \delta(q, \epsilon, \epsilon) = \emptyset$, because along with $\delta(q, 1, \epsilon) = \emptyset$ from the previous paragraph, no input can be processed. Therefore, either $\delta(q, 0, \epsilon) \neq \emptyset$ or $\delta(q, \epsilon, \epsilon) \neq \emptyset$.
- If $(q, 0) \in \delta(q, 0, \epsilon)$, then 010 would be accepted. If $(q, 0) \in \delta(q, \epsilon, \epsilon)$, then 1 would be accepted. But neither of 010 or 1 should be accepted. Thus we have $(q, 0) \notin \delta(q, 0, \epsilon)$ and $(q, 0) \notin \delta(q, \epsilon, \epsilon)$. This means before reading any 1, no 0 can be pushed to the stack, and thus 1 can never be processed. Thus it is impossible to have only one state with $\Gamma = \{0\}$.

(c) See the following diagram and table.



Input	0		1		ϵ	
Stack	0	ϵ	0	ϵ	0	ϵ
q_0		$(q_0, 0)$	(q_1, ϵ)			
q_1		(q_r, ϵ)	(q_1, ϵ)			
q_r						(q_r, ϵ)

Table 2: δ. Blank entries signify $\emptyset$.

For the first row, from $\delta(q_0, 0, \epsilon) \neq \emptyset$ and $\delta(q_0, 1, 0) \neq \emptyset$, all other entries are $\emptyset$. For the second row, from $\delta(q_1, 1, 0) \neq \emptyset$, we have

Input	0		1		ϵ	
Stack	0	ϵ	0	ϵ	0	ϵ
q_1			(q_1, ϵ)	$\emptyset$	$\emptyset$	$\emptyset$

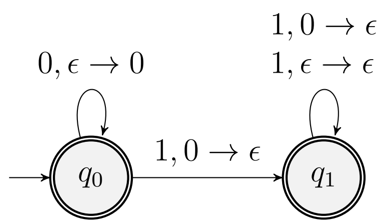
Table 3:

Thus we can have either $\delta(q_1, 0, 0) \neq \emptyset$ or $\delta(q_1, 0, \epsilon) \neq \emptyset$, but not both. For the third row, by $\delta(q_r, \epsilon, \epsilon) \neq \emptyset$, all other entries are $\emptyset$.

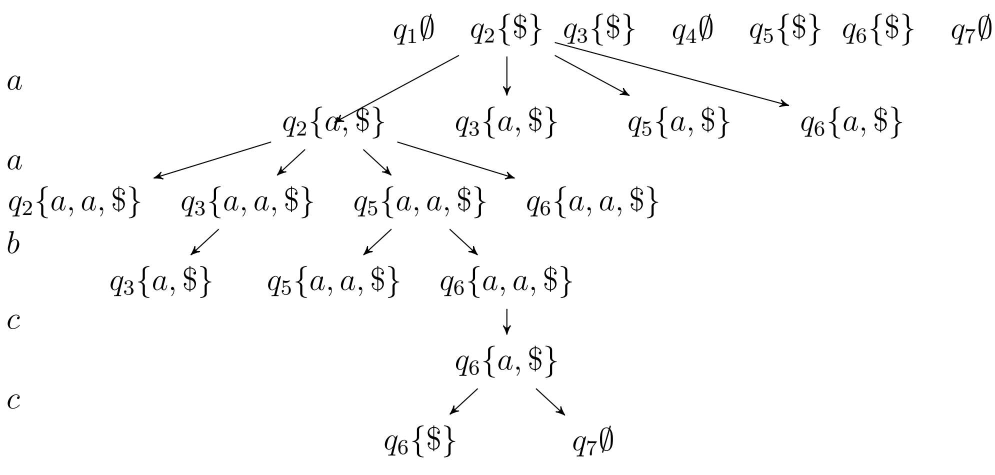
Problem 3 (20 pts). Consider $\Sigma = \{0, 1\}$ and the language

$$C = \{0^m 1^n \mid 0 \leq m \leq n\}.$$

Some propose the following PDA P_3.



(a) (5 pts) Check whether P_3 works for 000111 by drawing the simulation tree. Recall that in our PDA (part 2) slides, a tree is drawn like the following with input $aabcc$.

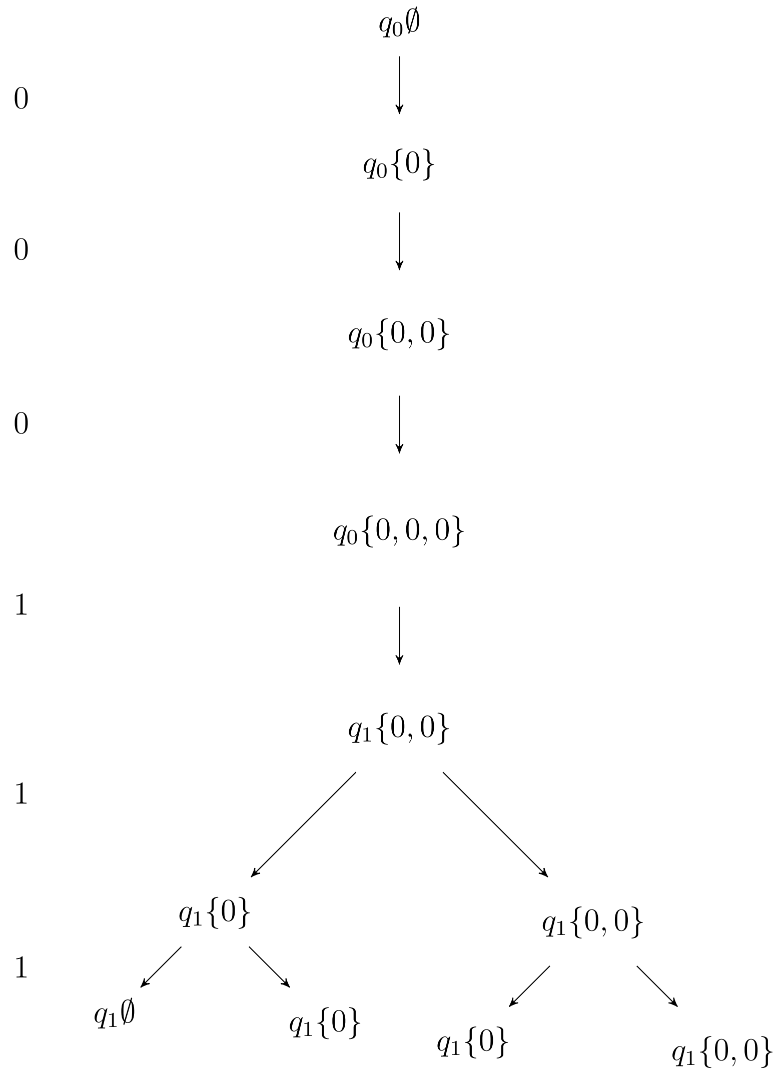


(b) (5 pts) Show that the PDA P_3 is actually wrong by finding one string s on which P_3 fails, i.e., $s \notin C$ but P_3 accepts s, or $s \in C$ but P_3 rejects s. Then draw a simulation tree like the one in (a).

(c) (10 pts) Construct a PDA P_3' with ≤ 4 states to recognize C. P_3' is restricted to have only **one accept state**. Draw the state diagram, and simulate P_3' on your string considered in (b).

Solution.

(a) See the following tree.



Note: We intend to check if you understand the non-deterministic setting. Thus if you drew a path instead of a tree, then you do not get any point.

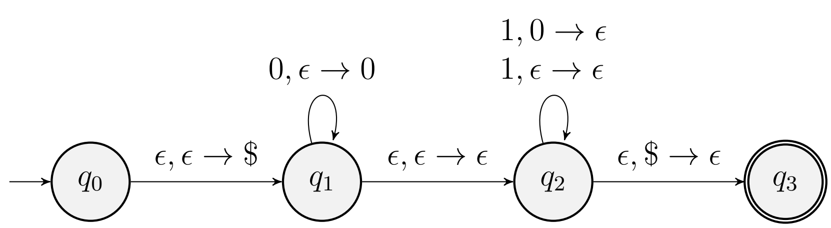
(b) $001 \notin C$ but P_3 accepts it. See the following tree.

$$
\begin{array}{cc}
 & q_0\emptyset \\
0 & \downarrow \\
 & q_0\{0\} \\
0 & \downarrow \\
 & q_0\{0,0\} \\
1 & \downarrow \\
 & q_1\{0\}
\end{array}
$$

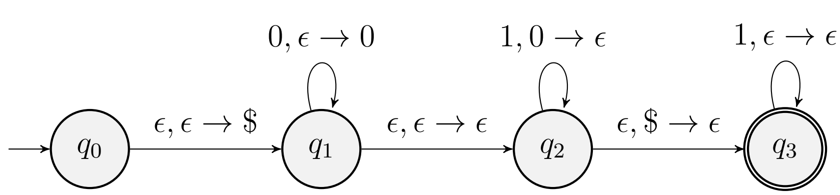
A simpler example is that $0 \notin C$ but P_3 accepts it. See the following tree.

$$
\begin{array}{cc}
 & q_0\emptyset \\
0 & \downarrow \\
 & q_0\{0\}
\end{array}
$$

(c) Either



or



See the following trees that show the rejection of 001.

Either

$$
\begin{tikzcd}
 & q_0\emptyset \arrow[r] & q_1\{\$\} \arrow[dd] & & \\
0 & & & & \\
 & & q_1\{0, \$\} \arrow[dd] & & \\
0 & & & & \\
 & & q_1\{0, 0, \$\} \arrow[r] & q_2\{0, 0, \$\} \arrow[dl] \arrow[dr] & \\
1 & & q_2\{0, \$\} & & q_2\{0, 0, \$\}
\end{tikzcd}
$$

or

$$
\begin{tikzcd}
 & q_0\emptyset \arrow[r] & q_1\{\$\} \arrow[dd] & \\
0 & & & \\
 & & q_1\{0, \$\} \arrow[dd] & \\
0 & & & \\
 & & q_1\{0, 0, \$\} \arrow[r] & q_2\{0, 0, \$\} \arrow[dd] \\
1 & & & \\
 & & & q_2\{0, \$\}
\end{tikzcd}
$$

See the following tree that shows the rejection of 0.

$$
\begin{tikzcd}
 & q_0\emptyset \arrow[r] & q_1\{\$\} \arrow[dd] \\
0 & & \\
 & & q_1\{0, \$\}
\end{tikzcd}
$$

Common mistakes:

- ϵ is rejected
- 1 is rejected

Problem 4 (10 pts). Consider the following CFG.

$$\begin{aligned} S &\to A \mid 0A \\ A &\to SAS \mid \epsilon \end{aligned}$$

Transform it to CNF by following the procedures in Theorem 2.9.

(i): Add a new start variable S_0 and the rule $S_0 \to S$.

(ii): Take care of all ϵ-rules.

(iii): Handle all unit rules.

(iv): Convert all remaining rules into the proper form.

You need to show details of the steps.

Solution.

Add S_0

$$\begin{aligned} S_0 &\to S \\ S &\to A \mid 0A \\ A &\to SAS \mid \epsilon \end{aligned}$$

Remove $A \to \epsilon$

$$\begin{aligned} S_0 &\to S \\ S &\to A \mid 0A \mid \epsilon \mid 0 \\ A &\to SAS \mid SS \end{aligned}$$

Remove $S \to \epsilon$

$$\begin{aligned} S_0 &\to S \mid \epsilon \\ S &\to A \mid 0A \mid 0 \\ A &\to SAS \mid SS \mid AS \mid SA \mid S \end{aligned}$$

Remove $S \to A$

$$\begin{aligned} S_0 &\to S \mid \epsilon \\ S &\to 0A \mid 0 \mid SAS \mid SS \mid AS \mid SA \mid S \\ A &\to SAS \mid SS \mid AS \mid SA \mid S \end{aligned}$$

Remove $A \to S$

$$\begin{aligned} S_0 &\to S \mid \epsilon \\ S &\to 0A \mid 0 \mid SAS \mid SS \mid AS \mid SA \\ A &\to SAS \mid SS \mid AS \mid SA \mid 0A \mid 0 \end{aligned}$$

Remove $S_0 \to S$

$$\begin{aligned} S_0 &\to 0A \mid 0 \mid SAS \mid SS \mid AS \mid SA \mid \epsilon \\ S &\to 0A \mid 0 \mid SAS \mid SS \mid AS \mid SA \\ A &\to 0A \mid 0 \mid SAS \mid SS \mid AS \mid SA \end{aligned}$$

Remove SAS

$$\begin{aligned} S_0 &\to 0A \mid 0 \mid SB \mid SS \mid AS \mid SA \mid \epsilon \\ S &\to 0A \mid 0 \mid SB \mid SS \mid AS \mid SA \\ A &\to 0A \mid 0 \mid SB \mid SS \mid AS \mid SA \\ B &\to AS \end{aligned}$$

Add $U \to 0$

$$\begin{aligned} S_0 &\to UA \mid 0 \mid SB \mid SS \mid AS \mid SA \mid \epsilon \\ S &\to UA \mid 0 \mid SB \mid SS \mid AS \mid SA \\ A &\to UA \mid 0 \mid SB \mid SS \mid AS \mid SA \\ B &\to AS \\ U &\to 0 \end{aligned}$$

Common mistakes:

- In removing $A \to \epsilon$, many forgot to add the rule $A \to SS$.
- In removing $S \to \epsilon$, many forgot to add the rule $A \to S$.

Problem 5 (35 pts). For $w = w_1 \cdots w_n$, we denote its reverse by $w^R = w_n \cdots w_1$. Consider

$$\Sigma = \{0, 1, \#\}$$

and the language

$$D = \{w\#w^R \mid w \in \{0,1\}^*\}.$$

For all following subproblems, **the number of states includes q_{accept} and q_{reject}**; however, you do not need to show q_{reject} and the transitions going to q_{reject}. **You do not get any point if more states than the specified limit are used**.

(a) (13 pts) Draw a TM diagram with ≤ 9 states to decide D by the outside-in strategy

Step 1: Replace the leftmost 0 or 1 in w with $\sqcup$.

Step 2: Move rightwards until the first $\sqcup$ (skipping all non-$\sqcup$ symbols) in or after w^R. Move left and check whether the element matches the previously seen leftmost one. If so, replace it with $\sqcup$. Otherwise, reject.

Step 3: Move back to find the first non-$\sqcup$ in w. Go to Step 1.

Step 4: In Step 3, if no more 0/1 in the left, check whether the head is pointing to a # and there is also nothing in the right.

and simulate two strings 01#10 and 010#0 by this TM. You are required to have

$$\Gamma = \{0, 1, \#, \sqcup\}.$$

For the simulation, please show the sequence of configurations for each step, and write the result (accept or reject). **You must finish the simulation in order to get points.**

(b) (12 pts) Draw a TM diagram with ≤ 8 states to decide D and follow the inside-out strategy:

Step 1: To know whether we are at the beginning of the tape, replace the first 0, 1 with $\$_0$, $\$_1$ respectively.

Step 2: Find the first #.

Step 3: Find the first 0 or 1 after # that has not been marked as $\times$ (crossing with $\times$), and check whether it matches the symbol before # that has not been marked as $\times$ (also crossing with $\times$). If the two do not match, reject. Otherwise, repeat this step.

Step 4: In the end, if we find the matching $\$_0/\$_1$ in the left (instead of matching 0/1), check whether there is nothing in the right except for $\times$ and #.

and simulate the same strings used in (a). You are required to have

$$\Gamma = \{0, 1, \#, \$_0, \$_1, \times, \sqcup\}.$$

Note that $\$_0, \$_1, \times$ are special symbols that never appear in the input string. Please use this property to save the number of states. **You must finish the simulation in order to get points.**

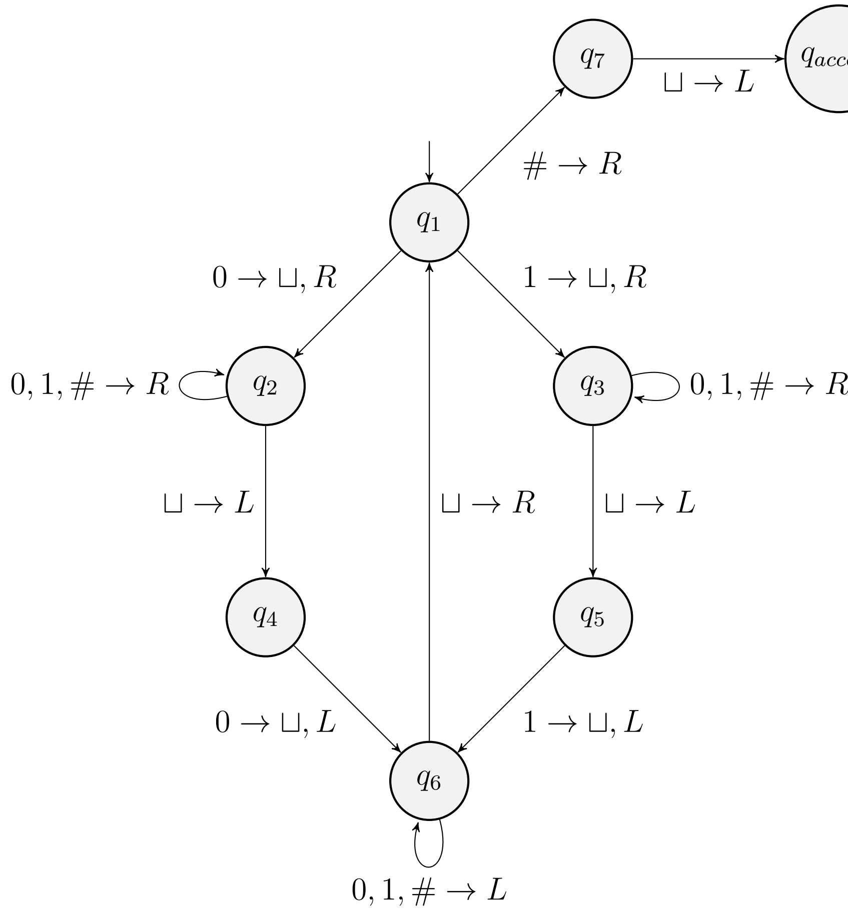
(c) (10 pts) Draw a two-tape TM diagram with ≤ 4 states to decide D, and simulate the same strings used in (a). You are required to have

$$\Gamma = \{0, 1, \#, \sqcup\}.$$

You must finish the simulation in order to get points.

Solution.

(a) See the following diagram. For all the following diagrams, we assume that the head moves right in each of the transitions to q_{reject}.



Simulation for 01#10 and 010#0:

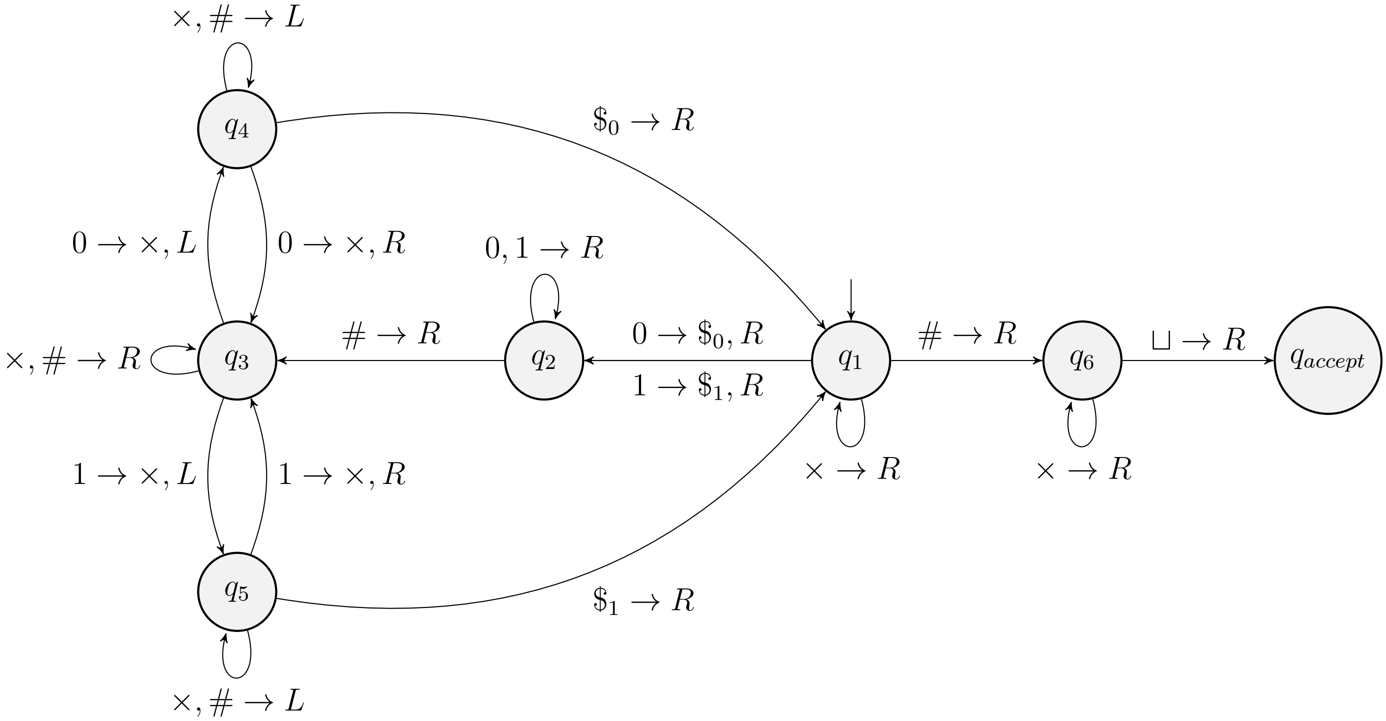
- 01#10:

$$
\begin{array}{lllllll}
 & q_1 01\#10 & \Rightarrow \sqcup q_2 1\#10 & \Rightarrow \sqcup 1 q_2 \#10 & \Rightarrow \sqcup 1\# q_2 10 & \Rightarrow \sqcup 1\#1 q_2 0 \\
\Rightarrow & \sqcup 1\#10 q_2 \sqcup & \Rightarrow \sqcup 1\#1 q_4 0 & \Rightarrow \sqcup 1\# q_6 1 & \Rightarrow \sqcup 1 q_6 \#1 & \Rightarrow \sqcup q_6 1\#1 \\
\Rightarrow & q_6 \sqcup 1\#1 & \Rightarrow \sqcup q_1 1\#1 & \Rightarrow \sqcup \sqcup q_3 \#1 & \Rightarrow \sqcup \sqcup \# q_3 1 & \Rightarrow \sqcup \sqcup \#1 q_3 \sqcup \\
\Rightarrow & \sqcup \sqcup \# q_5 1 & \Rightarrow \sqcup \sqcup q_6 \# & \Rightarrow \sqcup q_6 \sqcup \# & \Rightarrow \sqcup \sqcup q_1 \# & \Rightarrow \sqcup \sqcup \# q_7 \sqcup \\
\Rightarrow & \sqcup \sqcup q_{accept} \# & \Rightarrow \text{Accept} & & &
\end{array}
$$

- 010#0:

$$
\begin{array}{llllll}
 & q_1 010\#0 & \Rightarrow \sqcup q_2 10\#0 & \Rightarrow \sqcup 1 q_2 0\#0 & \Rightarrow \sqcup 10 q_2 \#0 & \Rightarrow \sqcup 10\# q_2 0 \\
\Rightarrow & \sqcup 10\#0 q_2 \sqcup & \Rightarrow \sqcup 10\# q_4 0 & \Rightarrow \sqcup 10 q_6 \# & \Rightarrow \sqcup 1 q_6 0\# & \Rightarrow \sqcup q_6 10\# \\
\Rightarrow & q_6 \sqcup 10\# & \Rightarrow \sqcup q_1 10\# & \Rightarrow \sqcup \sqcup q_3 0\# & \Rightarrow \sqcup \sqcup 0 q_3 \# & \Rightarrow \sqcup \sqcup 0\# q_3 \sqcup \\
\Rightarrow & \sqcup \sqcup 0 q_5 \# & \Rightarrow \sqcup \sqcup 0\# q_{reject} \sqcup & \Rightarrow \text{Reject} & &
\end{array}
$$

(b) See the following diagram.



Note that we do not worry about multiple #'s because of the $\# \to R$ loop at q_3. In the end, we use $q_1 \to q_6$ to ensure that only a string with one # is accepted.

Simulation for 01#10 and 010#0:

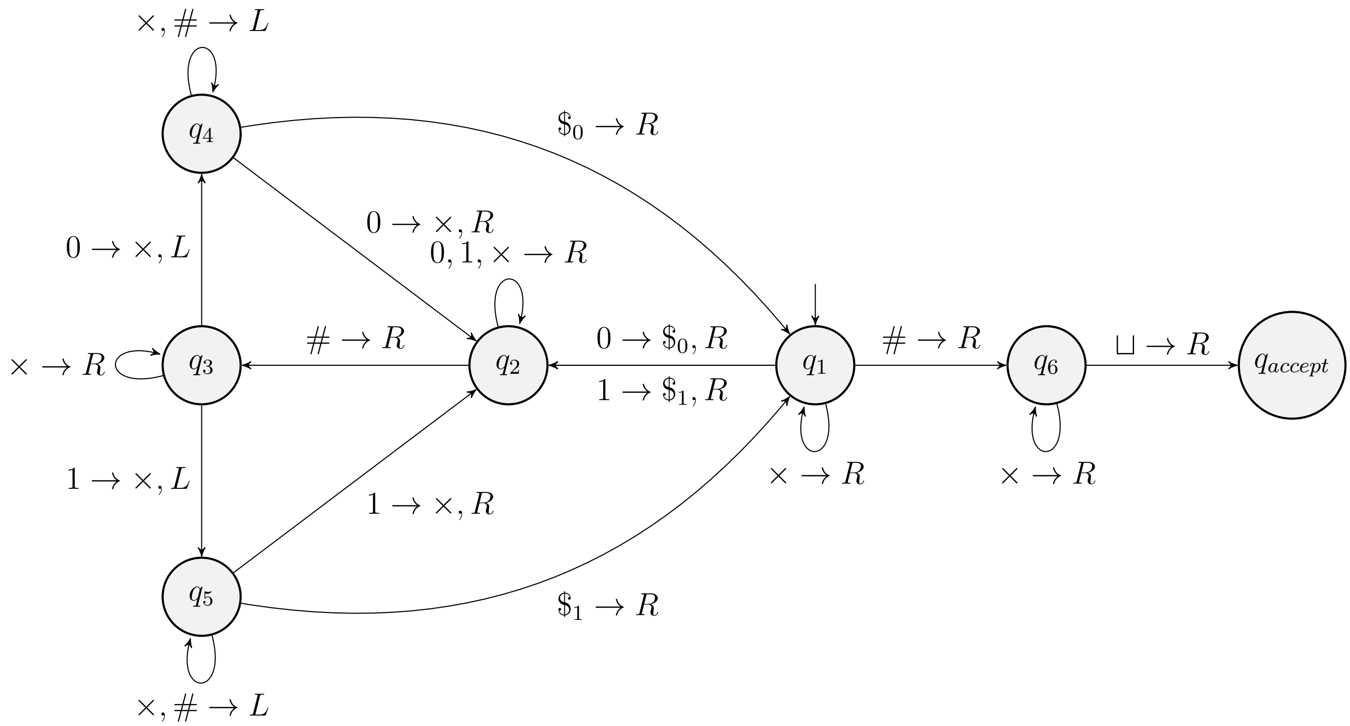
- 01#10:

$$\begin{array}{llll} & q_1 01\#10 & \Rightarrow \$_0 q_2 1\#10 & \Rightarrow \$_0 1 q_2 \#10 & \Rightarrow \$_0 1\# q_3 10 \\ \Rightarrow & \$_0 1 q_5 \# \times 0 & \Rightarrow \$_0 q_5 1\# \times 0 & \Rightarrow \$_0 \times q_3 \# \times 0 & \Rightarrow \$_0 \times \# q_3 \times 0 \\ \Rightarrow & \$_0 \times \# \times q_3 0 & \Rightarrow \$_0 \times \# q_4 \times \times & \Rightarrow \$_0 \times q_4 \# \times \times & \Rightarrow \$_0 q_4 \times \# \times \times \\ \Rightarrow & q_4 \$_0 \times \# \times \times & \Rightarrow \$_0 q_1 \times \# \times \times & \Rightarrow \$_0 \times q_1 \# \times \times & \Rightarrow \$_0 \times \# q_6 \times \times \\ \Rightarrow & \$_0 \times \# \times q_6 \times & \Rightarrow \$_0 \times \# \times \times q_6 \sqcup & \Rightarrow \$_0 \times \# \times \times \sqcup q_{accept} \sqcup & \Rightarrow \text{Accept} \end{array}$$

- 010#0:

$$\begin{array}{llll} & q_1 010\#0 & \Rightarrow \$_0 q_2 10\#0 & \Rightarrow \$_0 1 q_2 0\#0 & \Rightarrow \$_0 10 q_2 \#0 \\ \Rightarrow & \$_0 10\# q_3 0 & \Rightarrow \$_0 10 q_4 \# \times & \Rightarrow \$_0 1 q_4 0\# \times & \Rightarrow \$_0 1 \times q_3 \# \times \\ \Rightarrow & \$_0 1 \times \# q_3 \times & \Rightarrow \$_0 1 \times \# \times q_3 \sqcup & \Rightarrow \$_0 1 \times \# \times \sqcup q_{reject} \sqcup & \Rightarrow \text{Reject} \end{array}$$

Alternative solution:



Simulation for 01#10 and 010#0:

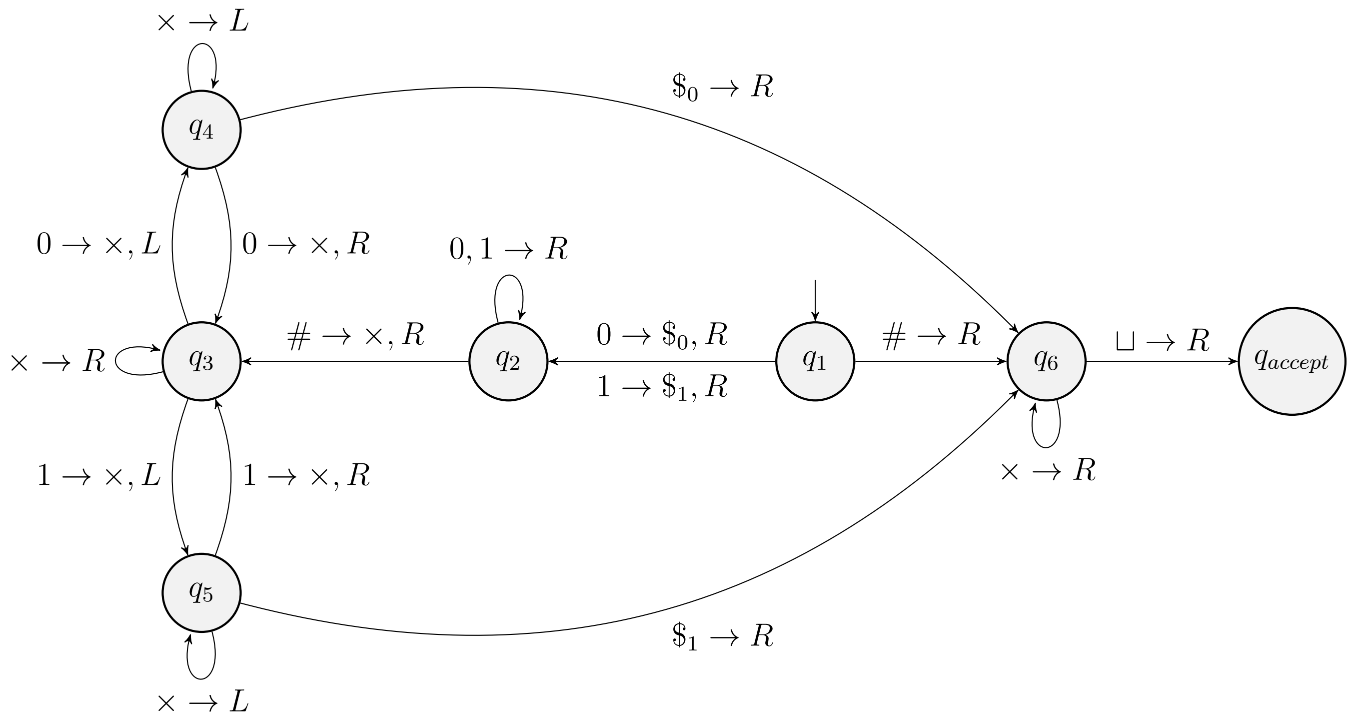
- 01#10:

$$\begin{array}{lllll}
 & q_1 01\#10 & \Rightarrow \$_0 q_2 1\#10 & \Rightarrow \$_0 1 q_2 \#10 & \Rightarrow \$_0 1\# q_3 10 \\
\Rightarrow & \$_0 1 q_5 \# \times 0 & \Rightarrow \$_0 q_5 1\# \times 0 & \Rightarrow \$_0 \times q_2 \# \times 0 & \Rightarrow \$_0 \times \# q_3 \times 0 \\
\Rightarrow & \$_0 \times \# \times q_3 0 & \Rightarrow \$_0 \times \# q_4 \times \times & \Rightarrow \$_0 \times q_4 \# \times \times & \Rightarrow \$_0 q_4 \times \# \times \times \\
\Rightarrow & q_4 \$_0 \times \# \times \times & \Rightarrow \$_0 q_1 \times \# \times \times & \Rightarrow \$_0 \times q_1 \# \times \times & \Rightarrow \$_0 \times \# q_6 \times \times \\
\Rightarrow & \$_0 \times \# \times q_6 \times & \Rightarrow \$_0 \times \# \times \times q_6 \sqcup & \Rightarrow \$_0 \times \# \times \times \sqcup q_{accept} \sqcup & \Rightarrow \text{Accept}
\end{array}$$

- 010#0:

$$\begin{array}{lllll}
 & q_1 010\#0 & \Rightarrow \$_0 q_2 10\#0 & \Rightarrow \$_0 1 q_2 0\#0 & \Rightarrow \$_0 10 q_2 \#0 \\
\Rightarrow & \$_0 10\# q_3 0 & \Rightarrow \$_0 10 q_4 \#\times & \Rightarrow \$_0 1 q_4 0\#\times & \Rightarrow \$_0 1 \times q_2 \#\times \\
\Rightarrow & \$_0 1 \times \# q_3 \times & \Rightarrow \$_0 1 \times \# \times q_3 \sqcup & \Rightarrow \$_0 1 \times \# \times \sqcup q_{reject} \sqcup & \Rightarrow \text{Reject}
\end{array}$$

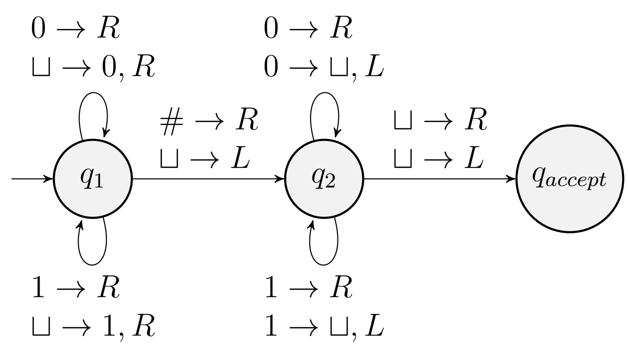
Another alternative solution (simulation omitted):



Common mistakes:

- Many wrongly accept 0#0#.
- Many wrongly reject #.

(c) See the following diagram.



Simulation for 01#10 and 010#0:

- 01#10:

$$\begin{aligned}
&\Rightarrow q_1 \begin{smallmatrix}0&1&\#&1&0\\ \sqcup&\sqcup&\sqcup&\sqcup&\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0\\0\end{smallmatrix} q_1 \begin{smallmatrix}1&\#&1&0\\ \sqcup&\sqcup&\sqcup&\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0&1\\0&1\end{smallmatrix} q_1 \begin{smallmatrix}\#&1&0\\ \sqcup&\sqcup&\sqcup\end{smallmatrix}\\
&\Rightarrow \begin{smallmatrix}0& &1&\#&q_2&1&0\\ 0&q_2&1&\sqcup& &\sqcup&\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix} &0&1&\#&1&q_2&0\\ q_2&0&\sqcup&\sqcup&\sqcup& &\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix} &0&1&\#&1&0&q_2&\sqcup\\ q_2&\sqcup&\sqcup&\sqcup&\sqcup&\sqcup& &\sqcup\end{smallmatrix}\\
&\Rightarrow \begin{smallmatrix} &0&1&\#&1&0&\sqcup&q_{accept}&\sqcup\\ q_{accept}&\sqcup&\sqcup&\sqcup&\sqcup&\sqcup&\sqcup& &\sqcup\end{smallmatrix} &&\Rightarrow \text{Accept}
\end{aligned}$$

- 010#0:

$$\begin{aligned}
&q_1 \begin{smallmatrix}0&1&0&\#&0\\ \sqcup&\sqcup&\sqcup&\sqcup&\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0\\0\end{smallmatrix} q_1 \begin{smallmatrix}1&0&\#&0\\ \sqcup&\sqcup&\sqcup&\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0&1\\0&1\end{smallmatrix} q_1 \begin{smallmatrix}0&\#&0\\ \sqcup&\sqcup&\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0&1&0\\0&1&0\end{smallmatrix} q_1 \begin{smallmatrix}\#&0\\ \sqcup&\sqcup\end{smallmatrix}\\
\Rightarrow &\begin{smallmatrix}0&1& &0&\#&q_2&0\\ 0&1&q_2&0&\sqcup& &\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0& &1&0&\#&0&q_2&\sqcup\\ 0&q_2&1&\sqcup&\sqcup&\sqcup& &\sqcup\end{smallmatrix} &&\Rightarrow \begin{smallmatrix}0&1& &0&\#&0&\sqcup&q_{reject}&\sqcup\\ 0&1&q_{reject}&\sqcup&\sqcup&\sqcup&\sqcup& &\sqcup\end{smallmatrix} &&\Rightarrow \text{Reject}
\end{aligned}$$