Comparative assessment for two wheels inverted pendulum mobile robot using robust control

Conference Paper · November 2010

DOI: 10.1109/ICCAS.2010.5669926 · Source: IEEE Xplore

3 authors, including:

Nor Maniha Ghani
Universiti Malaysia Pahang

Nadia Azmi
Curtin University

International Conference on Control, Automation and System 2010
Oct 27-30, 2010 in KINTEX, Gyeonggi-do, Korea

Comparative Assessment for Two Wheels Inverted Pendulum Mobile Robot Using Robust Control

N. M. Abdul Ghani[1], N. I. Mat Yatim[2], N. A. Azmi[3]

[1,2,3]Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang, 26600 Pekan, Pahang Malaysia
(E-mail: normaniha@ump.edu.my)

Abstract - Over the last decade, numbers of researcher has been increased in the two-wheeled inverted pendulum mobile robots or commonly known as balancing robot. This paper presents the linear modeling of two wheels inverted pendulum mobile robot and the robustness of sliding mode control. Linear modeling plant was obtained from the simplified nonlinear plant that was based on state-space approach. The dynamic model of the system has been established to design and analyze the control system, dynamic of the motors, chassis and wheels of balancing robot. Simulation on MATLAB/SIMULINK application is performed and to determine the performance and reliability of the Sliding Mode Controller.

Keywords- Two wheels balancing mobile robot, sliding mode controller.

1. INTRODUCTION

Balancing robots are characterized by the ability to balance on its two wheels without the assistance of other objects. The basic idea for a two-wheeled balancing robot is to drive the wheels in the direction that the upper part of the robot is falling. While the robot is moving, it can stay under its center of gravity, then the robot is remains balanced [1]. Anderson (2003) had build nBot by using a commercially available inertial sensor and position information from motor encoder to balance the system.

Felix Grasser (2002), a Mechanical Engineering student and researches at the Industrial Electronics laboratory at the Swiss Federal Institute of Technology has built a prototype robot which control is based on a Digital Signal Processor (DSP). Two sensors were used to measure position and speed of both wheels and a gyroscopic sensor to determine the position of the pendulum. This information is processed by a Digital Signal Processor .The signals from the driver through the remote control is taking into account and the torque applied to the two wheels is calculated to keep the system in a stable condition [2]. He named the two wheel balancing robot as JOE.

This two wheeled platform also to be tried in several human transporter applications. SEGWAY PT has been invented by Dean L. Kamen (2001). This robot is an electric, self-balancing human transporter with a complex, computer-controlled gyroscopic stabilization and control system [3]. The device balances on two parallel wheels and is controlled by moving body weight. By utilizing this technology, the user can traverse in small steps or curbs and allow easy navigation on various terrains.

The trajectory control for this type of robot has been successfully implanted in two dimensional by Koyanagi, E. Lida, S. and Yuta in 'A wheeled inverse pendulum type self-contained mobile robot and its two-dimensional trajectory control' [4]. The proposed controller has made the robot move autonomously, although quite slow. The work then has been further in 'Trajectory tracking control for navigation of the Inverse Pendulum Type Self-contained Mobile Robot' by Ha, Y. S and Yuta[5] where the robot is assumed to receive similar input on both wheels and the mathematical modeling was represented in one dimensional plane system.

Humanoid robots are the advanced technology in robotic. In order to remain stable, the robot's Center of Gravity must fall under its polygon of support. The polygon is basically the projection between all of its support points onto the surface. Sugihara modeled the walking motion of a human as an inverted pendulum in designing a real time motion generation method of a humanoid robot that controls the centre of gravity by indirect manipulation of Zero Moment Point (ZMP) [6]. The real time response of the method provides humanoid robots with high mobility. Example of humanoid robot is ASIMO.

A control system is a device or set of devices to manage, command, direct or regulate the behavior of other devices or systems. Over the years there are only two common classes of control systems being used by researchers in controlling system. These two types of control are categorized as linear and non-linear control.

Linear control systems use linear negative feedback to produce a control signal mathematically based on other variables, with a view to maintaining the controlled process within an acceptable operating range. Non-linear controller uses the unscathed dynamics model of the system in designing a controller. Generally, linear control system is the most popular control system as compared to non-linear.

The difference in balance control algorithm implemented depends mostly on how the system is modeled and how the tilt information is obtained.

Sliding Mode Control (SMC) is a nonlinear control method that alters the dynamics of a nonlinear system by application of a high-frequency switching control [7]. The state-feedback control law is not a continuous function of time. Instead, it switches from one continuous structure to another based on the current position in the state space. Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward a switching condition, and so the ultimate trajectory will not exist entirely within one control structure. Instead, the ultimate trajectory will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding (hyper) surface.

In the research title 'Variable Structure Control of Two Wheels Inverted Pendulum Mobile Robot' conducted by Nawawi S.W, Ahmad M.N, and Osman J.H.S (2006) showed the good control behaviour of the system to be control [8]. It can be seen in the result of the simulation. The characteristics of the SM appear to be more robust and good control compare to SFIC.

II. Dynamic Model

The dynamic performance of a balancing robot depends on the efficiency of the control algorithms and the dynamic model of the system [9]. By adopting the coordinate system shown in Fig. 1 using Newtonian mechanics, it can be shown that the dynamics of the T-WIP mobile robot under consideration is governed by the following motion equations [9] .The coordinate system for the robot is depicted in Fig. 1.

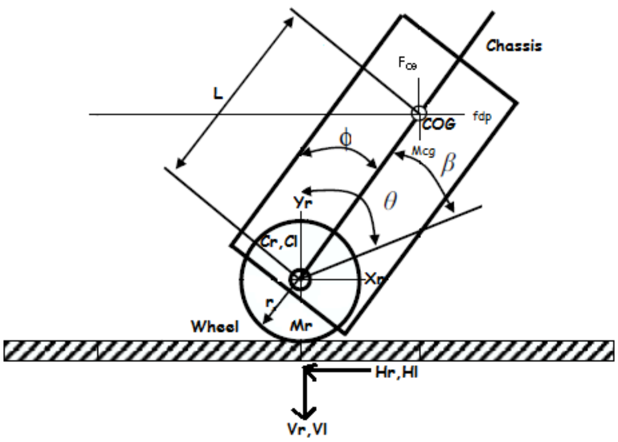

Fig.1.Coordinate system of the Two Wheels Mobile Robot

The following variables have been chosen to describe the vehicle:

For left hand wheel (analogous for right hand wheel):

$$\ddot{x}_{RL} M_r = H_{TL} - H_L + (f_{dRL} + F_{dRR}) \tag{1}$$

$$\ddot{y}_{RL} M_r = V_{TL-L} - M_r g \tag{2}$$

$$\ddot{\theta}_{RL} J_{RL} = C_L - H_{TL} r \tag{3}$$

$$\dot{x}_{RL} = r\,\dot{\theta}_{RL} \tag{4}$$

$$\dot{y}_p = -\ddot{\theta}_p L \sin\dot{\theta}_p \tag{5}$$

$$\dot{x}_p = \dot{\theta}_p L \cos\theta_p + ((\dot{x}_{RL} + \dot{x}_{RR}))/2 \tag{6}$$

$$\dot{\delta} = ((\dot{x}_{RL} + \dot{x}_{RR}))/2f \tag{7}$$

For the chassis, the equations:

$$\ddot{x}_p M_C = (H_R + H_L) + f_{dP} \tag{8}$$

$$\ddot{y}_p M_C = V_R + V_L - M_C g + Fc\theta \tag{9}$$

$$\ddot{\theta}_p J_p = (V_R + V_L) L \sin\theta_p - (H_R + H_L) L \cos\theta_p - (C_L + C_R) \tag{10}$$

$$\ddot{\delta} J_\delta = (H_L - H_R) * (D/2) \tag{11}$$

HTL, HTR, HL, HR, VTL, VTR, VL, VR represent reaction forces between the different free bodies.

Equations (1)-(11) can be represented in the state-space form as:

$$\dot{x}(t) = f(x) + g(x) \tag{12}$$

Where $x \in \Re^n$, $u \in \Re^m$ are respectively the state and the control. *F(x)* is nonlinear dynamic function matrix and *g(x)* is nonlinear input function matrix. The state, x of the system is defined as:

$$x = [X_r, \dot{X}_r, \theta_p, \dot{\theta}_p, \delta, \dot{\delta}] \tag{13}$$

Modifying the equations above and then linearizing the result around the operating point (θp=0, xr=0, δ=0) and de-coupling, the system's state space equations can be written in matrix form as:

$$\begin{bmatrix} \dot{X}r \\ \ddot{X}r \\ \dot{\theta}p \\ \ddot{\theta}p \\ \dot{\delta} \\ \delta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & A_{42} & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{62} & A_{63} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Xr \\ \dot{X}r \\ \theta p \\ \dot{\theta}p \\ \delta \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{21} & B_{22} \\ 0 & 0 \\ B_{41} & B_{42} \\ 0 & 0 \\ B_{61} & B_{62} \end{bmatrix} \begin{bmatrix} C_L \\ C_R \end{bmatrix} \tag{14}$$

Where:

*A_{22}=Ke*Km*(1+omega)*(1/(Mp*r^20+1/gamma)/(beta* alpha*gamma)* (1a)

*A_{23} = Mp*g*l^2/(beta*alpha*gamma)* (2a)

*A_{42}=Ke*Km*(1+omega)*(1+1/ (beta*r^2))/(alpha*R* gamma)* (3a)

*A_{43}=-Mp*g*l/(alpha*gamma)* (4a)

*A_{62}=-D*Ke*Km*(1-omega)*(1/r^2-(Mr+Jw/r^2))*((1+omega) /(beta*alpha))* (1/Mp*r^2)+1/gamma)/(2*Jpd*r^2*R)* (5a)

*B_{21}= (Km/(beta*alpha*R))*((1/Mp*r)+1/gamma)* (6a)

B_{21}=B_{22} (7a)

*B_{41}= (-Km/(alpha*R*gamma))*(1+1/(r*R))* (8a)

B_{41}=B_{42} (9a)

*B_{61}= (-D*Km/(2*Jpd*R))*(1/r+(Mr+Jw/r^2)*(1-omega)* (1/Mp*r+1/gamma))* (10a)

*B_{62}= (D*Km/(2*Jpd*R))*(1/r-(Mr+Jw/r^2)*(1-omega)*(1/Mp*r+1/gamma))* (11a)

*gamma= Jp + (Mp*l^2)* (12a)

*beta= 1+Mw/Mp+(Jw/(r^2*Mp))+omega*Mw/Mp+ omega*Jw/(r^2*Mp)* (13a)

*alpha =1 - (Mp*l^2/(beta*gamma))* (14a)

The T-WIP balancing model, namely equation (14) will be used throughout this work consequently.

III. SLIDING MODE CONTROL

Here the Sliding Mode Controller (SMC) will be shown and compare with uncontrolled to find which method will give the better stability toward the performance of two wheels mobile robots. For a Sliding Mode controller, the controller's gain were calculated and implemented. In this paper, there is no disturbance applied in the system.

Issues like settling time and damping ratio could be identified on the recorded responses and permitted fine tuning of the system. The parameter, Ca= [7.7265, 1.3914, 14.2175, 0.5106, -93.6363, 57.3035] , Cb = [12.0834, 2.3828, 38.8247, 1.6601, -446.5253, 150.5571] δ = -0.4376 and ρ= -0.1710 for SMC.

In this study, we utilized the sliding surface $\sigma(t)$ as follows:

$$\sigma(t) = Cx(t) \tag{15}$$

Then, the differential of Equation (15) gives,

$$\dot{\sigma}(t) = C\dot{x}(t) \tag{16}$$

In describing the method of equivalent control it will initially be assumed that the uncertain function is identically zero:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{17}$$

Substituting Equation (17) into Equation (16) yields,

$$\dot{\sigma}(t) = C[Ax(t) + Bu(t)] \tag{18}$$

Equating Equation (18) to zero gives the equivalent control, $u_{eq}(t)$ as follows,

$$u_{eq}(t) = -[CB]^{-1}CAx(t) \tag{19}$$

Substituting (19) into (17) gives the equivalent dynamics equation of the system in sliding mode as follows,

$$\dot{x}(t) = Ax(t) + B[-[CB]^{-1}CAx(t)]$$

$$\dot{x}(t) = [I_n - B[CB]^{-1}C]Ax(t) \tag{20}$$

Where I_n is an *nxn* identity matrix.

Remark. For the system with uncertainties satisfy the matching condition, i.e. rank [B|*f (t)*] = rank [B], then equation (20) can be reduced to $\dot{x}(t) = (A + BK)x(t)$. Thus asymptotic stability on the sliding surface *σ(t) =0* during sliding mode is assured.

We now design the control scheme that drives the state trajectories of the system onto the sliding surface *σ(t) =0* and the system remains in it thereafter. In order to fulfill the reaching condition, the control input *u(t)* is divided into two part as described in equation (21);

$$u(t) = u_{eq}(t) + u_{nl}(t) \tag{21}$$

And the following switching control $u_{nl}(t)$ is used in this study:

$$u_{nl}(t) = -(CB)^{-1}\rho\frac{\sigma(t)}{|\sigma(t)| + \delta} \tag{22}$$

Finally for the uncertain system in equation (8) satisfying assumptions (i) and (ii), the following control law is used:

$$u(t) = -(CB)^{-1}CAx(t) - (CB)^{-1}\rho\frac{\sigma(t)}{|\sigma(t)| + \delta} \tag{23}$$

Where δ and ρ are the positive constants.

<u>Theorem 1:</u> The hitting condition of the sliding surface (15) is satisfied if

$$\|A + BK\|\|x(t)\| \geq \|f(t)\| \tag{24}$$

Proof: In the hitting phase $\sigma^T(t)\sigma(t) \geq 0$; using the Lyapunov function candidate $V(t) = 1/2\sigma^T(t)\sigma(t)$,
The reaching condition is evaluated as follows,

$$\dot{V}(t) = \sigma^T(t)\dot{\sigma}(t)$$

$$\begin{aligned}\sigma^T(t)\dot{\sigma}(t) &= \sigma^T(t)[C\dot{x}(t)]\\ &= \sigma^T(t)[C\{Ax(t) + Bu(t)\}]\\ &= \sigma^T(t)[CAx(t) + CBu(t)]\end{aligned} \tag{25}$$

Substituting Equation (17) into Equation (19), gives

$$\sigma^T(t)\dot{\sigma}(t) = \sigma(t)[CAx(t) + (CB)\{-(CB)^{-1}CAx(t) - (CB)^{-1}\rho\frac{\sigma(t)}{|\sigma(t)|\delta}\}] \tag{26}$$

It follows that $\dot{V}(t) \leq 0$ if condition (24) is satisfied. Thus the reaching condition is satisfied.

A. Sliding Mode Controller (SMC)

For SMC, value C was obtained by applying the equation:

$$u(t) = -(CB)^{-1} CAx(t) - (CB)^{-1} \rho \frac{\sigma(t)}{|\sigma(t)| + \delta}$$

$$\sigma(t) = Cx(t)$$

By referring to the linear state space for two wheels mobile robot, the matrix for controller input has two inputs. Each u is 1 X 1and matrix for state vector, x is 1 X 6. By analyze the matrix form we find out that state feedback gain matrix, C is 6 X 1. This is very important to make sure error not occurs when control the whole system of two wheel mobile robot.

$$[C_L] = [C_1 C_2 C_3 C_4 C_5 C_6] \begin{bmatrix} X_r \\ \dot{X}_r \\ \theta_p \\ \dot{\theta}_p \\ \delta \\ \dot{\delta} \end{bmatrix}$$

$$C_L = C_1 X_r + C_2 \dot{X}_r + C_3 \theta_p + C_4 \dot{\theta}_p + C_5 \delta + C_6 \dot{\delta}$$

$$[C_R] = [C_7 C_8 C_9 C_{10} C_{11} C_{12}] \begin{bmatrix} X_r \\ \dot{X}_r \\ \theta_p \\ \dot{\theta}_p \\ \delta \\ \dot{\delta} \end{bmatrix}$$

$$C_R = C_7 X_r + C_8 \dot{X}_r + C_9 \theta_p + C_{10} \dot{\theta}_p + C_{11} \delta + C_{12} \dot{\delta}$$

III. Simulation And Results

In process to compare the Sliding Mode controller between uncontrolled, The MATLAB application is applied by running the simulation to analyze the impact of controller when control the balancing of two wheels mobile robot. The performance of uncontrolled and Sliding Mode Controller (SMC) is analyzed in term of position, speed, and angle. Two controllers are compared by applied Ca = [7.7265, 1.3914, 14.2175, 0.5106, -93.6363, 57.3035] and Cb = [12.0834, 2.3828, 38.8247, 1.6601, -446.5253, 150.5571] for SMC.

Figure 3, 4, 5, 6, 7. and 8 shows comparison of between SMC and uncontrolled system, position, speed, and angle performance respectively.

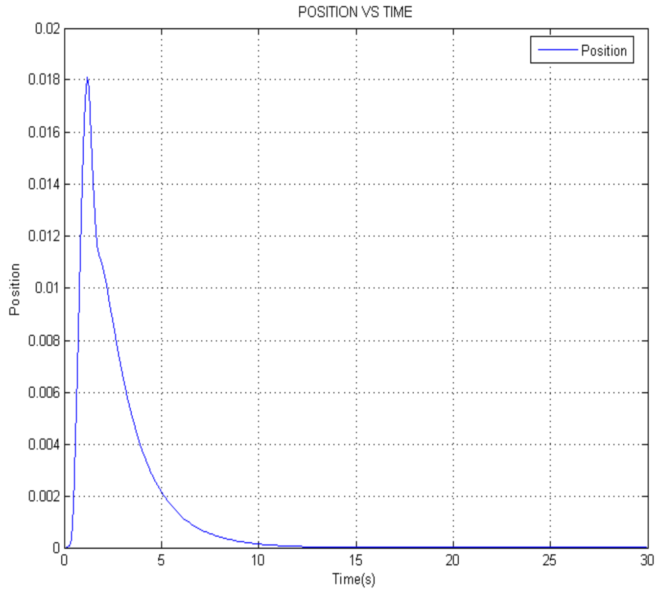

Fig. 3 Position vs Time (SMC)

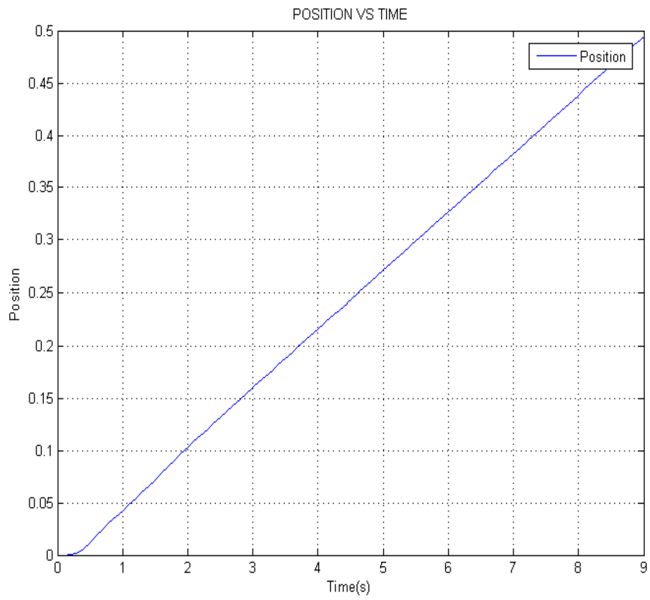

Fig.4 Position vs Time (Uncontrolled system)

Based on the figure 3 and 4, we can see that Sliding Mode Controller (SMC) has given the best performance in position. For SMC, the peak reached 0.018 at 2s and settled at 10.28s. The uncontrolled has linear signal but not stable in short time range.

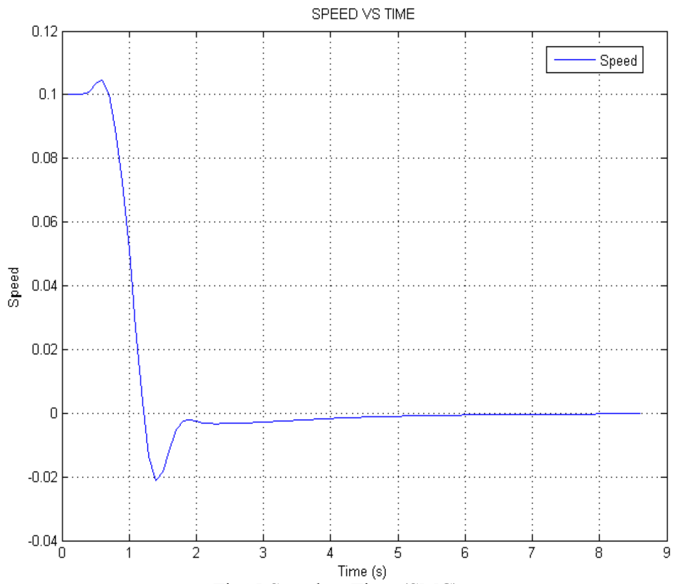

Fig. 5 Speed vs Time (SMC)

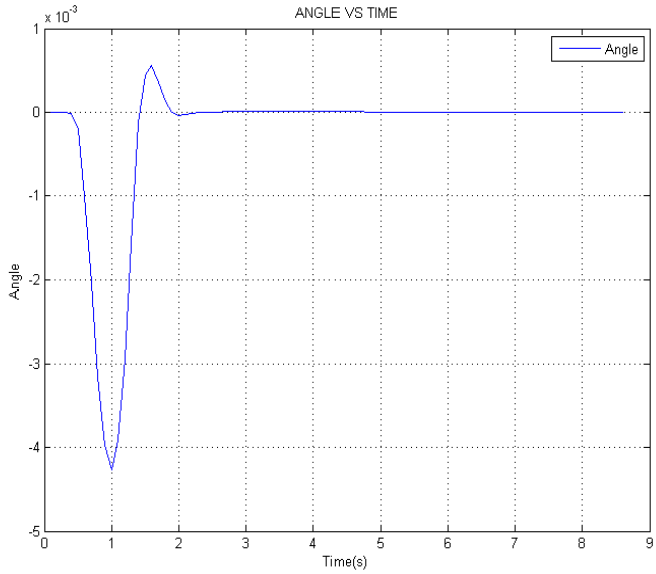

Fig. 7 Angel vs Time (SMC)

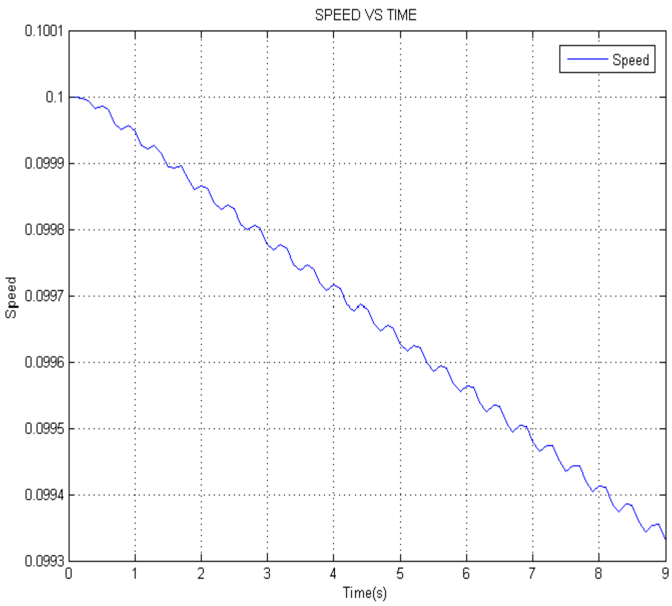

Fig. 6 Speed vs Time (Uncontrolled system)

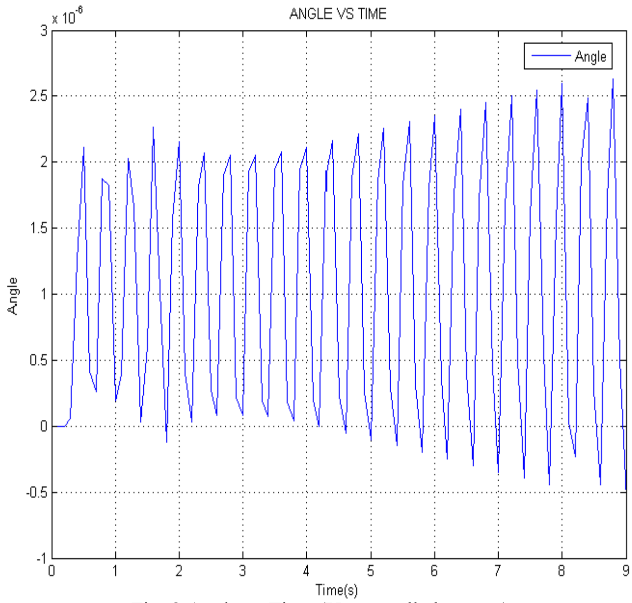

Fig. 8 Angle vs Time (Uncontrolled system)

For speed, the starting point has been chosen at 0.1. The result that been observed from above shows that SMC has better performance compare to uncontrolled in speed point of view. This can be observing through Figure 5 and 6. The signal speed from uncontrolled happens to be infinite oscillated linear that cannot be settled in such short time. And when using SMC, the signal comes out with desired output.

Sliding Mode Controller (SMC) has shown better performances compared to uncontrolled. It has settled with short time which is at 2.7s. Figure 8 shows uncontrolled systems that has very small oscillated signal that can be considered as 0 values while Figure 7 produce desired output response for SMC.

TABLE II
SPECIFICATION FOR UNCONTROLLED AND SLIDING MODE CONTROLLER

Parameters	Settling Time (s)	
	Uncontrolled	SMC
Position	Infinite	8.7
Speed	Infinite	8.6
Angle	0	2.3

IV. CONCLUSIONS

The comparison between Sliding Mode Controller and uncontrolled for two wheels mobile robot has been presented. The performances of the control schemes have been evaluated in terms of time response specifications and stability. Acceptable performance in disturbance suppression has been achieved with both control strategies. Moreover, a significant result has been achieved with Sliding Mode Controller. A comparison of the results has demonstrated that Sliding Mode Controller provide higher level of disturbance reduction as compared to uncontrolled. It is concluded that the result from this work show that the Sliding Mode Controller (SMC) based on MATLAB capable of delivering the desired outcome.

ACKNOWLEDGMENT

The authors gratefully acknowledged the contributions of S.W.Nawawi, for work on the original version of this document and Ministry of Science and Technology (MOSTI) for giving fund for this project.

REFERENCES

[1] Anderson, D.P, 2003, 'Nbot, a two wheel balancing robot', Available from :< http://www.geology.smu.edu/~dpa-www/robo/nbot/ >
[2] Felix Grasser, Aldo D'Arrigo, Silvio Colombi, and Alfred C,. Rufer. 2002 "JOE: A Mobile, Inverted Pendulum', *IEEE Transactions on Industrial Electronics*, OL. 49, No. 1, February 2002.
[3] Dean kamen, 2001
Available from : <http://www.segway.com>
[4] Koyanagi, E. , Lida, S. and Yuta, S. ,"A wheeled inverse pendulum type self-contained mobile robot and its two-dimensional trajectory control", *Proceeding of ISMCR92*, pp.891-898, 1992.
[5] Ha, Y.S. and Yuta, S, "Trajectory Tracking Control for Navigation of the Inverse Pendulum Type Self-contained Mobile Robot". *Robotic and*
Autonomous System, 17, 1996 pp. 65-80.
[6] Sugihara, T, & H. Kpbayashi., 2008. *A Handy Robot Navigation By Non-interuptive Switching of Guided Point and Synergetic Points,* 8th IEEES-RAS International Conference on Humanoid Robots.
[7] en.wikipedia.org/wiki/Sliding_mode_control from Wikipedia website
[8] S.W. Nawawi, Ahmad M. N, and Osman J.H.S, "Variable Structure Control of Two-Wheels Inverted Pendulum Mobile Robot", Regional Postgraduate Conference on Engineering and Science (RPCES 2006), Johore, 26-27 July
[9] S. W. Nawawi, M. N. Ahmad, and J. H. S. Osman, "Real-Time Control of a Two-Wheeled Inverted Pendulum Mobile Robot", proceeding a world academy of science, engineering and technology volume 29 May 20008 ISSN 1307-6884