Math 124 - Programming for Mathematical Applications

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Project 2 - Random Maze

Due Friday, March 3

Description

In this project, you will write a computer code to generate a random maze using a recursive algorithm. You will also write a code to find a path between two points in a given maze.

The integer `n` specifies the size of the `n`-by-`n` array of cells in the maze. Note the matrix indices `i,j` specify the x and y-coordinates, respectively (see plot below).

The horizontal and the vertical *interior* walls of the maze are described by the arrays:

- `H`, Bool array of size `n`-by-`n-1`
- `V`, Bool array of size `n-1`-by-`n`

These arrays specify if there is a wall or not between two neighboring cells.

An example is given below, with $n = 6$:

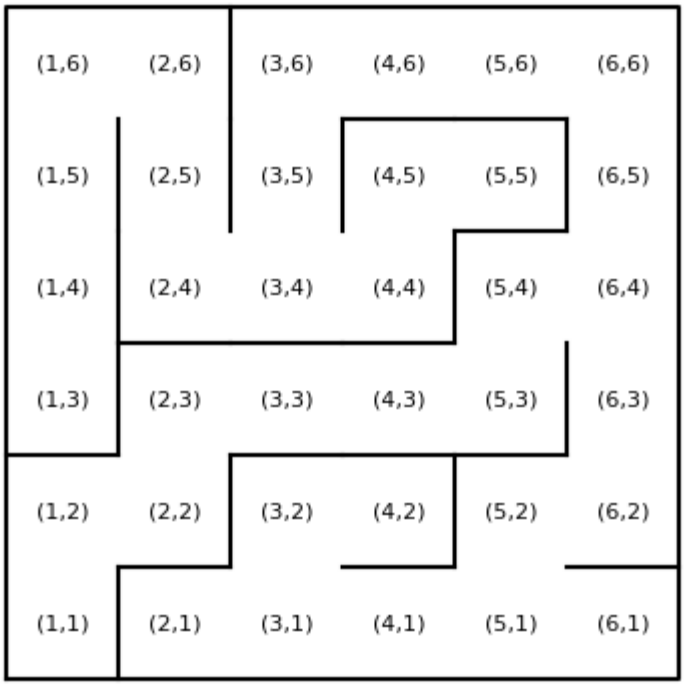
In [1]:
```
H = Bool[0 1 0 0 0; 1 0 1 0 0; 0 1 1 0 0; 1 1 1 0 1; 0 1 0 1 1; 1 0 0 0 0];
V = Bool[1 0 1 1 1 0; 0 1 0 0 1 1; 0 0 0 0 1 0; 0 1 0 1 0 0; 0 0 1 0 1 0];
```

and the following helper functions can be used to plot the maze:

In [2]:
```
using PyPlot, Random

function plot_maze(H,V)
    clf()
    axis("off")
    axis("equal")
    n = size(H,1)
    plot([0,n,n,0,0], [0,0,n,n,0], color="k")

    for x = 1:n-1, y = 1:n
        if V[x,y]
            plot([x,x], [y-1,y], color="k")
        end
    end
    for x = 1:n, y = 1:n-1
        if H[x,y]
            plot([x-1,x], [y,y], color="k")
        end
    end
end

function plot_cell_indices(n)
    for i = 1:n
        for j = 1:n
            text(i-0.5, j-0.5, "($i,$j)",
                horizontalalignment="center",
                verticalalignment="center",
                fontsize=8)
        end
    end
end
```

Out[2]:
```
plot_cell_indices (generic function with 1 method)
```

In [3]:
```
plot_maze(H,V)
plot_cell_indices(size(H,1))
```

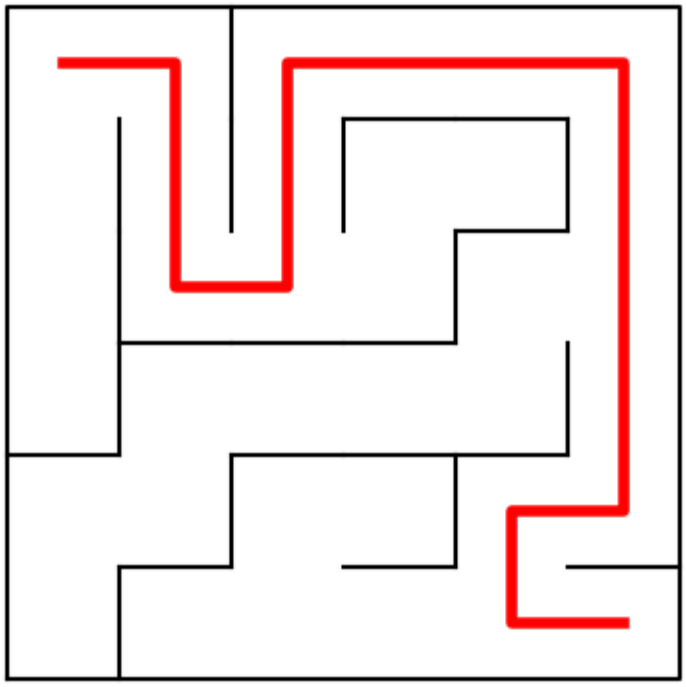


In addition, we will find paths between the points `1,n` and `n,1`, which can be stored in two arrays of integers. For the example above, this path is given by

In [4]:
```
x = [6, 5, 5, 6, 6, 6, 6, 6, 5, 4, 3, 3, 3, 2, 2, 2, 1];
y = [1, 1, 2, 2, 3, 4, 5, 6, 6, 6, 6, 5, 4, 4, 5, 6, 6];
```

and it can be plotted along with the maze using the commands:

In [5]:
```
plot_maze(H,V);
plot(x .- 0.5, y .- 0.5, color="r", linewidth=4);
```



Problem 1 - Generate random maze

Write a function with the syntax

```
H,V = make_maze(n)
```

which produces a random maze of size `n` -by- `n` using the following algorithm:

1. Initialize `H` and `V` to matrices of `trues` (that is, assume all cells have walls on all sides)
2. Also initialize an array `visit` to a matrix of `falses` , to keep track of cells that have been visited
3. Create a function `dig(x,y)` which loops over the four directions (Right, Left, Up, Down) in a random order. For each direction, if the neighbor cell is valid and not visited, remove the corresponding wall from `H` or `V` and run the `dig` function recursively on the neighbor cell.
4. Call `dig(1,1)` and return `H,V`

In [6]:
```
using Random
function make_maze(n)
    #1 = true, 0 = false
    V = trues(n-1,n)
    H = trues(n, n-1)
    visited = falses(n,n)

    x = 1
    y = 1

    function dig(x,y)
        visited[x,y] = true
        up, down, left, right = randperm(4)

        if up == 1 && y+1 ≤ n && !visited[x, y+1]
            H[x,y] = false
            visited[x, y+1] = true
            dig(x,y+1)

        elseif down == 1 && y-1 ≥ 1 && !visited[x, y-1]
            H[x,y-1] = false
            visited[x,y] = true
            dig(x,y-1)

        elseif left == 1 && x-1 ≥ 1  && !visited[x-1, y]
            V[x-1,y] = false
            visited[x-1, y] = true
            dig(x-1,y)

        elseif right == 1 && x+1 ≤ n && !visited[x+1, y]
            V[x,y] = false
            visited[x+1, y] = true
            dig(x+1,y)
        end

        if up == 2 && y+1 ≤ n && !visited[x, y+1]
            H[x,y] = false
            visited[x, y+1] = true
            dig(x,y+1)

        elseif down == 2 && y-1 ≥ 1 && !visited[x, y-1]
            H[x,y-1] = false
            visited[x, y-1] = true
            dig(x,y-1)

        elseif left == 2 && x-1 ≥ 1 && !visited[x-1, y]
            V[x-1,y] = false
            visited[x-1, y] = true
            dig(x-1,y)

        elseif right == 2 && x+1 ≤ n && !visited[x+1, y]
            V[x,y] = false
            visited[x+1, y] = true
            dig(x+1,y)
        end
```

```
        if up == 3 && y+1 ≤ n && !visited[x, y+1]
            H[x,y] = false
            visited[x, y+1] = true
            dig(x,y+1)

        elseif down == 3 && y-1 ≥ 1 && !visited[x, y-1]
            H[x,y-1] = false
            visited[x, y-1] = true
            dig(x,y-1)

        elseif left == 3 && x-1 ≥ 1 && !visited[x-1, y]
            V[x-1,y] = false
            visited[x-1, y] = true
            dig(x-1,y)

        elseif right == 3 && x+1 ≤ n && !visited[x+1, y]
            V[x,y] = false
            visited[x+1, y] = true
            dig(x+1,y)
        end

        if up == 4 && y+1 ≤ n && !visited[x, y+1]
            H[x,y] = false
            visited[x, y+1] = true
            dig(x,y+1)

        elseif down == 4 && y-1 ≥ 1 && !visited[x, y-1]
            H[x,y-1] = false
            visited[x, y-1] = true
            dig(x,y-1)

        elseif left == 4 && x-1 ≥ 1 && !visited[x-1, y]
            V[x-1,y] = false
            visited[x-1, y] = true
            dig(x-1,y)

        elseif right == 4 && x+1 ≤ n && !visited[x+1, y]
            V[x,y] = false
            visited[x+1, y] = true
            dig(x+1,y)
        end
    end

    dig(1,1)
    return H,V
end
```

Out[6]: make_maze (generic function with 1 method)

Problem 2 - Find path from `1,n` to `n,1`

Next, write a function with the syntax

```
pathx, pathy = find_path(H,V)
```

which finds a path in the maze `H,V` between the coordinates `1,n` and `n,1` using the following algorithm:

1. Again create an array `visit` to keep track of visited cells
2. Also initialize empty vectors `pathx,pathy` to store the final path
3. Create a recursive function `recur(x,y)` which performs the following:
   A. If the position `x==n` and `y==1` is found, insert these values into `pathx,pathy` and return `true`
   B. Otherwise, consider each neighbor of `x,y`. If the cell is valid, the maze has no wall in that direction, and the cell has not been visited, apply `recur` to the neighbor cell.
   C. If any of the calls to `recur` returns `true`, insert `x,y` into `pathx,pathy` and return `true`
4. Call `recur(1,n)` and return `pathx,path`

In [7]:
```
function find_path(H,V)
    n = size(H, 1)
    visited = falses(n,n)
    pathx = Int64[]
    pathy = Int64[]

    found = false

    function recur(x,y)
        visited[x,y] = true

        if x == n && y == 1
            push!(pathx, x)
            push!(pathy, y)
            return true

        else
            if x-1 ≥ 1 && !visited[x-1,y] && !V[x-1,y] #LEFT??
                visited[x-1,y] = true
                found = recur(x-1,y)
                if found
                    push!(pathx, x-1)
                    push!(pathy, y)
                    return true
                end
            end

            if x+1 ≤ n && !visited[x+1,y] && !V[x,y] #RIGHT
                visited[x+1,y] = true
                found = recur(x+1,y)
                if found
                    push!(pathx, x+1)
                    push!(pathy, y)
                    return true
                end
            end

            if y-1 ≥ 1 && !visited[x,y-1] && !H[x,y-1] #DOWN
                visited[x,y-1] = true
                found = recur(x,y-1)
                if found
                    push!(pathx, x)
                    push!(pathy, y-1)
                    return true
                end
            end

            if y+1 ≤ n && !visited[x,y+1] && !H[x,y] #UP
                visited[x,y+1] = true
                found = recur(x,y+1)
                if found
                    push!(pathx, x)
                    push!(pathy, y+1)
                    return true
                end
            end
```

```
            if found
                return true
            else
                return false
            end
        end

        return found
    end

    #found = recur(1,n)
    recur(1,n)

    push!(pathx, 1)
    push!(pathy, n)
    return pathx, pathy
end
```

Out[7]: find_path (generic function with 1 method)

Problem 3 - Large maze test

Finally, run the code below to illustrate your codes.

In [8]:
```
n = 25
H,V = make_maze(n);
plot_maze(H,V);
x, y = find_path(H,V)
plot(x .- 0.5, y .- 0.5, color="r", linewidth=4);
```