Mihel HW2

1a) $\begin{pmatrix} .45 \\ .95 \\ .2 \\ -.05 \\ .6 \end{pmatrix} = x_{n-1}$ $\begin{pmatrix} .5 \\ .9 \\ .3 \\ -.1 \\ .5 \end{pmatrix} = x_n$ Absolute error $= \|\hat{x} - x^*\|_p$ $p=1$

$|.5-.45|+|.9-.95|+|.3-.2|+|-.1-(-.05)|+|.5-.6| = \underline{.35} = \text{abs err}$

$\frac{.35}{.45+.95+.2+(-.05)+.06} = \underline{.156} = \text{rel Err}$

b) $p=2$ abserr $= \sqrt{|.45-.5|^2+|.9-.95|^2+|.3-.2|^2+|-.\blacksquare-(-.05)|^2+|.5-.6|^2} = \underline{.166}$

$\frac{.166}{\sqrt{|.5|^2+|.9|^2+|.3|^2+|-.1|^2+|.5|^2}} = \underline{.140} = \text{rel err}$

c) $p=\infty$, so take max value diff $.95-.9 = \underline{.05} = \text{abs err}$ relerr $= \frac{.05}{.95} = \underline{.0526}$

d) least restrictive is the lowest error, which converges faster, so $p=\infty$ is least restrictive

$\begin{pmatrix} .49 \\ .92 \\ .4 \\ -.09 \\ .51 \end{pmatrix} = x_{n-1}$ $p=1$: $|.5-.49|+|.9-.92|+|.3-.4|+|-.1-(-.09)|+|.5-.51| = \underline{.15} = \text{abs err}$

$\frac{.15}{.49+.92+.4+(-.09)+.51} = \underline{.062} = \text{rel err}$

$p=2$ $\sqrt{|.5-.49|^2+|.9-.92|^2+|.3-.4|^2+|-.1+(-.09)|^2+|.5-.51|^2} = \underline{.103} = \text{abs err}$

$\frac{.103}{\sqrt{|.5|^2+|.9|^2+|.3|^2+|-.1|^2+|.5|^2}} = \underline{.087} = \text{rel err}$

$p=\infty$ greatest difference $|.3-.4| = \underline{.1} = \text{abs err}$ $\frac{.1}{.92} = \underline{.108} = \text{rel err}$

p_∞ almost doubled; relerr increases as p increases. p_1 now has the lowest error, which means p_1 is least restrictive. This means no error sampling is best, and so it is wise, if time permits, to choose multiple values.

$\rightarrow$

3a) $I = \int_2^4 \frac{x}{\sqrt{x^2-1}}\,dx$

$u = x^2 - 1$ $\quad$ $x=2 \quad u=3$

$du = 2x\,dx$ $\quad$ $x=4 \quad u=15$

$x\,dx = \frac{1}{2}\,du$

Substitute new values in:

$$I = \int_3^{15} u^{-\frac{1}{2}} \cdot \frac{1}{2}\,du \rightarrow \frac{1}{2}\int_3^{15} u^{-\frac{1}{2}}\,du = \frac{1}{2}\cdot\left[2u^{\frac{1}{2}}\right]_3^{15} = \underline{\underline{\sqrt{15} - \sqrt{3}}}\ \text{ANS}$$