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P&P Pset

1 To obtain the marginal joint distribution from the multivariate normal random variables (T_i, ϵ_i) the irrelevant variables are dropped from the mean vector and the covariance matrix. Thus the joint distribution of (T_i, ϵ_i) is multivariate normal and can be expressed as

$$\begin{pmatrix} T_i \\ \epsilon_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

From this we can see that the two variables T_i and ϵ_i are uncorrellated as $\rho = 0$ in this case. As a result it is true that $T_i \perp \epsilon_i$. However, in the case of X_i it is unknown if that variable is correlated with either T_i or ϵ_i and so it is unknown if $T_i \perp X_i$. This does not change the relationship between T_i and ϵ_i.

2 .

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2 $\begin{bmatrix} X_i \\ T_i \end{bmatrix} \sim N\left(0, \Omega\right) \quad \Omega = \begin{bmatrix} 1 & \rho_2 \\ \rho_2 & 1 \end{bmatrix}$

$X_i = aT_i + u_i \qquad = \begin{bmatrix} E[X_i^2] & E[X_i \cdot T_i] \\ E[X_i \cdot T_i] & E[T_i^2] \end{bmatrix}$

$$\rho_2 = E[X_i \cdot T_i]$$

$$E[T_i(aT_i + u_i)]$$

$$E[aT_i^2 + T_i^2 u_i]$$

$$E[aT_i^2] + E[T_i^2 u_i]$$

$$aE[T_i^2] + E[T_i^2]E[u_i]$$

$$a(1) + (1)(0)$$

$$\boxed{\rho_2 = a}$$

3

$$Y_i = \gamma T_i + \beta(\alpha T_i + u_i) + \epsilon_i$$

$$= \gamma T_i + \alpha\beta T_i + \beta u_i + \epsilon_i$$

$$Y_i = T_i(\gamma + \alpha\beta) + \beta u_i + \epsilon_i$$

$$E[Y_i] = E[T_i(\gamma+\alpha\beta) + \beta u_i + \epsilon_i]$$

$$= E[(\gamma+\alpha\beta)T_i] + E[\beta u_i] + E[\epsilon_i]$$

$$E[Y_i] = (\gamma + \alpha\beta)E[T_i]$$

$$\boxed{E[Y_i | T_i] = (\gamma + \rho_2\beta)T_i}$$

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4 From above $E[Y_i \mid T_i = t_i] = (\gamma + \rho_2 \beta) t_i$

The regression coefficient $\hat{\beta} = (\gamma + \rho_2 \beta)$ and only provides an unbiased estimate of γ if the term $\rho_2 \beta = 0$ which can only be the case if $\rho_2 = 0$. We can see this condition as ~~a~~ further evidence that the relationship betwee X_i and T_i is expressed through the coefficient ρ_2.

5

$$\epsilon_i = bT_i + cX_i + v_i$$

5 $$Cov(X_i, \epsilon_i) = \begin{pmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{pmatrix} = \begin{pmatrix} E[X_i^2] & E[X_i \cdot \epsilon_i] \\ E[\epsilon_i \cdot X_i] & E[\epsilon_i] \end{pmatrix}$$

$$Cov(T_i, \epsilon_i) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} E[T_i^2] & E[T_i \epsilon_i] \\ E[T_i \epsilon_i] & E[\epsilon_i^2] \end{pmatrix}$$

$$0 = E[T_i \cdot \epsilon_i] = E[T_i(bT_i + cX_i + v_i)]$$

$$= bE[T_i^2] + cE[T_iX_i] + E[v_i]E[T_i]$$

From 2

$$\boxed{0 = b + c(\rho_2)}$$

$$\rho_1 = E[X_i \cdot \epsilon_i] = E[X_i(bT_i + cX_i + v_i)]$$

$$= bE[T_i \cdot X_i] + cE[X_i^2] + E[X_i]E[v_i]$$

$$\boxed{\rho_1 = b\rho_2 + c}$$

6 $$Y_i = \gamma T_i + \beta X_i + \epsilon_i = \gamma T_i + \beta X_i + bT_i + cX_i + v_i$$

$$E[Y_i] = E[\gamma T_i + \beta X_i + bT_i + cX_i + v_i]$$

$$= E[(\gamma + b)T_i] + E[(\beta + c)X_i] + E[v_i]$$

$$E[Y_i] = (\gamma + b)E[T_i] + (\beta + c)E[X_i]$$

$$E[Y_i \mid X_i = x, T_i = t] = (\gamma + b)t + (\beta + c)x$$

$$\boxed{\blacksquare = (\gamma + b) \qquad \blacklozenge = (\beta + c)}$$

7 The regression coefficient $= \gamma + b$ identifies the true value of γ only if the term b is equal to zero which from the previous equation in problem 5 we can see is only true when the two variables, X_i and T_i, are uncorrelated or the term p_2 is equal to zero.

8 This is not always true.

$\varphi = (\gamma + p_2\beta)$ $= (\gamma + b)$

If $|p_2\beta| < |b|$ then it is better to exclude the X_i as a condition

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