The Kinematic Bicycle Model

Course 1, Module 3, Lesson 2

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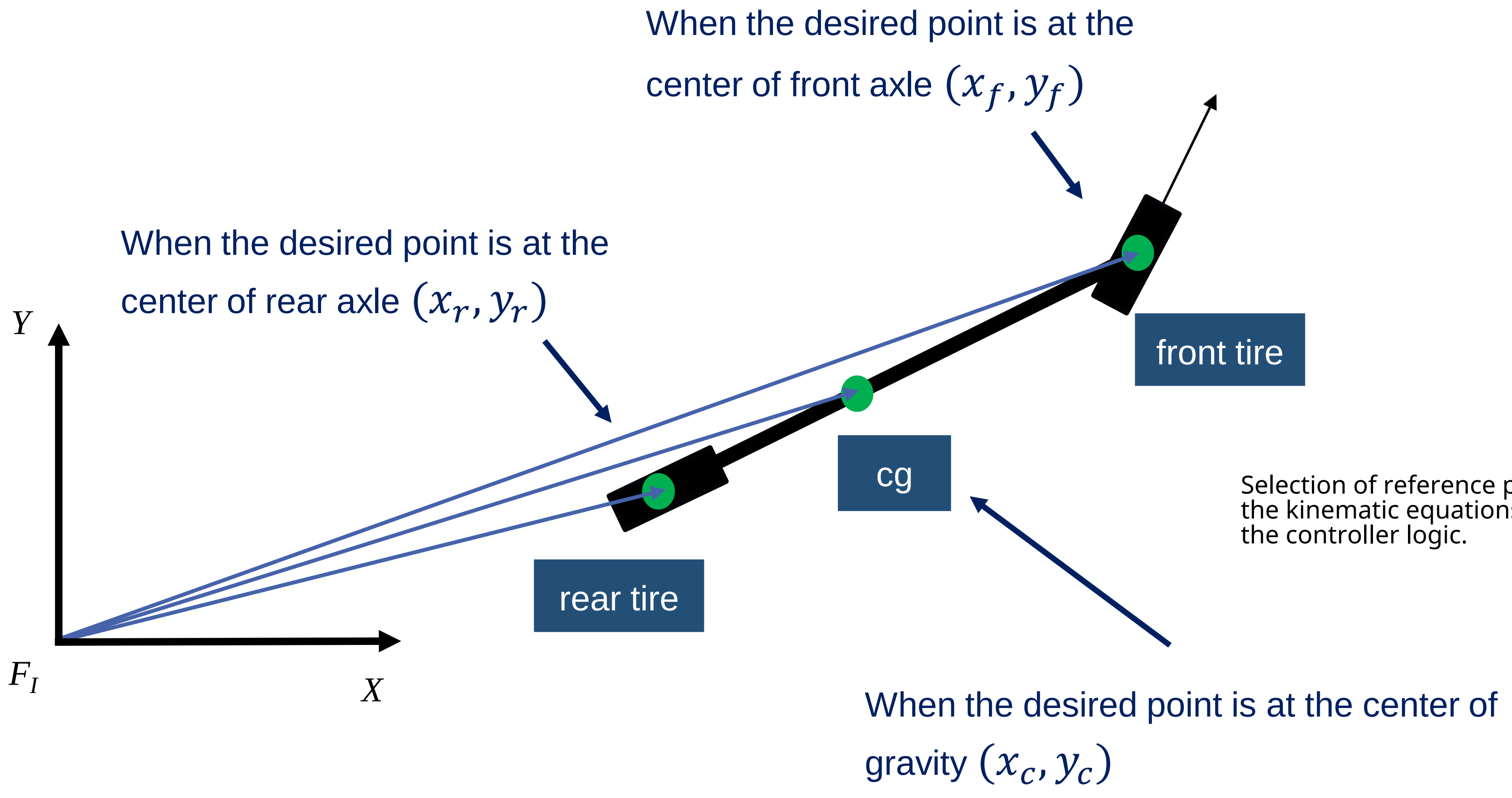
In this video...

- Learn about slip angle
- Develop the kinematic bicycle model

Bicycle Kinematic Model

- 2D bicycle model (simplified car model)
- Front wheel steering



Selection of reference points changes the kinematic equations and hence the controller logic.

Two-Wheeled Robot Kinematic Model

- Rear Wheel Reference Point
  - Apply Instantaneous Center of Rotation (ICR)

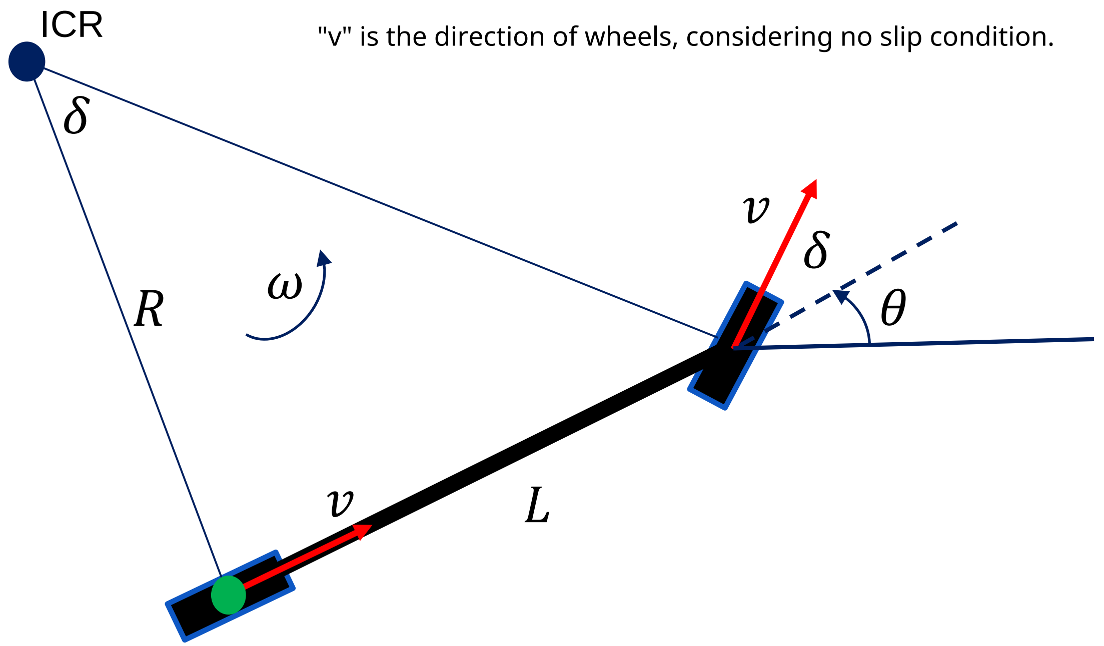
$$\dot{\theta} = \omega = \frac{v}{R}$$

  - Similar triangles

$$\tan\delta = \frac{L}{R}$$

  - Rotation rate equation

$$\dot{\theta} = \omega = \frac{v}{R} = \frac{v\tan\delta}{L}$$



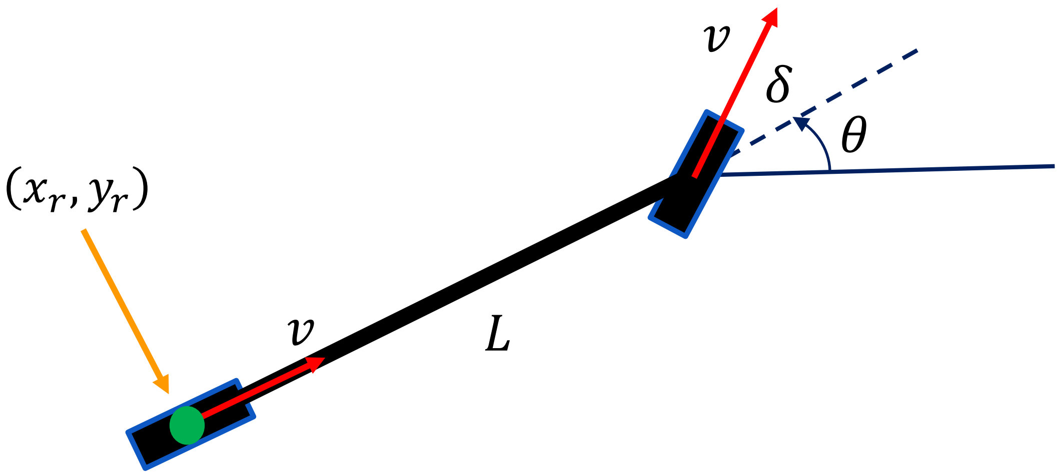
Rear Axle Bicycle Model

- If the desired point is at the center of the rear axle

$$\dot{x}_r = v\cos\theta$$

$$\dot{y}_r = v\sin\theta$$

$$\dot{\theta} = \frac{v\tan\delta}{L}$$



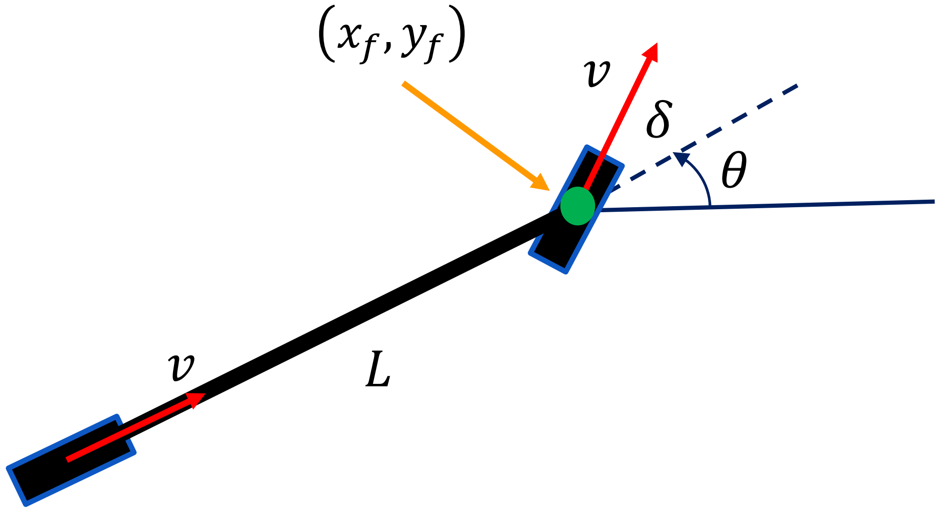
Bicycle Kinematic Model

- If the desired point is at the center of the front axle

$$\dot{x}_f = v\cos(\theta + \delta)$$

$$\dot{y}_f = v\sin(\theta + \delta)$$

$$\dot{\theta} = \frac{v\sin\delta}{L}$$



Bicycle Kinematic Model

- If the desired point is at the center of the gravity (cg)

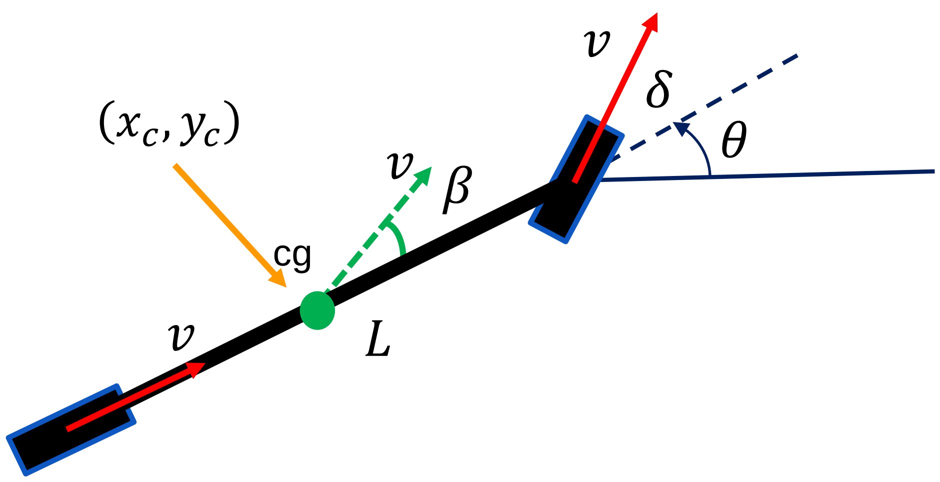
$$\dot{x}_c = v\cos(\theta + \beta)$$

$$\dot{y}_c = v\sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v\cos\beta\tan\delta}{L}$$

$$\beta = \tan^{-1}\left(\frac{l_r\tan\delta}{L}\right)$$

Direction of motion is different at CG than the heading and this difference can be represented as side slip angle beta. It is the difference between the velocity direction at CG and the heading angle.



State-space Representation

- Modify CG kinematic bicycle model to use steering rate input
  - State: $[x, y, \theta, \delta]^T$ Inputs: $[v, \varphi\ ]^T$

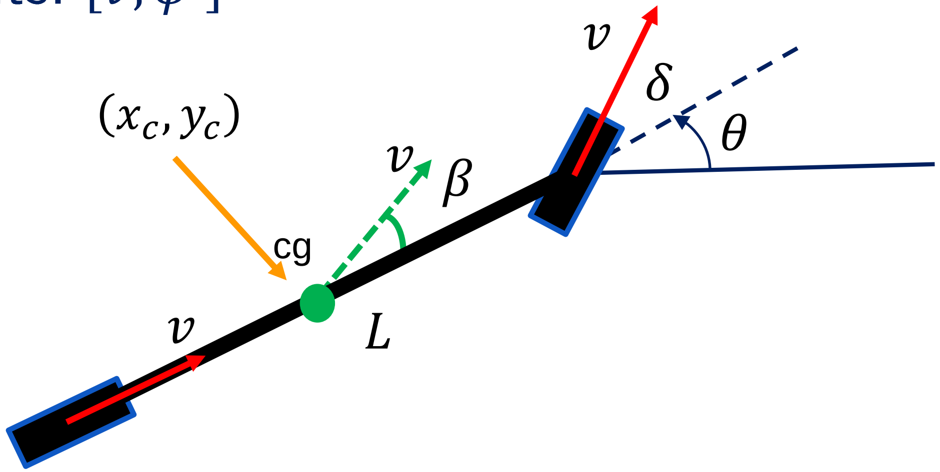
$$\dot{x}_c = v\cos(\theta + \beta)$$

$$\dot{y}_c = v\sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v\cos\beta\tan\delta}{L}$$

$$\dot{\delta} = \varphi$$

Modified Input: rate of change of steering angle



Summary

- What we have learned from this lesson:
  - Formulation of a kinematic bicycle model
  - State-space representation.
- What is next?
  - We will explore the Dynamics in 2D and how to define a vehicle model as a 2D dynamic system.