

Geometric steering control II – Stanley Controller

Course 1, Module 6, Lesson 3



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Learning Objectives

- Derive the Stanley path tracking controller
- Analyze the evolution of heading and crosstrack errors
- Evaluate convergence from arbitrary starting points

Stanley Controller Approach

- Stanley method is the path tracking approach used by Stanford University's Darpa Grand Challenge team
 - Uses the center of the front axle as a reference point
 - Look at both the error in heading and the error in position relative to the closest point on the path
 - Define an intuitive steering law to
 - Correct heading error
 - Correct position error
 - Obey max steering angle bounds



Heading control law

- Combine three requirements:
 - Steer to align heading with desired heading (proportional to heading error)

$$\delta(t) = \psi(t)$$

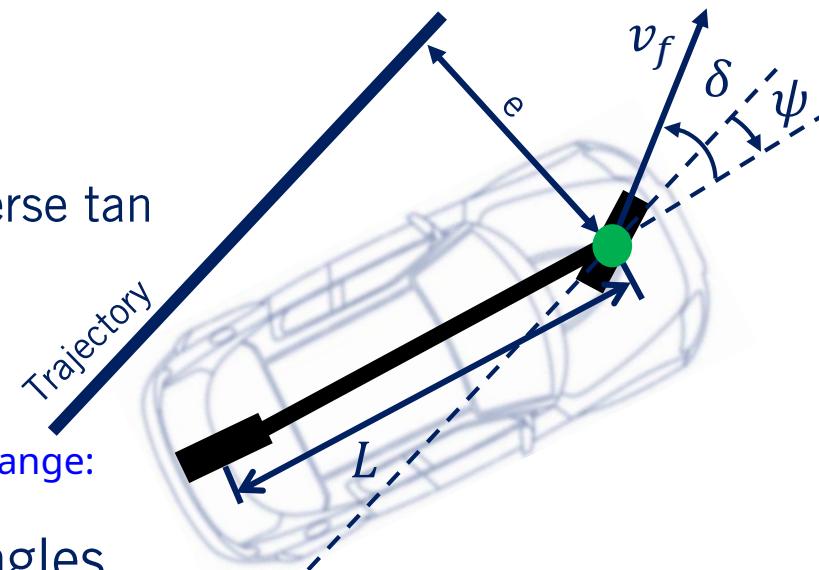
- Steer to eliminate crosstrack error
 - Essentially proportional to error
 - Inversely proportional to speed
 - Limit effect for large errors with inverse tan
 - Gain k determined experimentally

$$\delta(t) = \tan^{-1} \left(\frac{ke(t)}{v_f(t)} \right)$$

Angular range:
[-pi, pi]

- Maximum and minimum steering angles

$$\delta(t) \in [\delta_{min}, \delta_{max}]$$

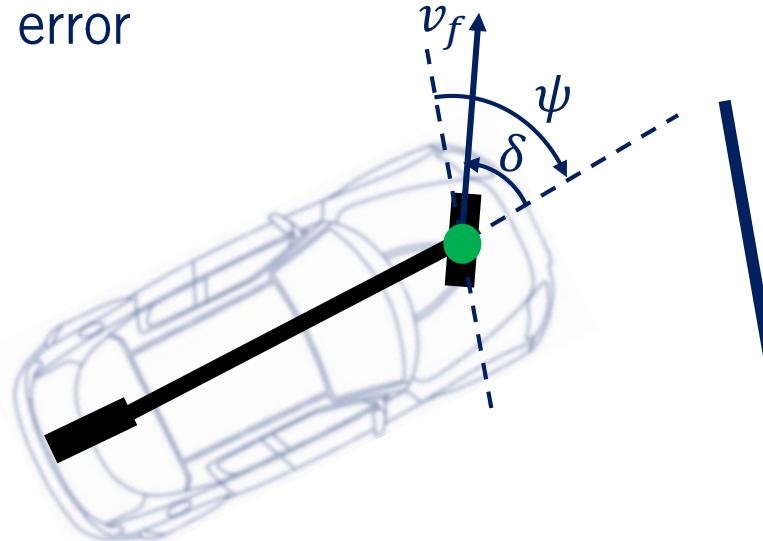


Combined steering law

- Stanley Control Law

$$\delta(t) = \psi(t) + \tan^{-1} \left(\frac{ke(t)}{v_f(t)} \right), \quad \delta(t) \in [\delta_{min}, \delta_{max}]$$

- For large heading error, steer in opposite direction
 - The larger the heading error, the larger the steering correction
 - Fixed at limit beyond maximum steering angle, assuming no crosstrack error

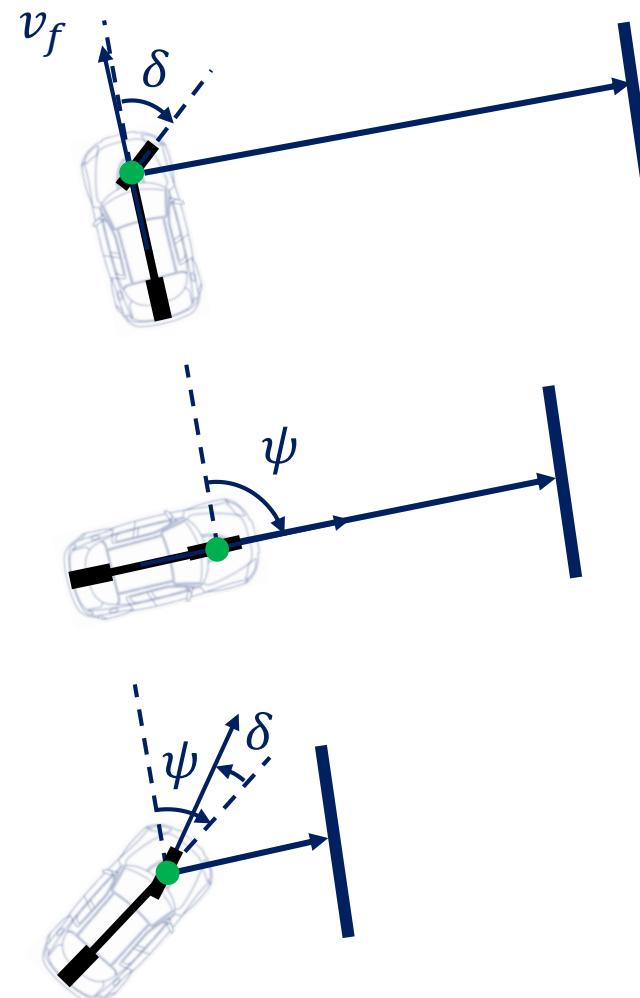


Combined steering law

- For larger positive crosstrack error

$$\tan^{-1} \left(\frac{ke(t)}{v_f(t)} \right) \approx \frac{\pi}{2} \rightarrow \delta(t) \approx \psi(t) + \frac{\pi}{2}$$

- As heading changes due to steering angle, the heading correction counteracts the crosstrack correction, and drives the steering angle back to zero
- The vehicle approaches the path, crosstrack error drops, and steering command starts to correct heading alignment.



Error Dynamics

- The error dynamics when not at maximum steering angle are:

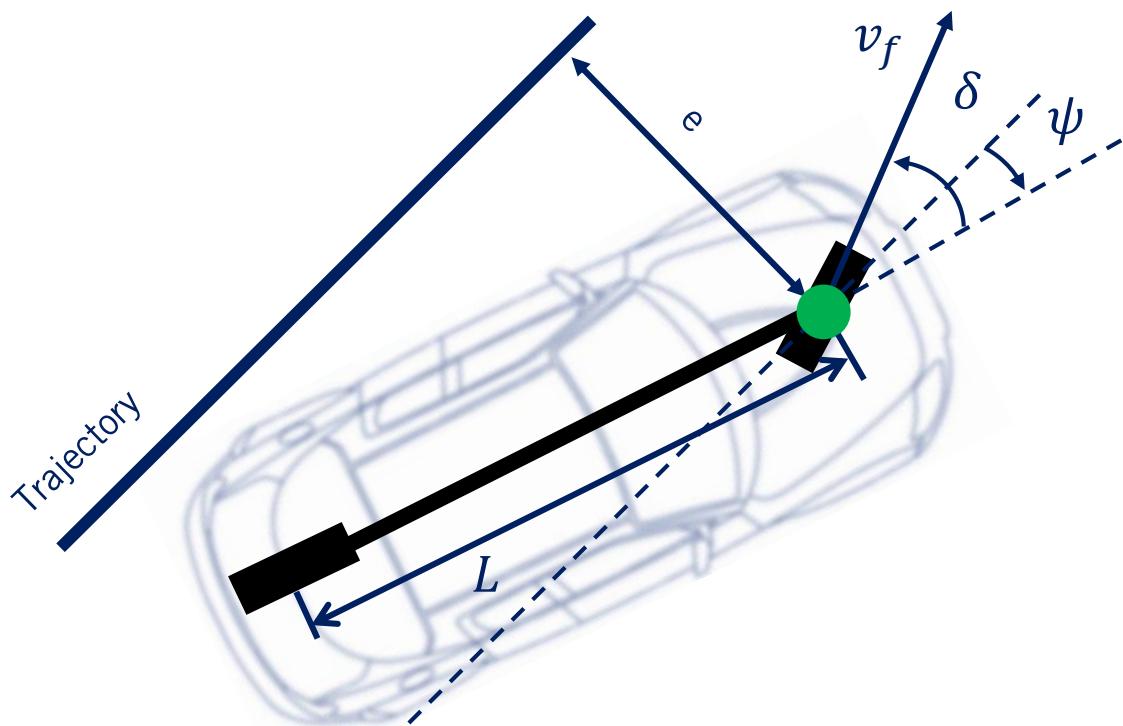
$$\begin{aligned}\dot{e}(t) &= -v_f(t) \sin(\psi(t) - \delta(t)) = -v_f(t) \sin\left(\tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right)\right) \\ &= \frac{-ke(t)}{\sqrt{1 + \left(\frac{ke(t)}{v_f}\right)^2}}\end{aligned}$$

- For small crosstrack errors, leads to exponential decay characteristics

$$\dot{e}(t) \approx -ke(t)$$

Case Study

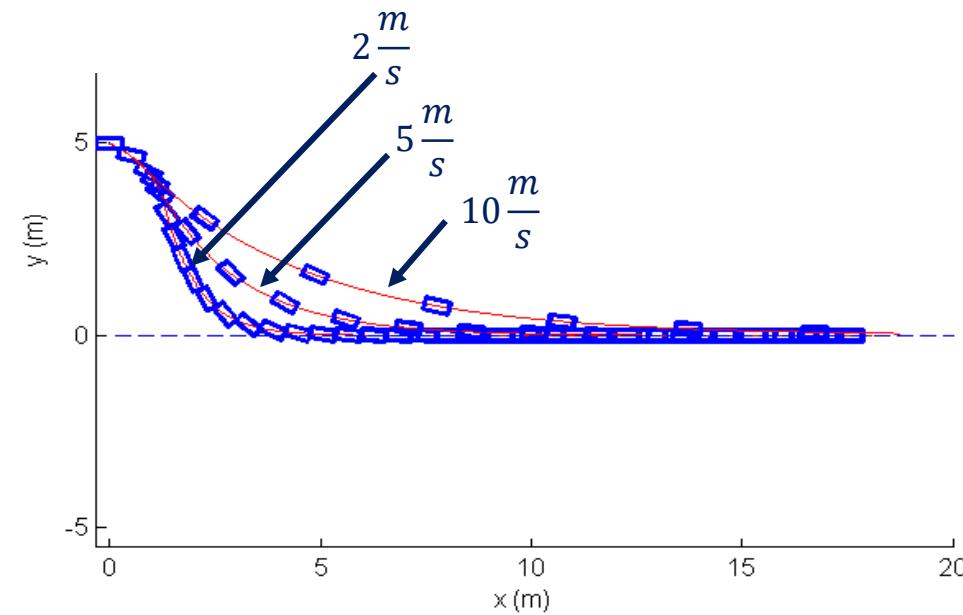
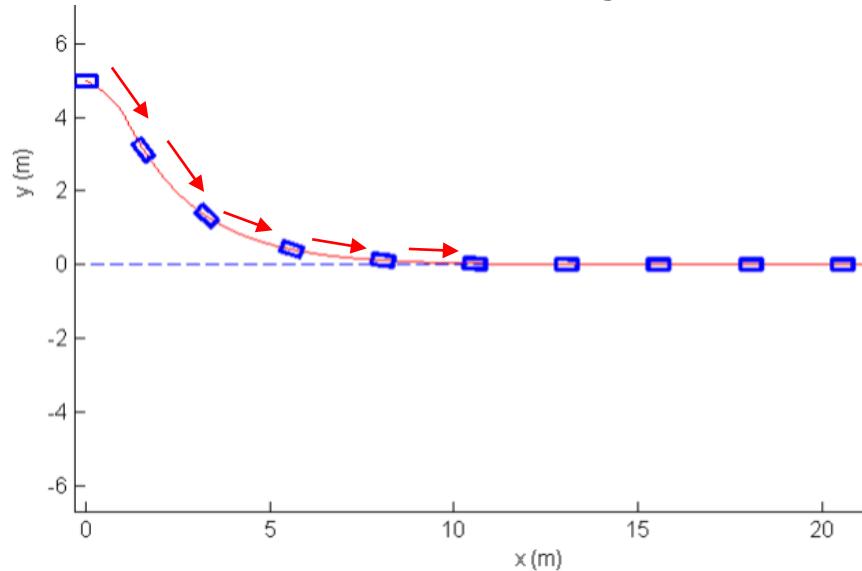
- Two scenarios:
 - Large initial crosstrack error
 - Large initial heading error



Case Study 1

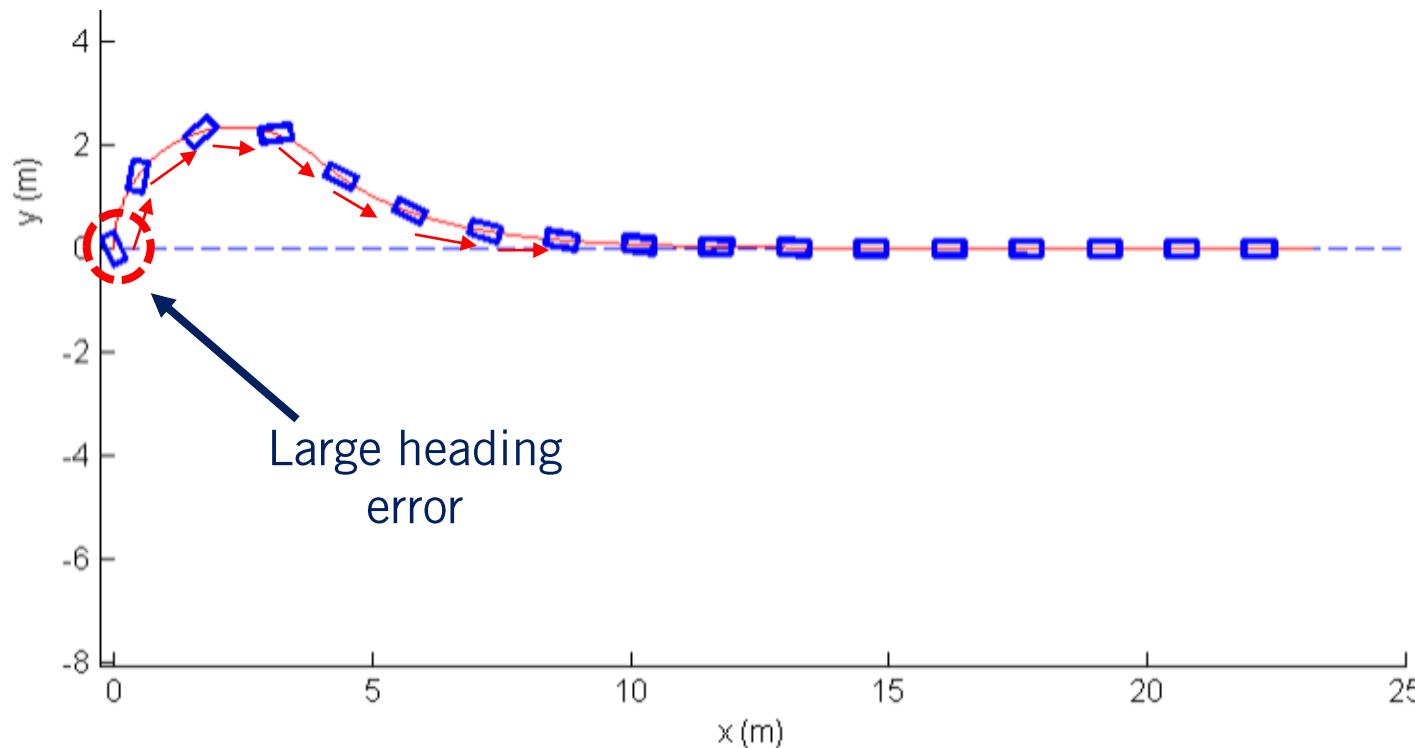
- Large initial crosstrack error

- Crosstrack error of 5 meters
- Max steer $\delta = 25^\circ$, forward speed of $v_f = 5 \frac{m}{s}$
- Gain $k = 2.5$, length $L = 1 m$
- Effect of speed variation
 - $v_f = 2, 5, 10 \frac{m}{s}$



Case Study 2

- Large initial heading error
 - Max steer $\delta = 25^\circ$, forward speed of $v_f = 5 \frac{m}{s}$
 - Gain $k = 2.5$, length $L = 1m$



Adjustment

- Low speed operation
 - Inverse speed can cause numerical instability
 - Add softening constant to controller

$$\delta(t) = \psi(t) + \tan^{-1} \left(\frac{ke(t)}{k_s + v_f(t)} \right)$$

- Extra damping on heading
 - Becomes an issue at higher speeds in real vehicle
- Steer into constant radius curves
 - Improves tracking on curves by adding a feedforward term on heading

Summary

In this lesson, you learned

- How to apply the Stanley controller to path tracking
- What the convergence properties of the Stanley controller are
- How to improve real-world performance of the Stanley controller

What is next?

- Advanced control strategies such as Model Predictive Control (MPC) for lateral control