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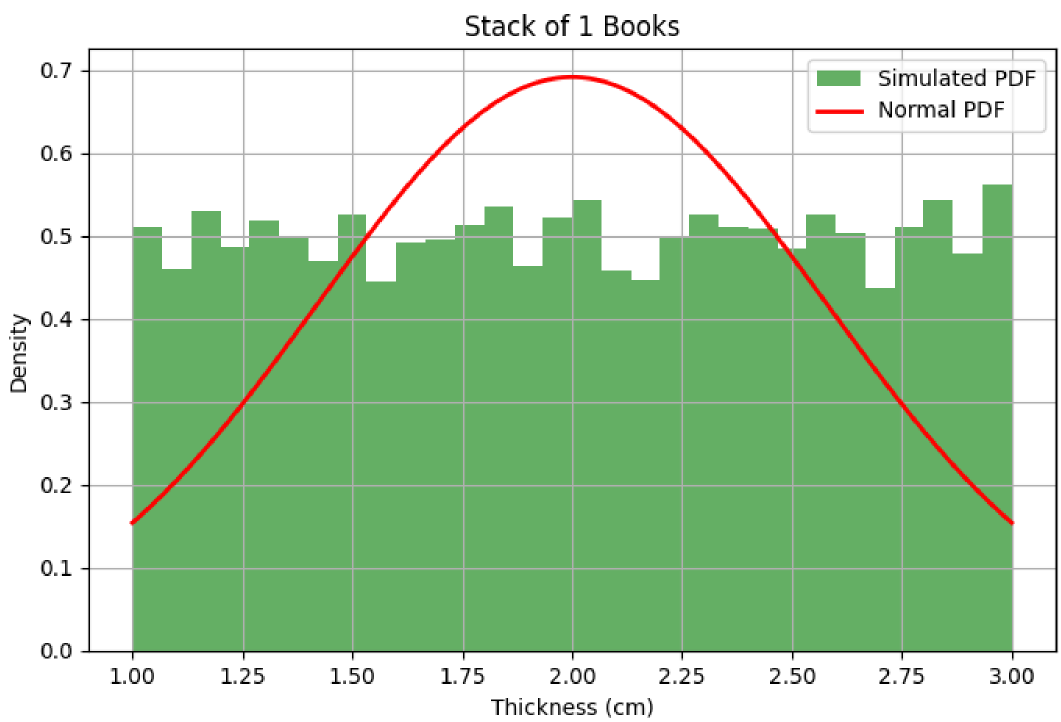
EE381
11/17/24

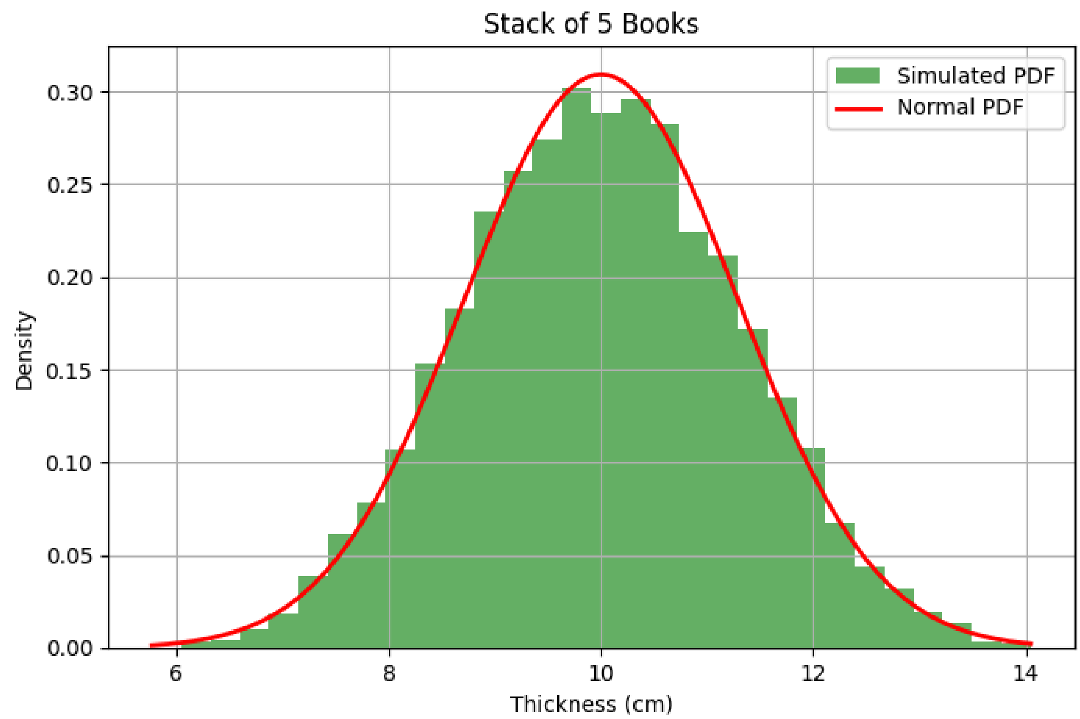
Project 4: Central Limit Theorem
Simulate RVs with Arbitrary Distributions

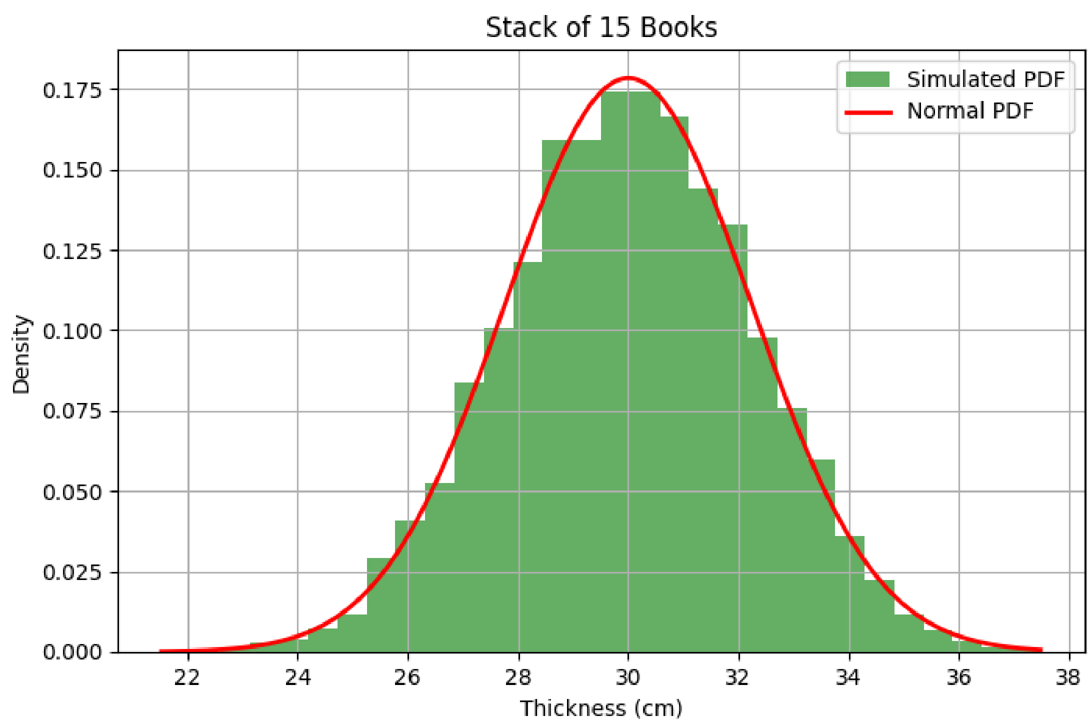
1. The Central Limit Theorem

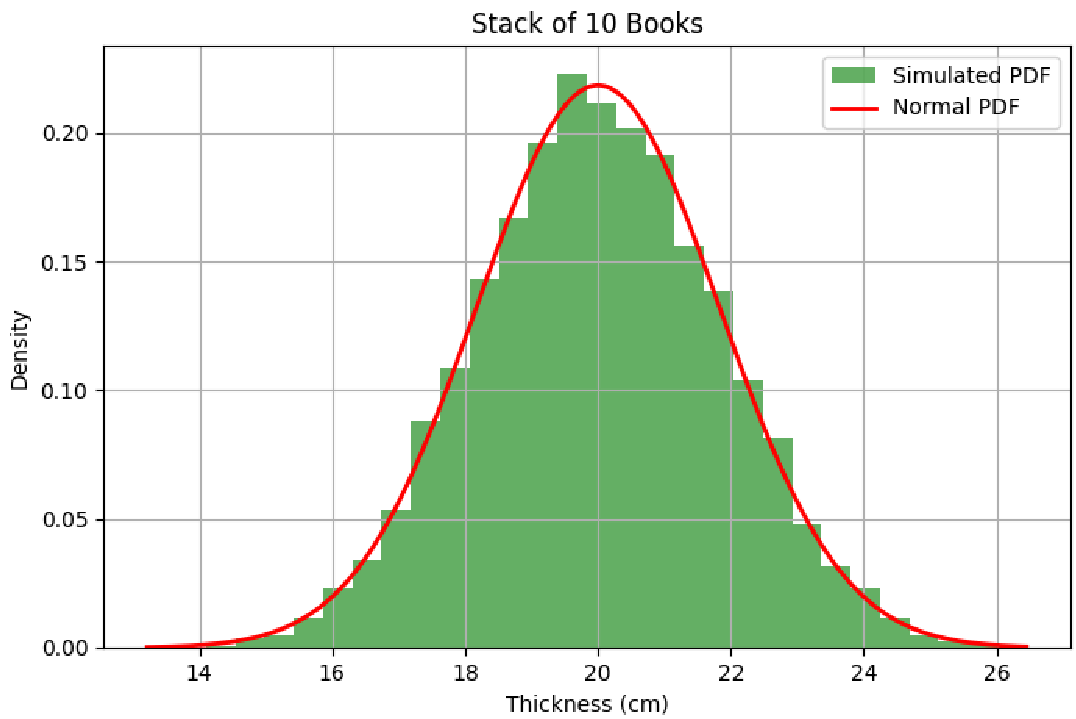
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import norm

a, b = 1, 3
N = 10000

mu_w = (a + b) / 2
sigmaW= np.sqrt((b - a) ** 2 / 12)

stack_sizes = [1, 5, 10, 15]
results = []

for n in stack_sizes:
    mu_s = n * mu_w
   sigmaS = sigmaW * np.sqrt(n)
   results.append((n, mu_s, sigmaS))

   stackThicknesses = [np.sum((b - a) * np.random.rand(n) + a) for _ in range(N)]
   plt.figure(figsize=(8, 5))
   plt.hist(stackThicknesses, bins=30, density=True, alpha=0.6, color='g',
label="Simulated PDF")

   x = np.linspace(min(stackThicknesses), max(stackThicknesses), 1000)
   plt.plot(x, norm.pdf(x, mu_s, sigmaS), 'r-', lw=2, label="Normal PDF")

   plt.title(f"Stack of {n} Books")
   plt.xlabel("Thickness (cm)")
   plt.ylabel("Density")
   plt.legend()
```

```
    plt.grid()
    plt.show()

results_df = pd.DataFrame(results, columns=["Number of Books (n)", "Mean Thickness (cm)", "Std. Dev. Thickness (cm)"])
print(results_df)
```




Stack of 5 Books
Simulated PDF
Normal PDF
Density
0.00
0.05
0.10
0.15
0.20
0.25
0.30
6
8
10
12
14
Thickness (cm)

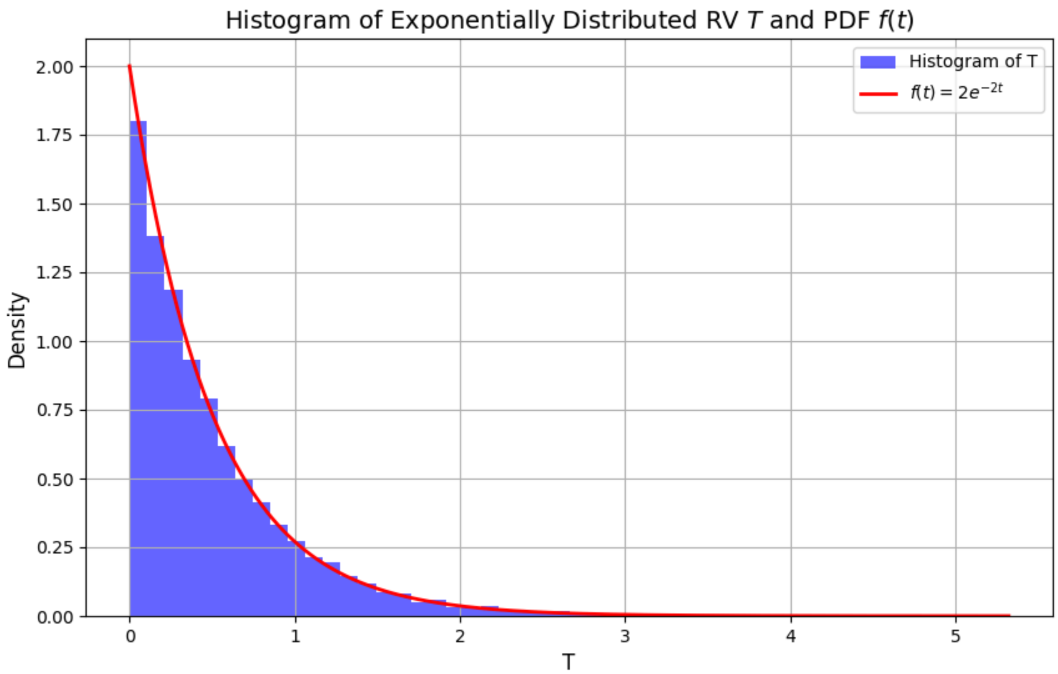

Stack of 15 Books
Simulated PDF
Normal PDF
Density
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
22
24
26
28
30
32
34
36
38
Thickness (cm)



Number of books n	Mean thickness of a stack of n books (cm)	Standard deviation of the thickness for n books
$n = 1$	$\mu_w =$ 2	$\sigma_w =$ 0.577
$n = 5$	$\mu_w =$ 10	$\sigma_w =$ 1.29
$n = 10$	$\mu_w =$ 20	$\sigma_w =$ 1.825
$n = 15$	$\mu_w =$ 30	$\sigma_w =$ 2.236

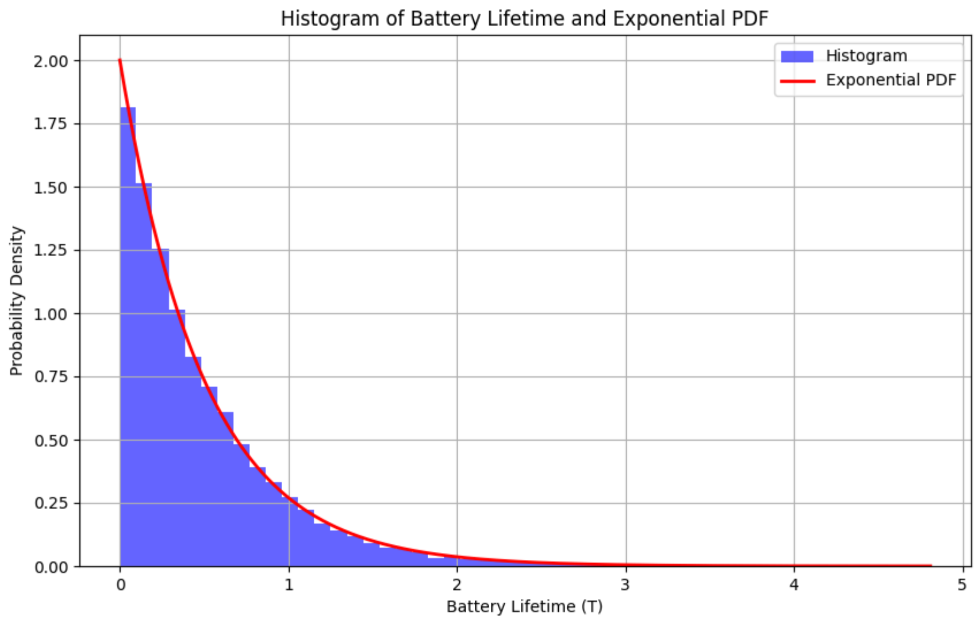
2. Exponentially Distributed Random Variables

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import norm

a, b = 1, 3
N = 10000

mu_w = (a + b) / 2
sigmaW= np.sqrt((b - a) ** 2 / 12)

stack_sizes = [1, 5, 10, 15]
results = []

for n in stack_sizes:
    mu_s = n * mu_w
   sigmaS = sigmaW * np.sqrt(n)
   results.append((n, mu_s, sigmaS))

   stackThicknesses = [np.sum((b - a) * np.random.rand(n) + a) for _ in range(N)]
   plt.figure(figsize=(8, 5))
   plt.hist(stackThicknesses, bins=30, density=True, alpha=0.6, color='g',
label="Simulated PDF")

   x = np.linspace(min(stackThicknesses), max(stackThicknesses), 1000)
   plt.plot(x, norm.pdf(x, mu_s, sigmaS), 'r-', lw=2, label="Normal PDF")

   plt.title(f"Stack of {n} Books")
   plt.xlabel("Thickness (cm)")
   plt.ylabel("Density")
   plt.legend()
   plt.grid()
   plt.show()

results_df = pd.DataFrame(results, columns=["Number of Books (n)", "Mean Thickness
(cm)", "Std. Dev. Thickness (cm)"])
print(results_df)
```


Histogram of Exponentially Distributed RV T and PDF $f(t)$
Histogram of T
$f(t) = 2e^{-2t}$
Density
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
1
2
3
4
5
T

3. Distribution of the Sum of RVs

```
import numpy as np
import matplotlib.pyplot as plt

N = 10000
lambda_param = 2
num_bins = 50

U = np.random.uniform(0, 1, N)
T = -np.log(1 - U) / lambda_param

plt.figure(figsize=(10, 6))
plt.hist(T, bins=num_bins, density=True, alpha=0.6, color='blue', label='Histogram')

t = np.linspace(0, np.max(T), 1000)
f_T = lambda_param * np.exp(-lambda_param * t)
plt.plot(t, f_T, 'r', lw=2, label='Exponential PDF')

plt.title('Histogram of Battery Lifetime and Exponential PDF')
plt.xlabel('Battery Lifetime (T)')
plt.ylabel('Probability Density')
plt.legend()
plt.grid(True)
plt.show()
```



1. From the histogram and overlay, it's clear that the probability for S > 1095 (3 years) is extremely low, near 0.
2. The probability that the carton lasts between 2.0 and 2.5 years is effectively 0 due to the rapid decay of the exponential distribution.

QUESTION	ANS.
1. Probability that the carton will last longer than 2.5 years	0.0
2. Probability that the carton will last between 1.50 and 2.0years	0.0