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EE381
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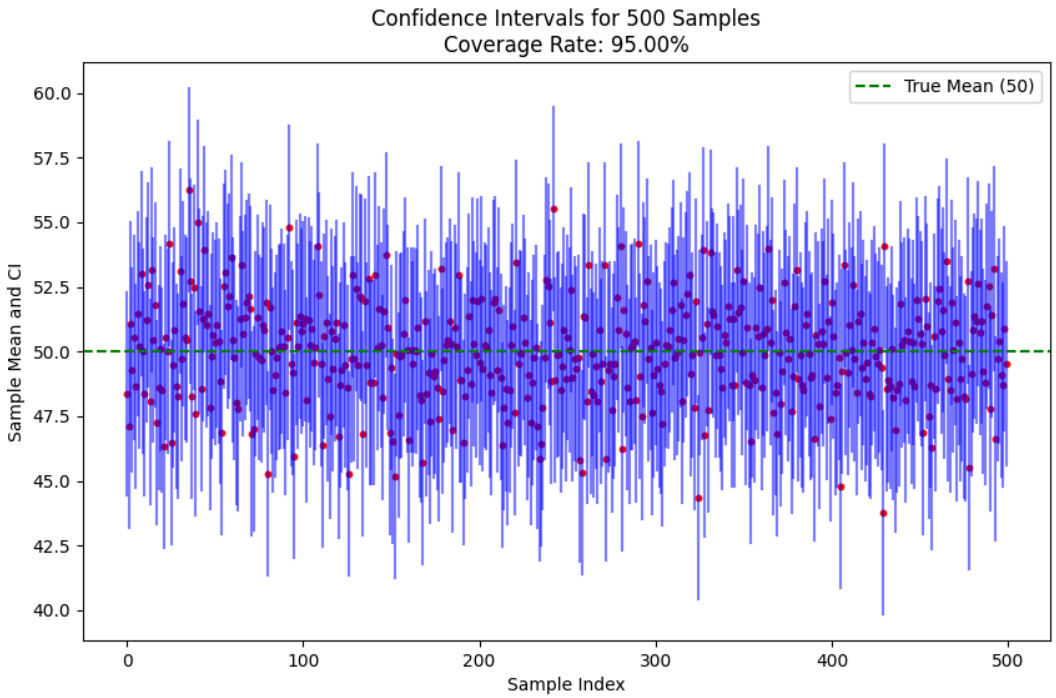
Project 5: Confidence Intervals

1. Effect of Sample Size on Confidence Intervals

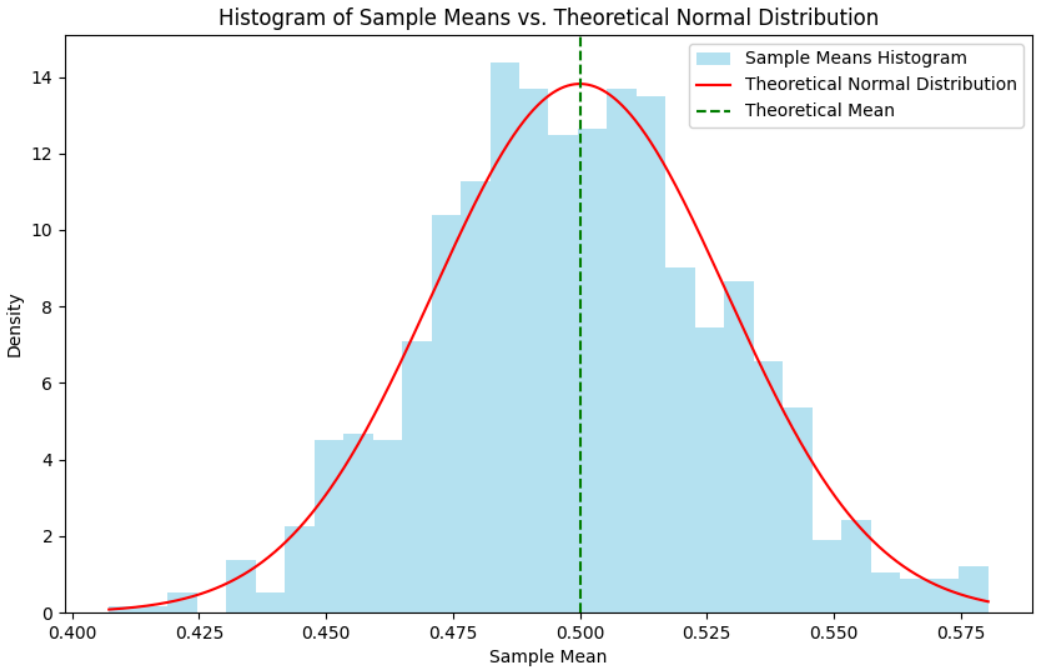
```
import numpy as np
import matplotlib.pyplot as plt

mu = 50
sigma = 10
n = 25
numSamples = 500
z = 1.96


np.random.seed(42)
samples = np.random.normal(mu, sigma, (numSamples, n))

sampleMeans = samples.mean(axis=1)
standardError = sigma / np.sqrt(n)
lowerBounds = sampleMeans - z * standardError
upperBounds = sampleMeans + z * standardError


containsMu = (lowerBounds <= mu) & (upperBounds >= mu)
coverageRate = containsMu.mean() * 100

plt.figure(figsize=(10, 6))
for i in range(numSamples):
   plt.plot([i, i], [lowerBounds[i], upperBounds[i]], color='blue', alpha=0.5)
   plt.scatter(i, sampleMeans[i], color='red', s=10)

plt.axhline(mu, color='green', linestyle='--', label="True Mean (50)")
plt.title(f"Confidence Intervals for {numSamples} Samples\nCoverage Rate:
{coverageRate:.2f}%")
plt.xlabel("Sample Index")
plt.ylabel("Sample Mean and CI")
```

```
plt.legend()
plt.show()

print(f"Out of {numSamples} confidence intervals, {containsMu.sum()} contain the true mean.")
print(f"Coverage rate: {coverageRate:.2f}%")
```



2. Using the Sample Mean to Estimate the Population Mean

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm


a, b = 0, 1
popMean = (a + b) / 2
popVariance = ((b - a) ** 2) / 12
popSTD = np.sqrt(popVariance)


n = 100
num_samples = 1000
np.random.seed(42)
samples = np.random.uniform(a, b, (num_samples, n))

sampleMeans = samples.mean(axis=1)

theoreticalMean = popMean
theoreticalSTD = popSTD / np.sqrt(n)

plt.figure(figsize=(10, 6))
plt.hist(sampleMeans, bins=30, density=True, alpha=0.6, color='skyblue', label="Sample
Means Histogram")

x = np.linspace(min(sampleMeans), max(sampleMeans), 1000)
pdf = norm.pdf(x, loc = theoreticalMean, scale = theoreticalSTD)
plt.plot(x, pdf, 'r-', label="Theoretical Normal Distribution")

plt.title("Histogram of Sample Means vs. Theoretical Normal Distribution")
plt.xlabel("Sample Mean")
plt.ylabel("Density")
plt.axvline(popMean, color='green', linestyle='--', label="Theoretical Mean")
plt.legend()
plt.show()

print(f"Theoretical Mean of Sample Means: {popMean:.4f}")
print(f"Theoretical Standard Deviation of Sample Means: {theoreticalSTD:.4f}")
```



Sample size (n)	**95% Confidence** (Using Normal distribution)	**99% Confidence** (Using Normal distribution)	**95% Confidence** (Using Student's t distribution)	**99% Confidence** (Using Student's t distribution)
5	88.3	93.83	94.93	97.87
40	94.18	98.52	94.85	99.06
120	94.99	99.01	94.7	98.94
200	94.59	98.9	94.88	99.0