ME 608 Fluid Dynamics, M. Wosnik

Problem 7.1
A streamlined, cylindrical body, which can be considered infinitely long into the drawing plane, is immersed into an oncoming flow with uniform velocity U_∞. The flow direction coincides with axis of symmetry, and hence the only force the body will experience is the drag Force F_D. Downstream of the body a "wake" exists, which is the region where the velocity u_2 is less than U_∞.
If u_2/U_∞ at location 2 is known, calculate the drag Force F_D per unit depth acting on the body.

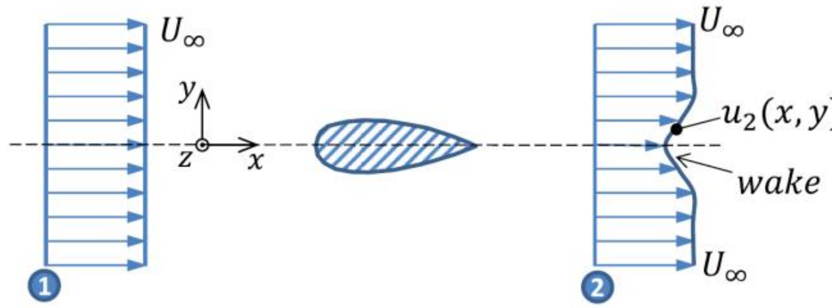
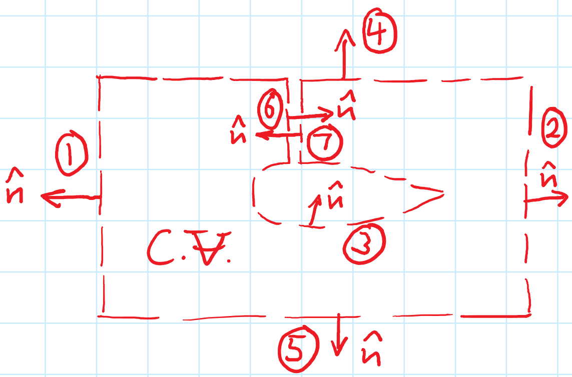


Figure 4: Sketch of streamlined body in uniform flow.

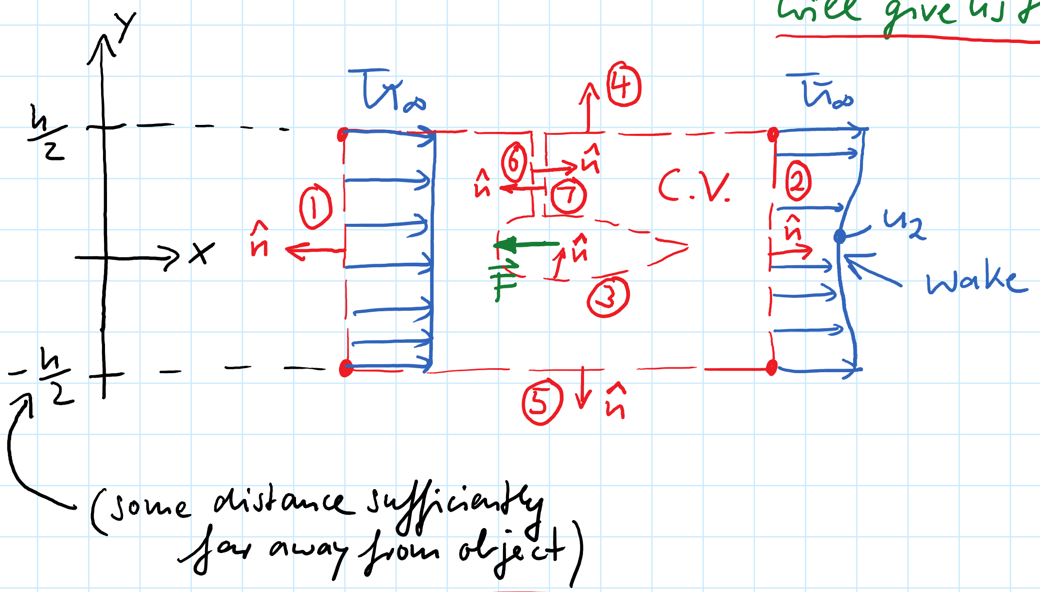
Solution: The key is to select a Control Volume that only contains fluid, but also surrounds the body of interest and includes the inflow and outflow planes where we know the velocity profile.

$\Longrightarrow$ We select a control volume that includes the body, and we extend it outward from the body using a narrow slit. We extend the C.V. in the up & down directions sufficiently far so that the body's presence is no longer felt, i.e. there are no tangential stresses and the pressure is ambient pressure.



Conceptually:
- "narrow slit" w/ surfaces 6 & 7
  - ∴ contributions from 6 & 7 to conservation of mass or momentum will always cancel
- 4 & 5 parallel to inflow direction U_∞.
  Careful: Since there is a wake, and since 4 and 5 are not solid walls, there will be some mass flow out of these control surfaces! (and therefore momentum flow, too.)
- This means we are left with

momentum flow (rate of change of momentum across ①, ②, ④, ⑤ and difference in surface force ① vs. ②, this will give us the force on ③



pressure outside of C.V. is uniform

$P_{①} = P_{②} = P_{④} = P_{⑤} = P_\infty$

Governing Equation

$$\boxed{\cancel{\frac{\partial}{\partial t}}\iiint_{C.V.} \vec{V}\rho\, d\forall + \iint_{C.S.} \vec{V}\rho(\vec{V}\cdot\hat{n})\,dA = \cancel{\vec{F}_B} + \vec{F}_s}$$

assumptions:

- steady state $\frac{\partial}{\partial t} = 0$
- neglect body forces F_B
- pressure uniform outside C.V., P_∞

$$\vec{F}_s = \iint_S \vec{t}\, dS$$

"stress vector", has components of pressure, normal stresses and tangential stresses.

$$\vec{F}_s = \underbrace{\iint_{C.S.①} -p_\infty dS_1 + \iint_{C.S.②} -p_\infty dS_2}_{\text{cancel}} + \underbrace{\iint_{C.S.③} \vec{t}\, dS_3}_{= \vec{F}} + \underbrace{\iint_{C.S.④} -p_\infty dS_4 + \iint_{C.S.⑤} -p_\infty dS_5}_{\text{cancel}}$$

force on body!

∴ conservation of momentum reduces to

$$\boxed{-\vec{F} = \iint_{C.S.} \vec{V}\rho(\vec{V}\cdot\hat{n})\,dA}$$

(Note: if pressure at ① and ② were different, we would have force contributions from $\iint -p\,dA$)

Drag ...

contributions from $\iint -p\,dA$)

Drag $\to$ x-component

$$-F_x = -\vec{F}\cdot\hat{\imath} = \iint_{C.S.\,1} \underbrace{u_1}_{=\bar{U}_\infty} \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=-\bar{U}_\infty} dS_1 + \iint_{C.S.\,2} \underbrace{u_2}_{\text{known}} \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=u_2} dS_2 +$$

$$+ \iint_{C.S.\,3} u \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=0,\ \text{since } \vec{V}\perp\hat{n} \text{ everywhere on body surface } 3} dS_3 +$$

$$+ \iint_{C.S.\,4} \underbrace{u}_{=\bar{U}_\infty} \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=?} dS_4 + \iint_{C.S.\,5} \underbrace{u}_{=\bar{U}_\infty} \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=?\ \to\ \text{conservation of mass}} dS_5$$

$$\underbrace{+ \iint_{C.S.\,6} u \rho \underbrace{(\vec{V}\cdot\hat{n})}_{\neq 0} dS_6 + \iint_{C.S.\,7} u \rho \underbrace{(\vec{V}\cdot\hat{n})}_{\neq 0} dS_7}_{\text{of equal magnitude, but opposite sign, so these two cancel}}$$

$$\therefore\ -F_x = \iint_{C.S.\,1} -\rho \bar{U}_\infty^2 dS_1 + \iint_{C.S.\,2} \rho u_2^2 dS_2 + \bar{U}_\infty \iint_{C.S.\,4} \rho(\vec{V}\cdot\hat{n}) dS_4 + \bar{U}_\infty \iint_{C.S.\,5} \rho(\vec{V}\cdot\hat{n}) dS_5$$

$$= \iint_{C.S.\,1} -\rho \bar{U}_\infty^2 dS_1 + \iint_{C.S.\,2} \rho u_2^2 dS_2 + \bar{U}_\infty \underbrace{\left\{ \iint_{C.S.\,4} \rho(\vec{V}\cdot\hat{n}) dS_4 + \iint_{C.S.\,5} \rho(\vec{V}\cdot\hat{n}) dS_5 \right\}}_{\text{evaluate w/ conservation of mass } *}$$

Conservation of mass

$$\iint_{C.S.\,1} \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=-\bar{U}_\infty} dS_1 + \iint_{C.S.\,2} \rho \underbrace{(\vec{V}\cdot\hat{n})}_{=u_2} dS_2 + \overbrace{\iint_{C.S.\,4} \rho(\vec{V}\cdot\hat{n}) dS_4 + \iint_{C.S.\,5} \rho(\vec{V}\cdot\hat{n}) dS_5}^{*} = \phi$$

$$\iint_{C.S.\,1} -\rho \bar{U}_\infty dS_1 + \iint_{C.S.\,2} \rho u_2 dS_2 + * = \phi$$

$$\therefore\ -F_x = \cancel{\iint_{C.S.\,1} -\rho \bar{U}_\infty^2 dS_1} + \iint_{C.S.\,2} \rho u_2^2 dS_2 + \cancel{\bar{U}_\infty \iint_{C.S.\,1} \rho \bar{U}_\infty dS_1} - \bar{U}_\infty \iint_{C.S.\,2} \rho u_2 dS_2$$

$$\therefore \quad -F_x = \iint_{C.S.①} -\rho U_\infty\, dS_1 + \iint_{C.S.②} \rho u_2\, dS_2 + U_\infty \iint_{C.S.①} \rho U_\infty\, dS_1 - U_\infty \iint_{C.S.②} \rho u_2\, dS_2$$

$$F_x = U_\infty \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho u_2\, dy - \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho u_2^2\, dh \qquad \text{(force per unit depth)}$$

$$= \rho U_\infty^2 \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{u_2}{U_\infty}\left(1 - \frac{u_2}{U_\infty}\right) dy$$

Note: Since the integrand is zero outside the wake, we can let the integration limits go to ∞.

$$\boxed{F_x = F_D = \rho U_\infty^2 \int_{-\infty}^{\infty} \frac{u_2}{U_\infty}\left(1 - \frac{u_2}{U_\infty}\right) dy}$$

Discussion: Since $\rho U_\infty^2 = \text{const.}$, the integral does not depend on x, even though $u_2(x,y)$ generally (u_2 depends on where you pick location 2.

The integral is a measure of the momentum loss caused by the body. (In boundary layer theory, this integral is called the "momentum thickness".)

If we define a "velocity deficit", $u_D = U_\infty - u_2$, we can write:

$$\boxed{\frac{F_D}{\rho U_\infty^2} = \int_{-\infty}^{+\infty} \frac{u_D}{U_\infty}\left(1 - \frac{u_D}{U_\infty}\right) dy}$$

Since far downstream of the body $\frac{u_D}{U_\infty} \ll 1$, we can simplify)

$$\frac{F_D}{\rho U_\infty^2} \cong \int_{-\infty}^{+\infty} \frac{u_D}{U_\infty}\, dy$$

7.2

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7.2.

(a) $\underbrace{\dot{Q} - \dot{W}_s - \dot{W}_{shear}}_{0} - \dot{W}_{other} = \underbrace{\frac{\partial}{\partial t}\int_{cv} e\rho\, d\forall}_{Steady} + \int_{cs}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V}\cdot d\vec{A}$

$$-\dot{W}_{other} = \int_{cs}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

$$= \int_{cs}\left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + gz_1\right)\rho\vec{V_1}\cdot d\vec{A_1} + \int_{cs}\left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + gz_2\right)\rho\vec{V_2}\cdot d\vec{A_2}$$

$$= -\left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + gz_1\right)\dot{m}_1 + \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + gz_2\right)\dot{m}_2$$

$\dot{m}_1 = \dot{m}_2 = \rho V_1 \frac{\pi D_1^2}{4}$

Assume: $u_1 = u_2$

$z_1 = z_2$. Same elevation

$$= \dot{m}_1\left(p_2 v_2 - p_1 v_1 + \frac{V_2^2 - V_1^2}{2}\right)$$

$$= \rho V_1 \frac{\pi D_1^2}{4}\left(-p_{1g} v + \frac{V_2^2 - V_1^2}{2}\right)$$

$$= 1000\,kg/m^3 \cdot 1.5\,m/s \cdot \frac{\pi (0.3\,m)^2}{4}\left(-\frac{100000\,Pa}{1000\,kg/m^3} + \frac{(6\,m/s)^2 - (1.5\,m/s)^2}{2}\right)$$

$$= -8.8136 \times 10^3\ J/s$$

$V_1 \frac{\pi D_1^2}{4} = V_2 \frac{\pi D_2^2}{4}$

$V_2 = V_1 \frac{\pi D_1^2}{4} \frac{4}{\pi D_2^2}$

$= V_1 \frac{D_1^2}{D_2^2}$

$= 6\,m/s$

(b) $\Delta \dot{KE} = -\frac{V_2^2}{2}\dot{m}_1$

$$= -\frac{(6\,m/s)^2}{2} \cdot 1000 \cdot 1.5\,m/s \cdot \frac{\pi \cdot (0.3\,m)^2}{4}$$

$$= -1.9085 \times 10^3\ J/s$$

7.3

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7.3.

Given: $\dot{V} = 50 L/s = 50\times10^{-3}\ m^3/s$

$H = 20m$

$D_t = 10cm = 0.1m$

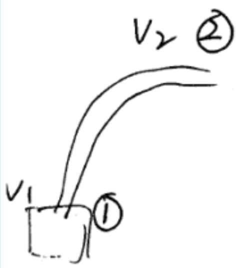
$P_{t\,gage} = 0.8bar = 80000\ Pa$

101000 Pa

0.8

80000

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{cv} e\rho\, d\forall + \int_{cs}(e+pv)\rho\vec{V}\cdot d\vec{A}$$



$V_1 = V_2$

$$-\dot{W}_{other} = \int_{cs}(e+pv)\rho\vec{V}\cdot d\vec{A}$$

$$= -(e_1 + p_1 v_1)\dot{m} + (e_2 + p_2 v_2)\dot{m}$$

$$= \left[(e_2 - e_1) + (p_2 v_2 - p_1 v_1)\right]\dot{m}$$

$$= \left(\underbrace{(u_2 - u_1)}_{0} + \underbrace{\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)}_{0} + g(z_2 - z_1)\right) + p_2 v_2 - p_1 v_1)\,]\dot{m}$$

$$= \left(gH + (p_2 - p_1)v\right)\dot{m}$$

$$= \left(gH + (-P_{gage})v\right)\dot{m}$$

$$= \left(gH\dot{m} - P_{gage}v\right)\dot{m}$$

$$= gH\rho_{water}\dot{V} - P_{gage}\dot{V}$$

$= 58.1\times10^3\ W$