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ME 643
Deliverable 1

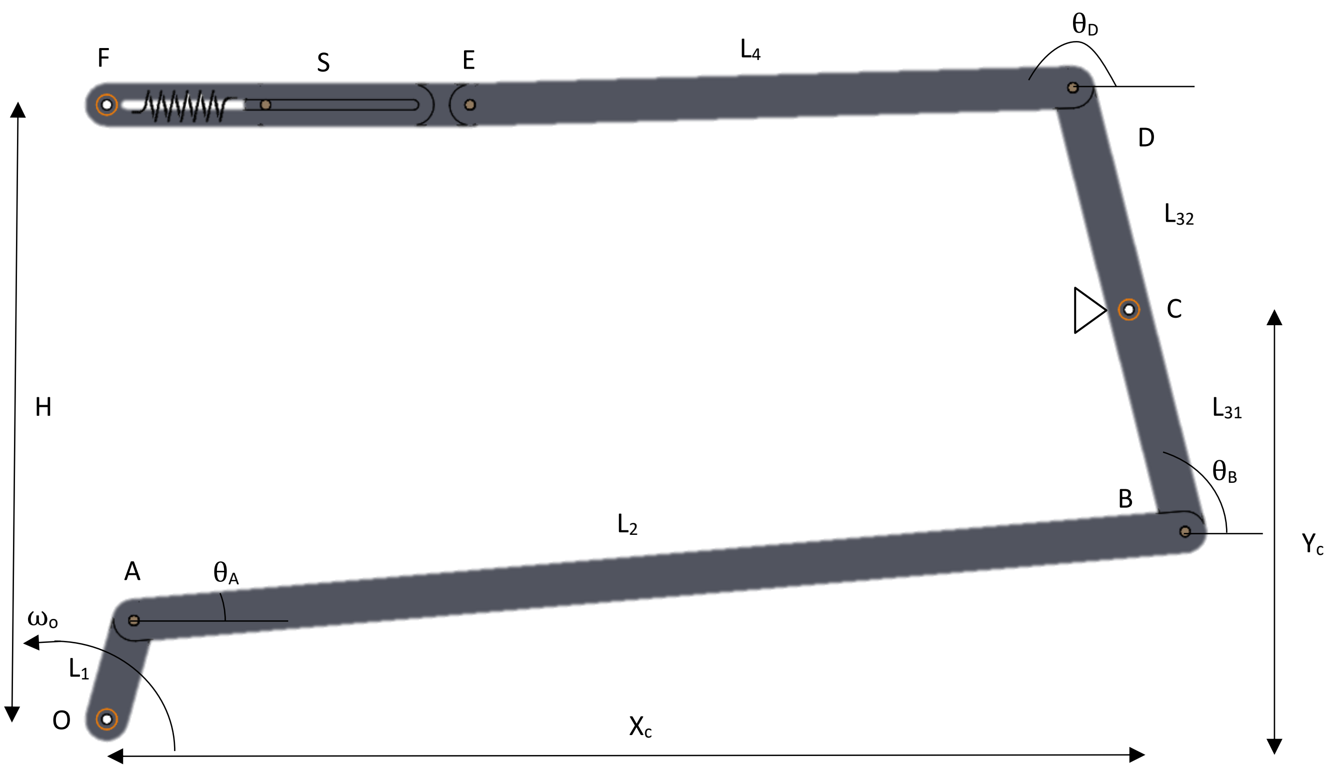


Figure #1: Problem Statement Diagram

Constants and Calculations

Constants

$$X_c = 5\ in$$
$$Y_c = 2\ in$$
$$H = 3\ in$$
$$s_{x,start} = 2.5\ in$$
$$s = 1\ in$$
$$L_1 = 0.5\ in$$
$$\omega_o = 50\ rpm$$
$$k = 1\ lbf/in$$

Geometry Calculations

$$r_{AB} = 5.160\ in$$
$$r_{CD} = 1.12\ in$$
$$r_{BC} = 1.12\ in$$
$$r_{DE} = 2.95\ in$$
$$r_{EF} = 2.5\ in$$
$$r_{OA} = L_1$$
$$r_{FO} = H$$
$$m_E = 0.05\ slugs$$

Calculations

Loop Definitions

Loop 1: $L_1 + L_2 + L_{31} - OC = 0$

Loop 2: $OC + L_{32} + L_4 - S - OF = 0$

Loop Parametrization

Loop 1

$$R_{OA} + R_{AB} + R_{BC} - R_{CO} = 0$$

$$R_{OA} = r_{OA}\,\{cos\theta, sin\theta\}$$

$$R_{AB} = r_{AB}\,\{cos\theta_A, sin\theta_A\}$$

$$R_{BC} = r_{BC}\,\{cos\theta_B, sin\theta_B\}$$

$$R_{CO} = \{X_C, Y_C\}$$

Loop 2

$$R_{OC} + R_{CD} + R_{DE} - R_{EF} - R_{FO} = 0$$

$$R_{CD} = r_{CD}\,\{cos\theta_B, sin\theta_B\}$$

$$R_{DE} = r_{DE}\,\{cos\ \theta_D, sin\theta_D\}$$

$$R_{EF} = r_{EF}\{1, 0\}$$

$$R_{FO} = r_{FO}\{0, 1\}$$

Position Loop Equations

<u>Loop 1</u>

$$r_{OA}cos(\theta) + r_{AB}cos(\theta_A) + r_{BC}cos(\theta_B) - X_C = 0 \qquad (X)$$

$$r_{OA}sin(\theta) + r_{AB}sin(\theta_A) + r_{BC}sin(\theta_B) - Y_C = 0 \qquad (Y)$$

<u>Loop 2</u>

$$X_C + r_{CD}cos(\theta_B) + r_{DE}cos(\theta_D) - r_{EF} = 0 \quad (X)$$

$$Y_C + r_{CD}sin(\theta_B) + r_{DE}sin(\theta_D) - r_{FO} = 0 \quad (Y)$$

θ_A, θ_B, θ_D, and r_{EF} are the unknowns to be solved. Taking the derivative of the position equations above with respect to time we get velocity equations.

Velocity Loop Equations

<u>Loop 1</u>

$$-r_{OA}\dot{\theta}\, sin(\theta) - r_{AB}\dot{\theta}_A sin(\theta_A) - r_{BC}\dot{\theta}_B sin(\theta_B) = 0 \qquad (X)$$

$$r_{OA}\dot{\theta}\, cos(\theta) + r_{AB}\dot{\theta}_A cos(\theta_A) + r_{BC}\dot{\theta}_B cos(\theta_B) = 0 \qquad (Y)$$

<u>Loop 2</u>

$$-r_{CD}\dot{\theta}_B sin(\theta_B) - r_{DE}\dot{\theta}_D sin(\theta_D) - \dot{r}_{EF} = 0 \qquad (X)$$

$$r_{CD}\dot{\theta}_B cos(\theta_B) + r_{DE}\dot{\theta}_D cos(\theta_D) = 0 \qquad (Y)$$

Where $\dot{\theta}_A$, $\dot{\theta}_B$, $\dot{\theta}_D$, and $\dot{r}_{EF}$ are our unknowns to be solved for. Taking derivative of the velocity equations above with respect to time we get our acceleration equations.

Acceleration Loop Equations

<u>Loop 1</u>

$$-r_{OA}\ddot{\theta}\, sin(\theta) - r_{OA}\dot{\theta}^2 cos(\theta) - r_{AB}\ddot{\theta}_A sin(\theta_A) - r_{AB}\dot{\theta}_A^2 cos(\theta_A) - r_{BC}\ddot{\theta}_B sin(\theta_B) - r_{BC}\dot{\theta}_B^2 cos(\theta_B) = 0 \qquad (X)$$

$$r_{OA}\ddot{\theta}\, cos(\theta) - r_{OA}\dot{\theta}^2 sin(\theta) + r_{AB}\ddot{\theta}_A cos(\theta_A) - r_{AB}\dot{\theta}_A^2 sin(\theta_A) + r_{BC}\ddot{\theta}_B cos(\theta_B) - r_{BC}\dot{\theta}_B^2 sin(\theta_B) = 0 \qquad (Y)$$

<u>Loop 2</u>

$$-r_{CD}\ddot{\theta}_B sin(\theta_B) - r_{CD}\dot{\theta}_B^2 cos(\theta_B) - r_{DE}\ddot{\theta}_D sin(\theta_D) - r_{DE}\dot{\theta}_D^2 cos(\theta_D) - \ddot{r}_{EF} = 0 \qquad (X)$$

$$r_{CD}\ddot{\theta}_B cos(\theta_B) - r_{CD}\dot{\theta}_B^2 sin(\theta_B) + r_{DE}\ddot{\theta}_D cos(\theta_D) - r_{DE}\dot{\theta}_D^2 sin(\theta_D) = 0 \qquad (Y)$$

Where $\ddot{\theta}_A$, $\ddot{\theta}_B$, $\ddot{\theta}_D$, and $\ddot{r}_{EF}$ are our unknowns to be solved for. The forces on each pin were then determined mathematically by using MATLAB.

Distance Definitions

To calculate the axial and transverse force we needed to project the forces onto x and y axis

Loop 1

$$R_{OA,x} = (\frac{R_{OA}}{2}) \cdot \cos\theta$$

$$R_{OA,y} = (\frac{R_{OA}}{2}) \cdot \sin\theta$$

$$R_{AB,x} = (\frac{R_{AB}}{2}) \cdot \cos\ \theta_A$$

$$R_{AB,y} = (\frac{R_{AB}}{2}) \cdot \sin\theta_A$$

$$R_{BC,x} = (\frac{R_{BC}}{2}) \cdot \cos\theta_B$$

$$R_{BC,y} = (\frac{R_{BC}}{2}) \cdot \sin\theta_B$$

Loop 2

$$R_{CD,x} = (\frac{R_{CD}}{2}) \cdot \sin\theta_B$$

$$R_{CD,y} = (\frac{R_{CD}}{2}) \cdot \cos\theta_B$$

$$R_{DE,x} = (\frac{R_{DE}}{2}) \cdot \cos\theta_D$$

$$R_{DE,y} = (\frac{R_{DE}}{2}) \cdot \sin\theta_D$$

$$R_{EF,x} = R_{OE,x}$$

$$R_{EF,y} = 3\ in$$

$$R_{FO,x} = 0$$

$$R_{FO,y} = 1.5\ in$$

Forces on Pins

<u>Member OA</u>

$$F_{Ax} + F_{Ox} = 0$$

$$F_{Oy} + F_{Ay} = 0$$

$$M_O + F_{Ay} \cdot R_{OA,y} - F_{Ax} \cdot R_{OA,x} - F_{Ox} \cdot R_{OA,y} + F_{Oy} \cdot R_{OA,x} = 0$$

<u>Member AB</u>

$$-F_{Ax} + F_{Bx} cos\theta_B = 0$$

$$-F_{Ay} + F_{Bx} sin\theta_B = 0$$

$$-F_{Ax} \cdot R_{AB,x} - F_{Ay} \cdot R_{AB,y} - F_{Bx} \cdot R_{AB,x} + F_{By} \cdot R_{AB,y} = 0$$

<u>Member BC</u>

$$F_{Cx} - F_{Bx} + F_{pin,x} = 0$$

$$F_{Cy} + F_{By} + F_{pin,y} = 0$$

$$F_{Bx} \cdot R_{BC,x} - F_{By} \cdot R_{BC,y} - F_{Cx} \cdot R_{BC,x} + F_{Cy} \cdot R_{BC,y} = 0$$

Member CD

$$-F_{Cx} + F_{Dx} = 0$$

$$-F_{Cy} + F_{Dy} = 0$$

$$-F_{Cy} \cdot R_{CD,x} - F_{Cx} \cdot R_{CD,y} - F_{Dy} \cdot R_{CD,x} + F_{Dx} \cdot R_{CD,y} = 0$$

Member DE

$$k_{spring}(2 - R_{OE,x}) - F_{Dx} - m_E \cdot a_E = 0$$

$$F_N + F_{Dy} - F_G = 0$$

Figures

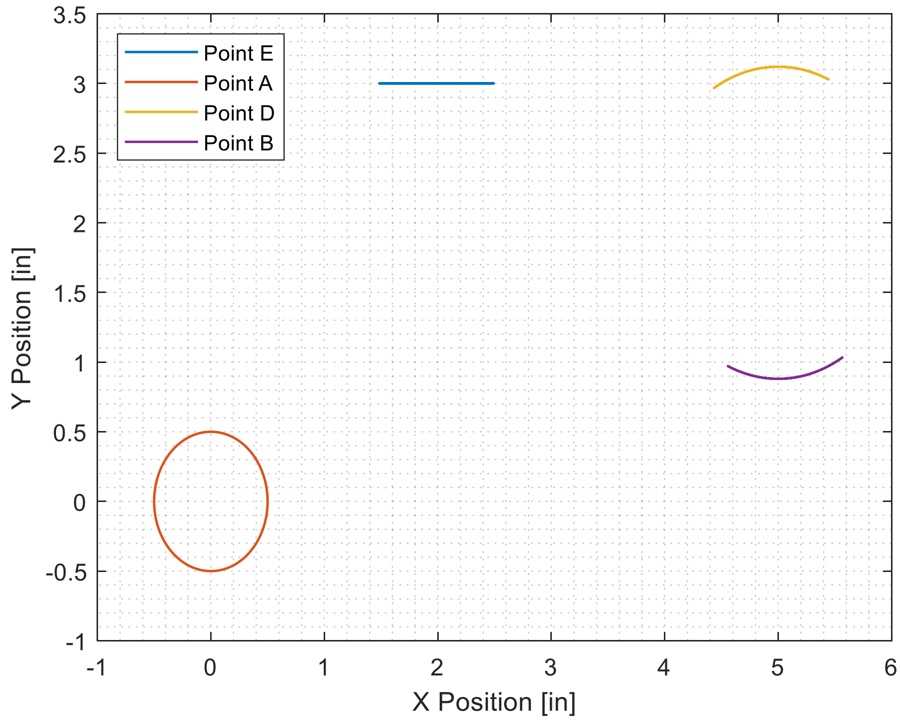


Figure #2: Trajectories of points A, B, D, and E

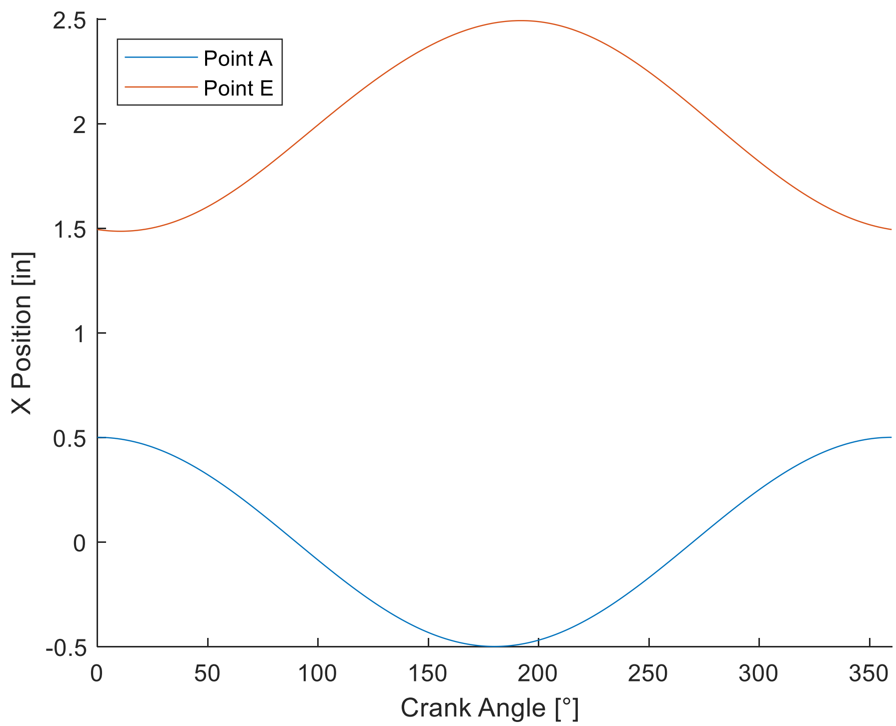


Figure #3: X position of points A and E Vs crank angle

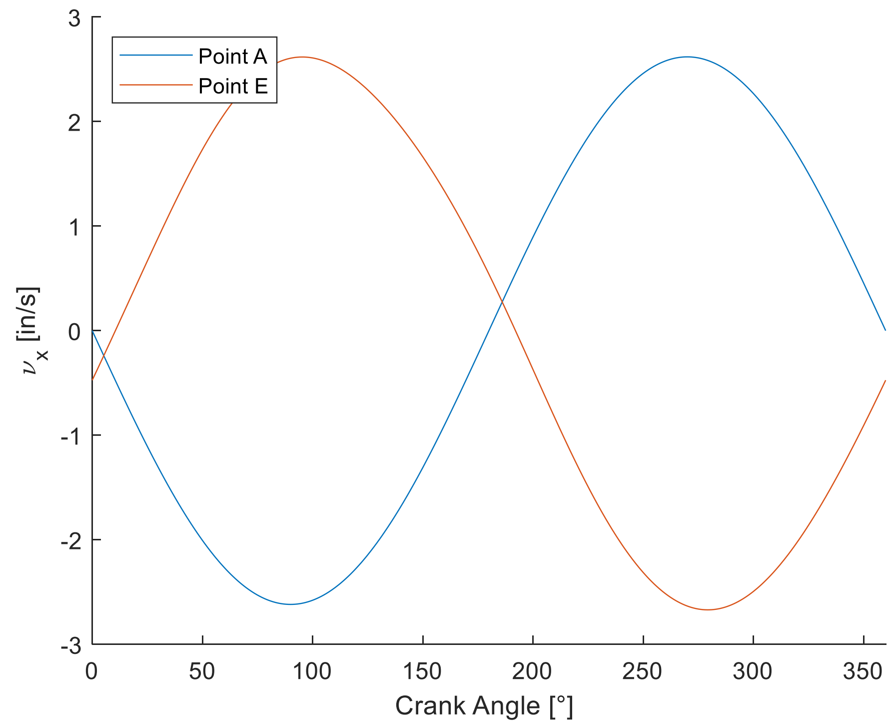


Figure #4: X component of linear velocities of points A and E Vs crank angle

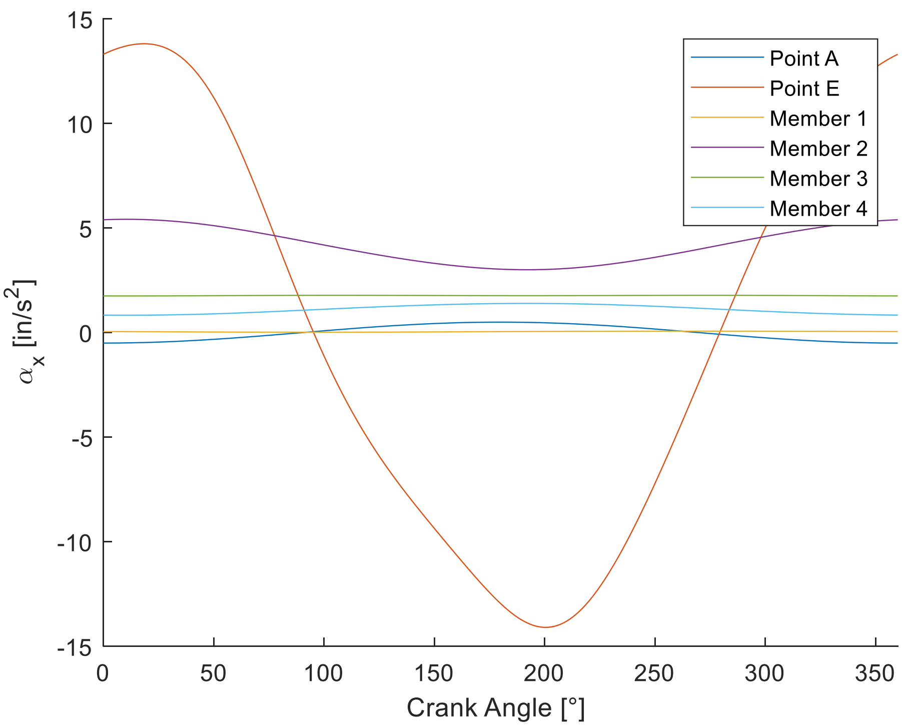


Figure #5 X component of linear acceleration of points A and E with the magnitude of linear accelerations of CM of members 1,2,3,4 Vs crank angle

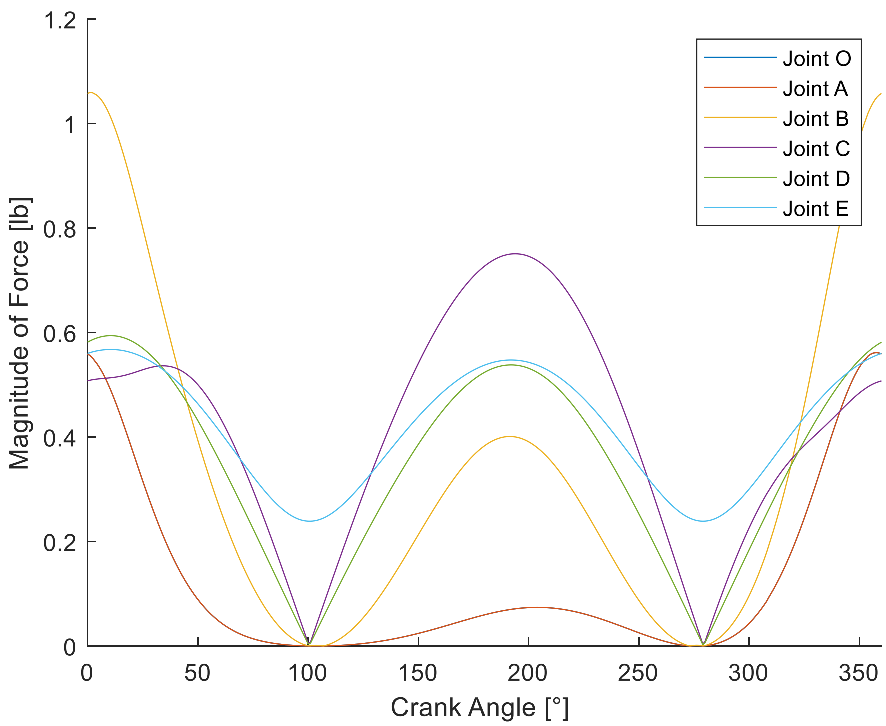
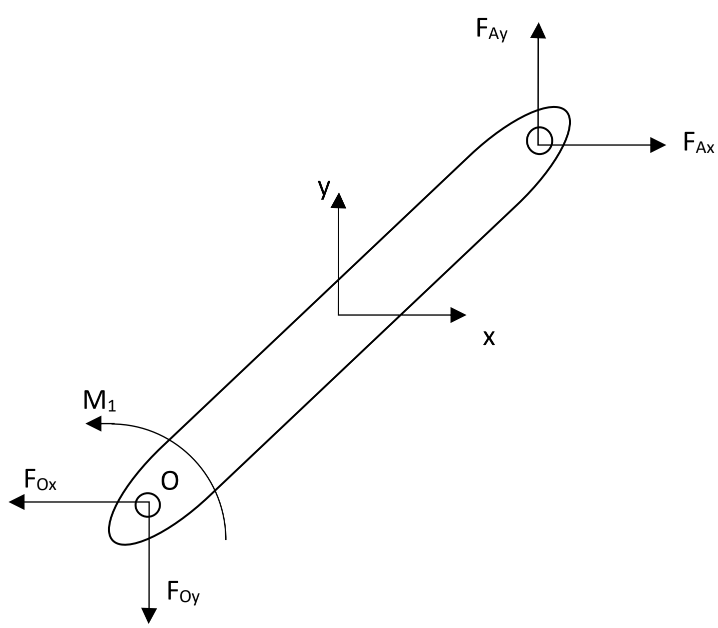


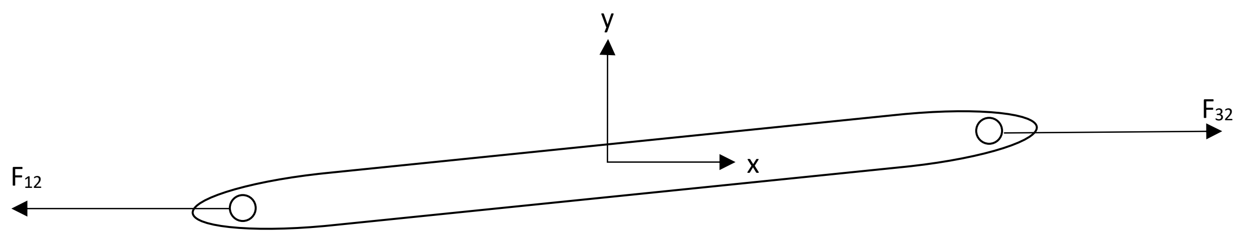
Figure #6 Magnitude in joints O, A, B, C, D and E Vs crank angle

Free Body Diagrams of Members #1-5

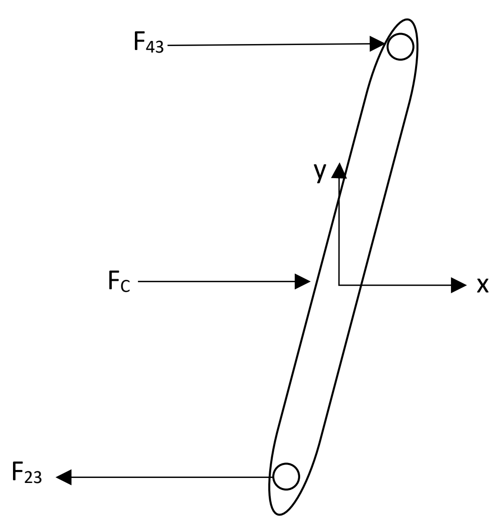
Member #1.



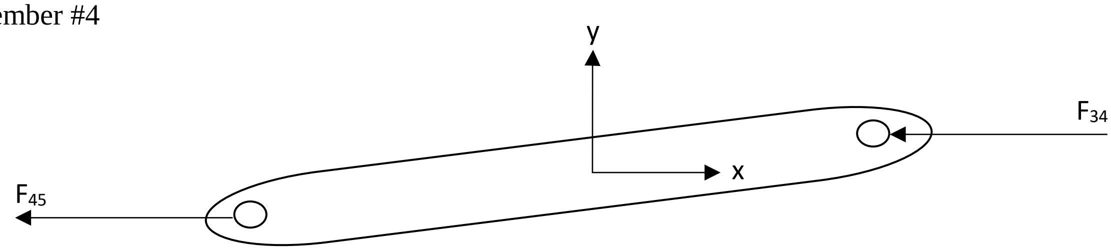
Member #2



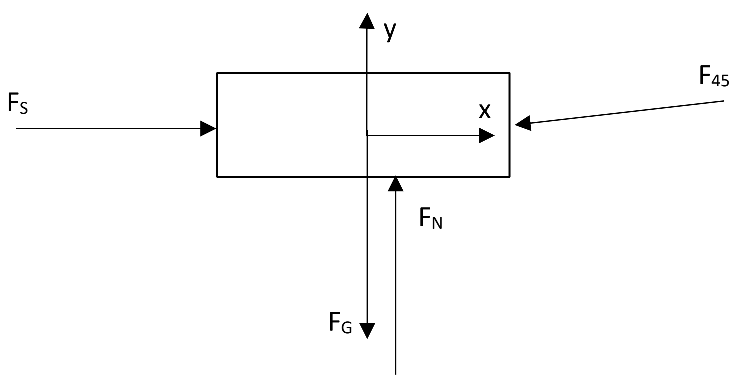
Member #3



Member #4



Member #5



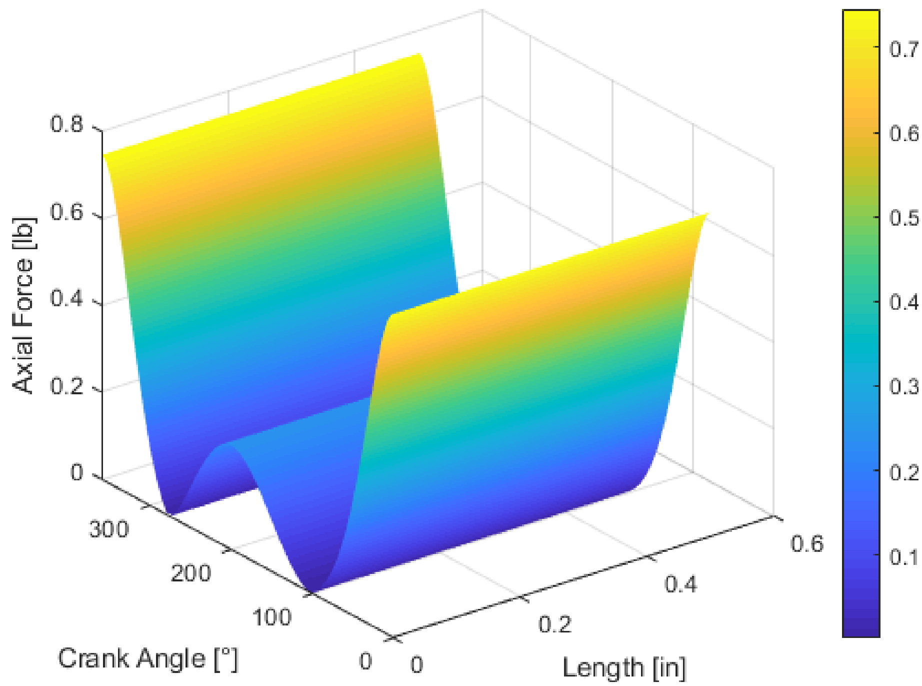


Figure #7: Axial force on member OA.

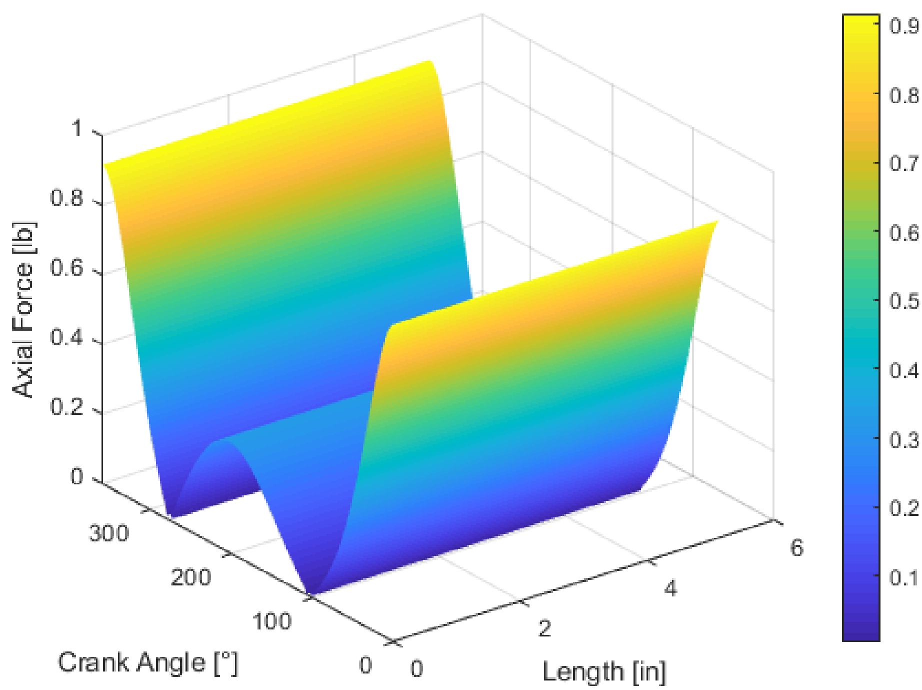


Figure #8: Axial force on member AB.

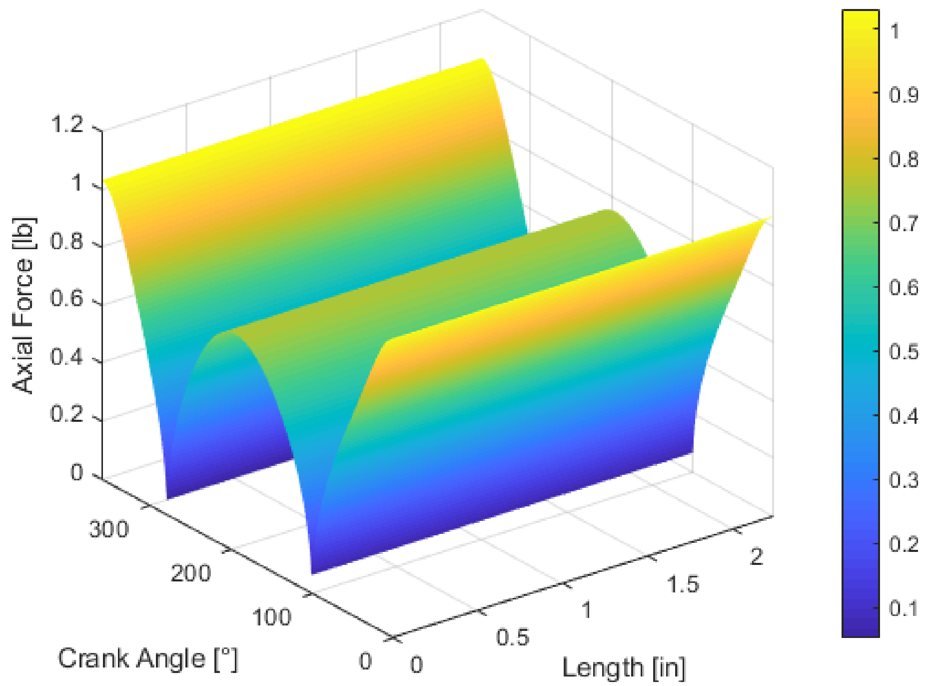


Figure #9: Axial force on member BD.

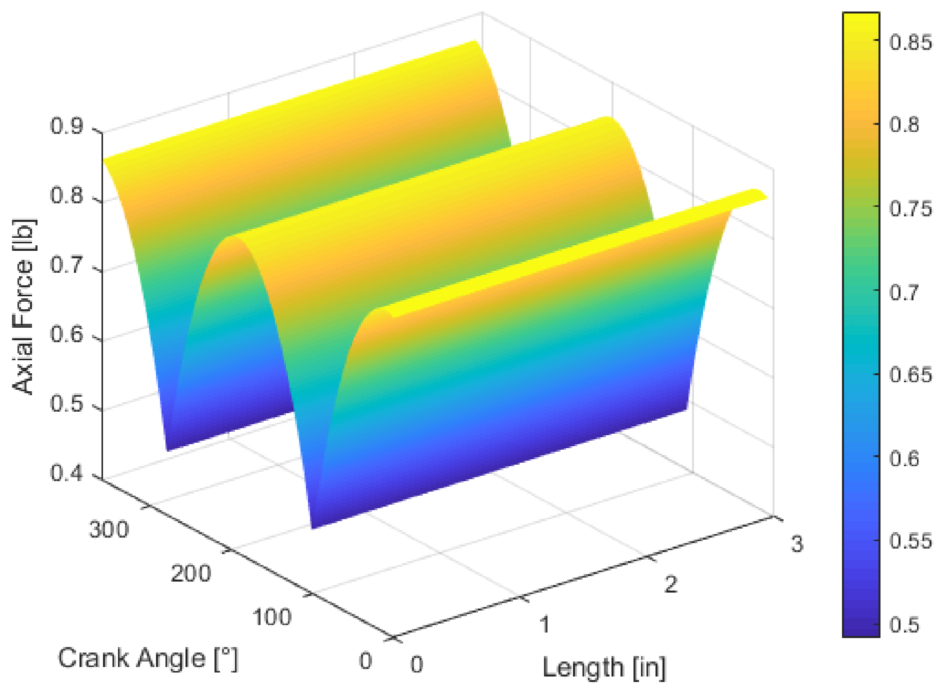


Figure #10: Axial force of member DE.

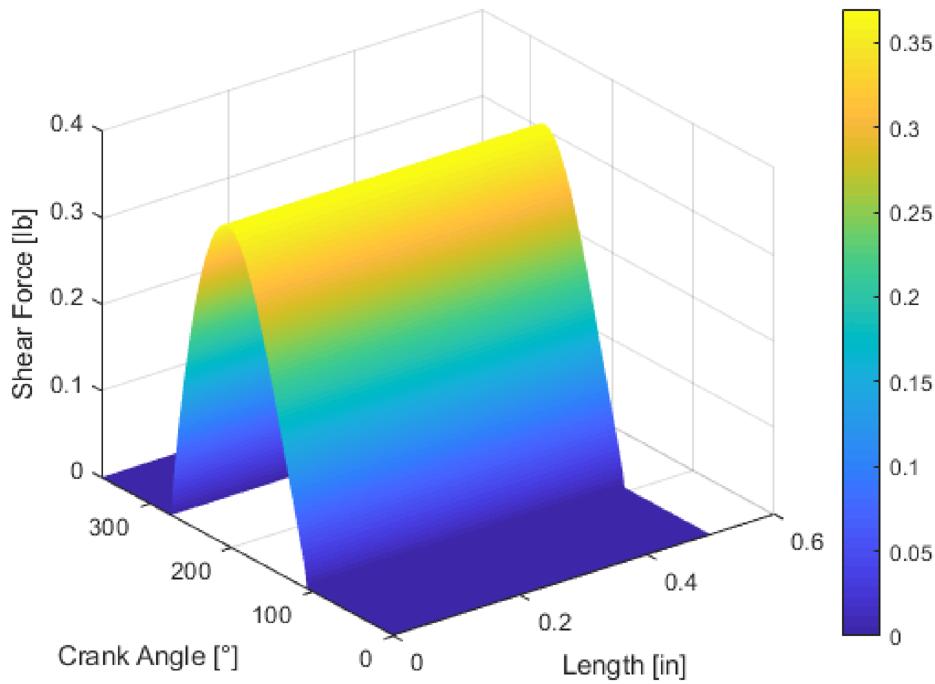


Figure #11: Shear force on member OA.

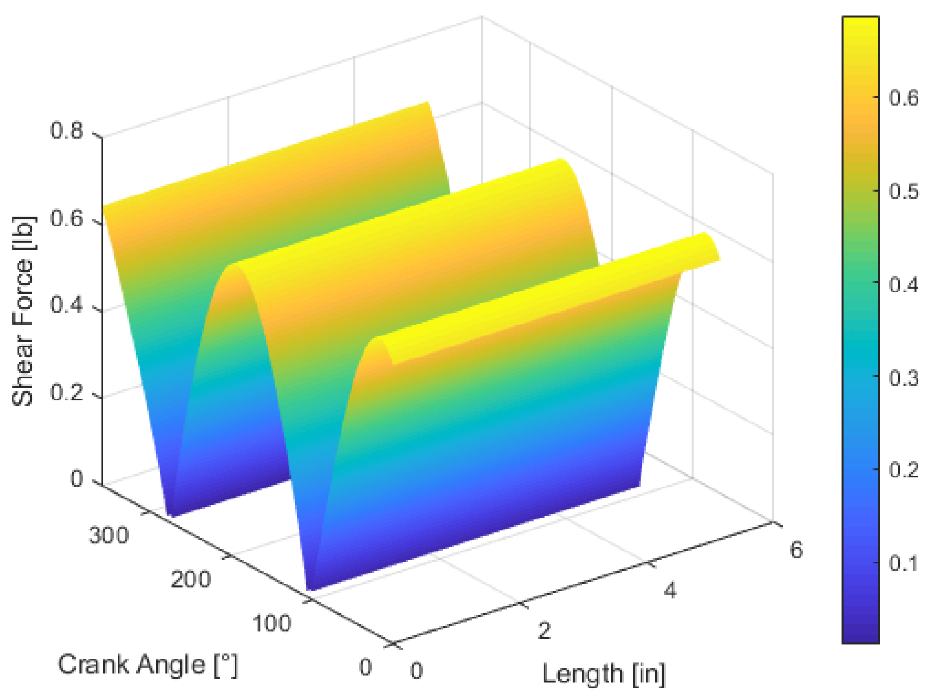


Figure #12: Shear force on member AB.

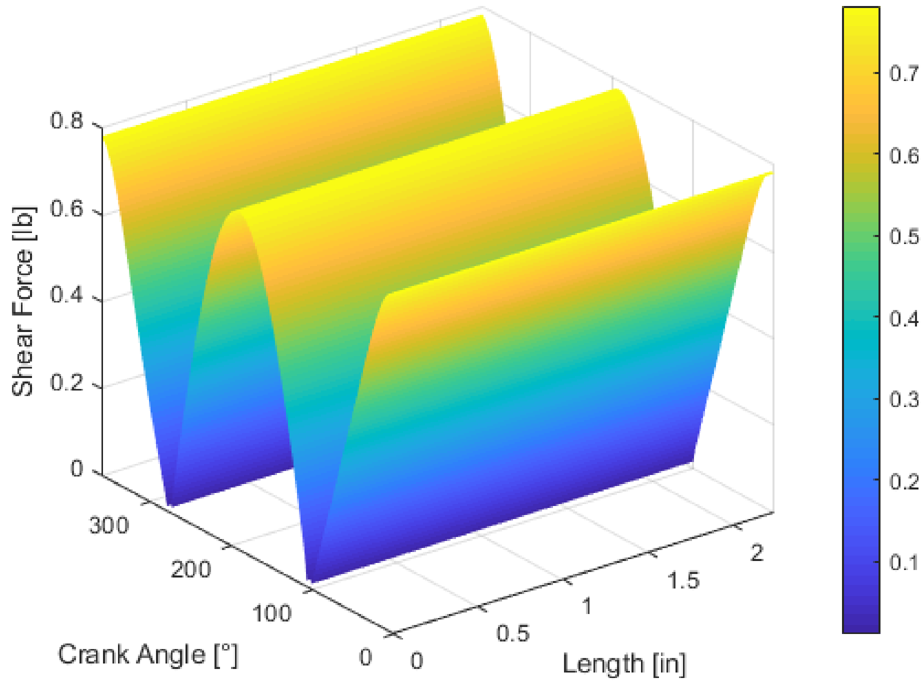


Figure #13: Shear force on member BD.

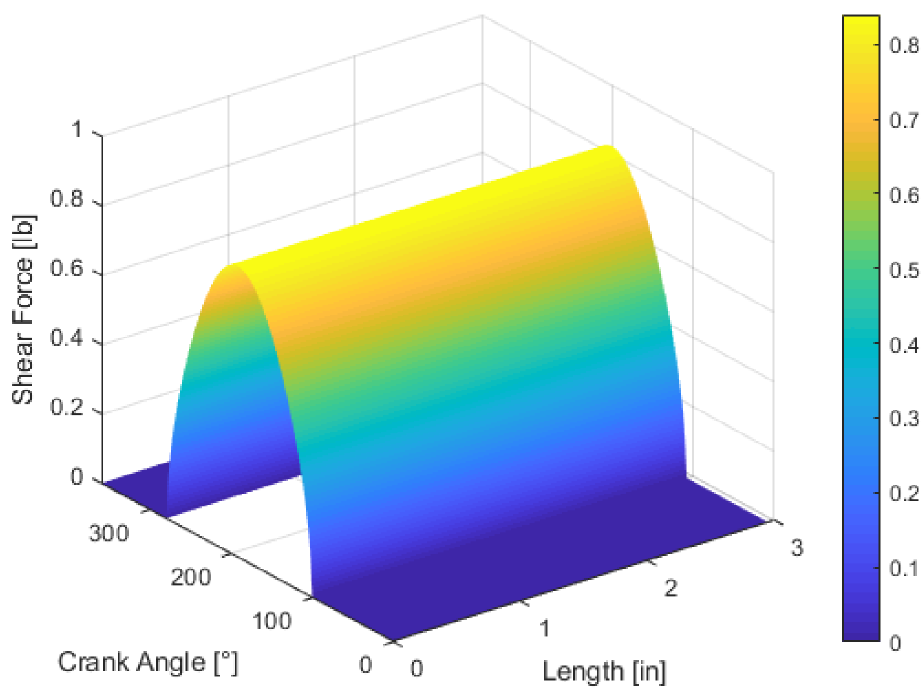


Figure #14: Shear force on member DE.

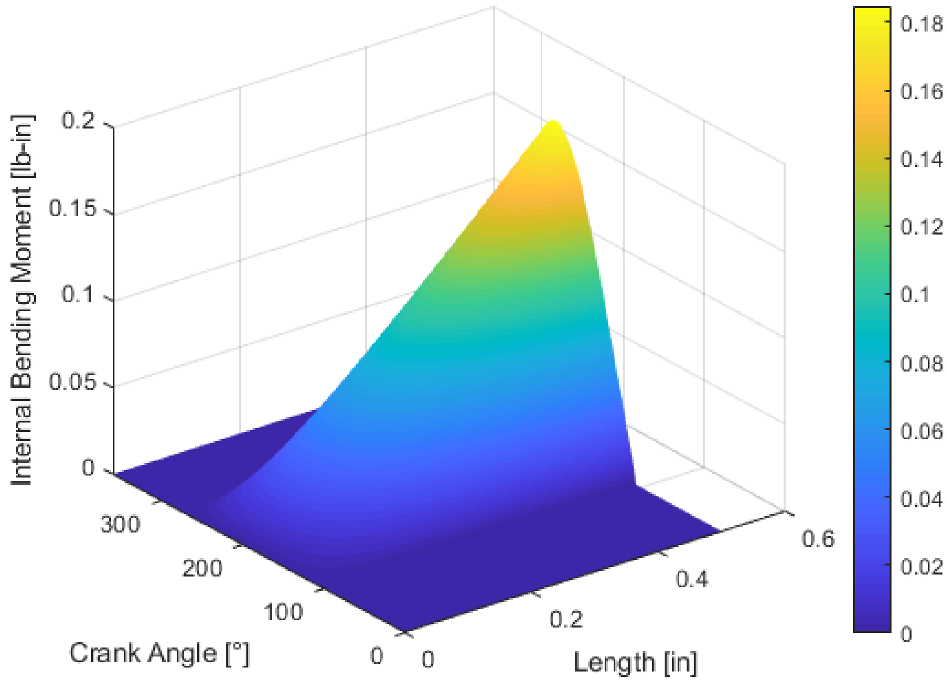


Figure #15: Internal bending moment on member OA.

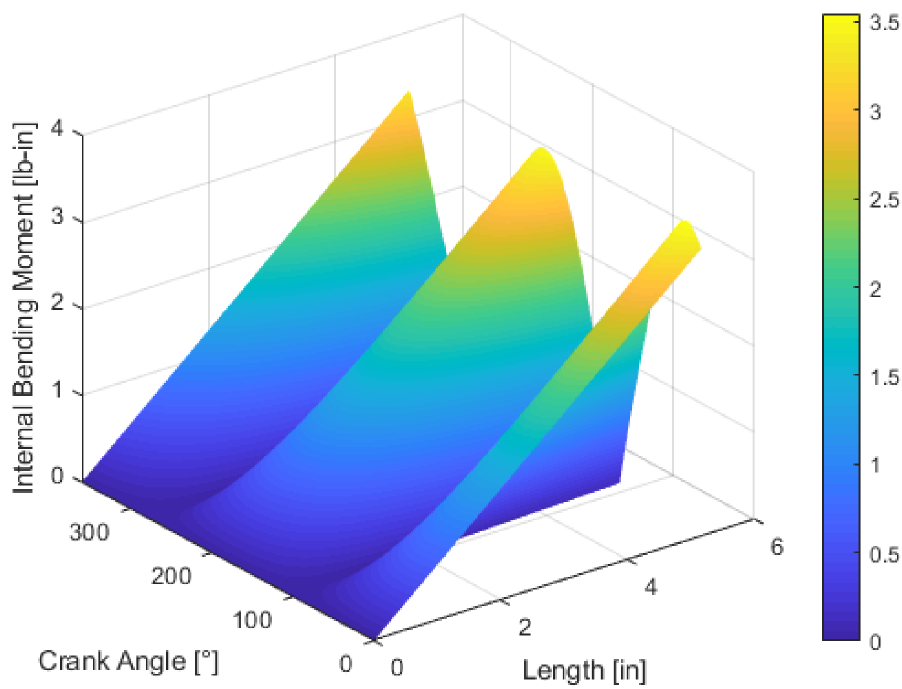


Figure #16: Internal bending moment on member AB.

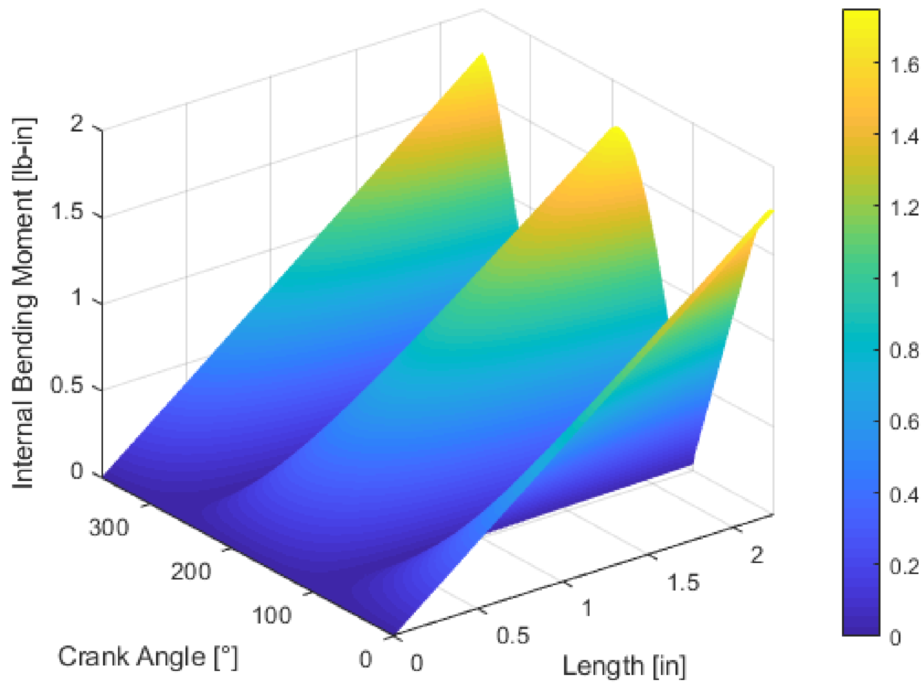


Figure #17: Internal bending moment on member BD.

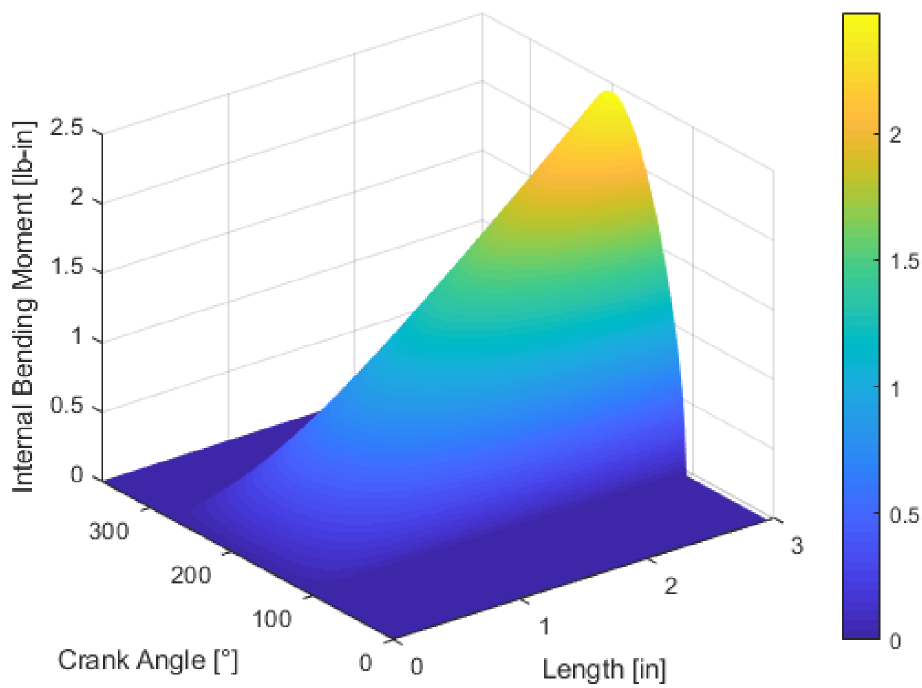


Figure #18: Internal bending moment on member DE.

Critical Values

Member	Force (lb)
Max Axial OA	0.745
Max Axial AB	0.916
Max Axial BD	1.031
Max Axial DE	0.867
Max Shear OA	0.370
Max Shear AB	0.687
Max Shear BD	0.782
Max Shear DE	0.840
Max Moment OA	0.185 lb in
Max Moment AB	3.545 lb in
Max Moment BD	1.751 lb in
Max Moment DE	2.477 lb in
Min Axial OA	0.002
Min Axial AB	0.004
Min Axial BD	0.052
Min Axial DE	0.491
Min Shear OA	0.000
Min Shear AB	0.012
Min Shear BD	0.011
Min Shear DE	0.000
Min Moment OA	0.000 lb in
Min Moment AB	0.000 lb in
Min Moment BD	0.000 lb in
Min Moment DE	0.000 lb in

Pin	Max Force (lb)
O	0.561422
A	0.561422
B	1.059321
C	0.75069
D	0.593995
E	0.567541

Results

From the compiled list of critical values of the dynamic system, the maximum axial force occurs in member BD of our system, whereas the minimum occurs in OA. From the shear stress calculations, the maximum value occurs in member DE, and minimum value, zero, occurs at various orientations of members OA and DE. The largest bending moment is in the longest member, AB, while all members exhibit the ability to operate without a bending moment at set orientations.

For the connections, pins O and A exhibit the same values, as member OA is a two force member with equal and opposite axial forces at each connection. These pins also exhibit the lowest maximum load. The connection at point B will need to be the sturdiest in our system, as it endures almost twice as much of a maximum load as pins O and A.