ME 709/809 Final [20 points]

You will have 72 hours to complete this assignment. You will be simulating a 2D, fully developed channel flow using the $k - \epsilon$ model.

- TASK 1 [2 points]: Without using index notation, show the governing equations for a 2D, time averaged $k - \epsilon$ model.
  - Show the time averaged continuity equation.
  - Show each of the components of the averaged Naveir-Stokes equations. Use the Boussinesq assumption as listed in section 6 of the notes. You can ignore the term in blue from the assumption.
  - Start from the modeled k and ϵ equations (equations 11.97 and 11.98 in `http://www.tfd.chalmers.se/~lada/postscript_files/solids-and-fluids_turbulent-flow_turbulence-modelling.pdf`) and show the modeled k and ω equations for a 2D, time-averaged flow.
- TASK 2 [2 points]: Simplify the equations from TASK1 for a 2D fully developed channel flow. Clearly indicate what terms you are setting to 0 and why. Prove that the governing equations reduce to:
  $0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial U}{\partial y}\right]$;
  $0 = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - \epsilon$;
  $0 = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial y}\right] + \frac{\epsilon}{k}(c_1 P_k - c_2\epsilon)$;
  where
  $P_k = \nu_t\left(\frac{\partial U}{\partial y}\right)^2$; and
  $\nu_t = c_\mu\frac{k^2}{\epsilon}$.
- TASK 3 [5 points]: Discretize the equations using central difference and a non-equidistant mesh. Note that you need to keep k and ϵ positive to avoid divergence of the solution. To achieve that, put all negative source terms in S_P where $S = S_P\Phi_P + S_U$. For example, the dissipation term in the k equations should be in the S_P and look similar to: $S_P = -\frac{\epsilon}{k}\Delta y$ or $S_P = -\frac{\epsilon}{k}\Delta V$ (depending on how you integrate the equations).
- TASK 4 [3 points]: Write down the steps of the algorithm for solving this problem in Matlab. For the boundary conditions use $U = k = \frac{\partial \epsilon}{\partial y} = 0$.
  Use channel height of 2 m, $\rho = 1$ kg/m^3, $u_\tau = 1$ m/s and use $-\frac{\partial P}{\partial x} = \tau_w$.
  Use a Gauss-Seidel solver.
- TASK 5 [8 points] Simulate this flow using Matlab.
  Plot the velocity profile.

  Please submit the following on Canvas under FINAL exam submission:
  - A pdf file containing the work for TASK 1, 2, 3, and 4; The plot from TASK 5, and a printout of the code from TASK 5. The work from TASKS 1 – 4 can be scanned papers, an exported one-note document or have any other form, as long as everything is legible and the submission is a single pdf file.
  - A zipped folder with all your work on the assignment, including the final .m file.