Math 740/840 Homework Assignment #4 Fall 2019
Due 11/6/2019
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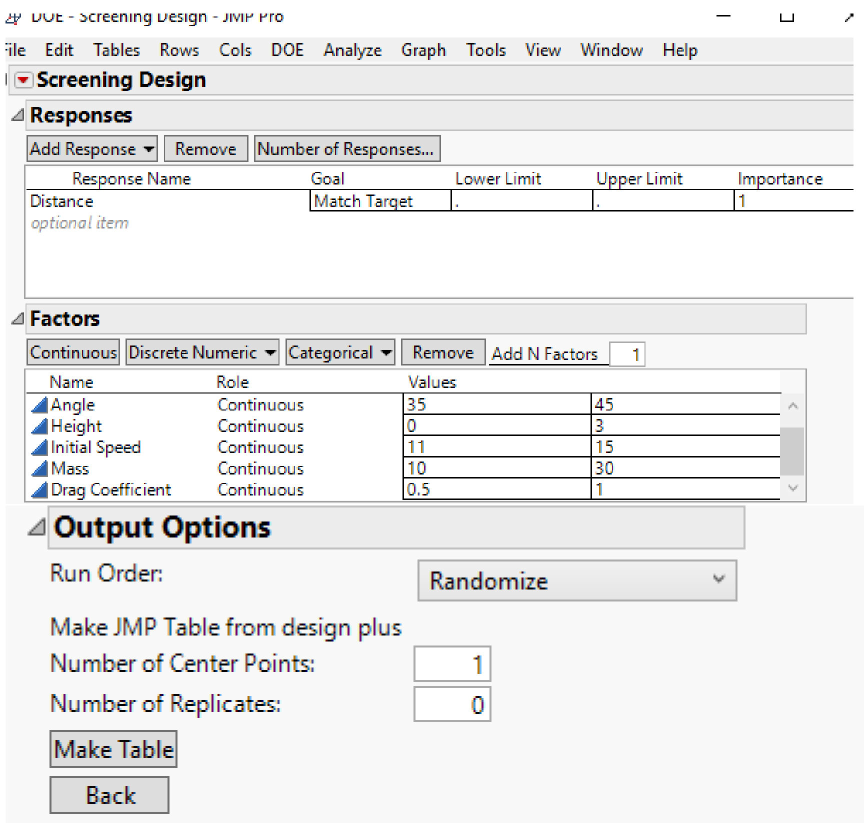
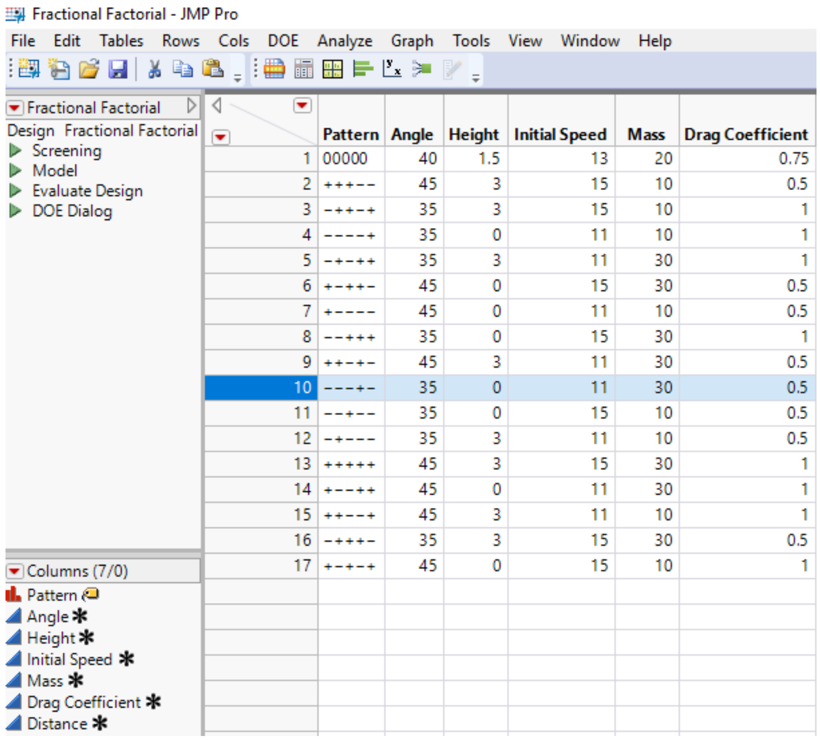
Part 1 – Designing and Performing a Fractional Factorial Experiment

1. (10 pts.) Go the Web site or use the HTML file and familiarize yourself with the operation of the projectile simulator. Once you have figured out how to manipulate the simulator to change settings and fire a shot you are ready to start designing the experiment.

    a. The goal of the experiment is to find settings of the catapult factors that will **hit a target of 20 meters**. During the experiment you do not want to actually hit 20 meters, what you want to do is find settings that result in launch distances well short of and well beyond 20 meters; this allows you to fit a model that predicts over a range that includes 20 meters. You may take up to 20 practice launches to find high and low settings for the five experimental factors; again we desire launch distances where some are short of 20 meters and some are beyond 20 meters, the goal at this point is not to hit the target. Once you have taken the practice shots decide on the high and low level for each of the five factors. **At this point the practice shot data is no longer of use and should be discarded**.

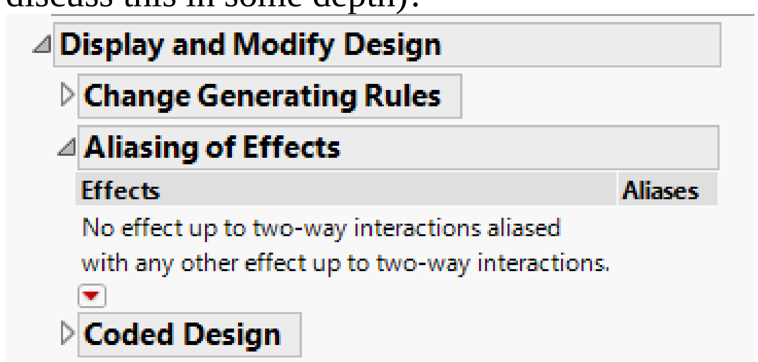
    b. Once you have decided on the high and low levels for each of the four factors, you are ready to design the experiment. In JMP use the **Screening** platform to design a 2^{5-1} (one half fraction of a full 2^5 factorial) fractional factorial experiment (**DOE →Classical → Screening Design**). In the **Factor Table** select 5 continuous factors each with 2 levels. Next type in the names of the four variables. Do not use the generic X1, X2, X3, X4, X5 format instead use the actual names of the simulator factors. Finally, type in the low and high settings for each factor that you determined in part a. **Do not** use the generic -1, 1 format, use the actual values you have determined for each factor. Finally, in the response table change **Y** to **Distance**. Important, to get credit for this part of the problem include a screenshot of the **Factor Table** properly formatted as instructed. Click the **Continue** button to access the list of fractional factorial designs and select the one with 16 runs that is resolution V. Specify the Number of Center Points as **1**. Finally click on the **Make Table** button to create the JMP data table. You should have a table with the 17 experimental trials in random order.





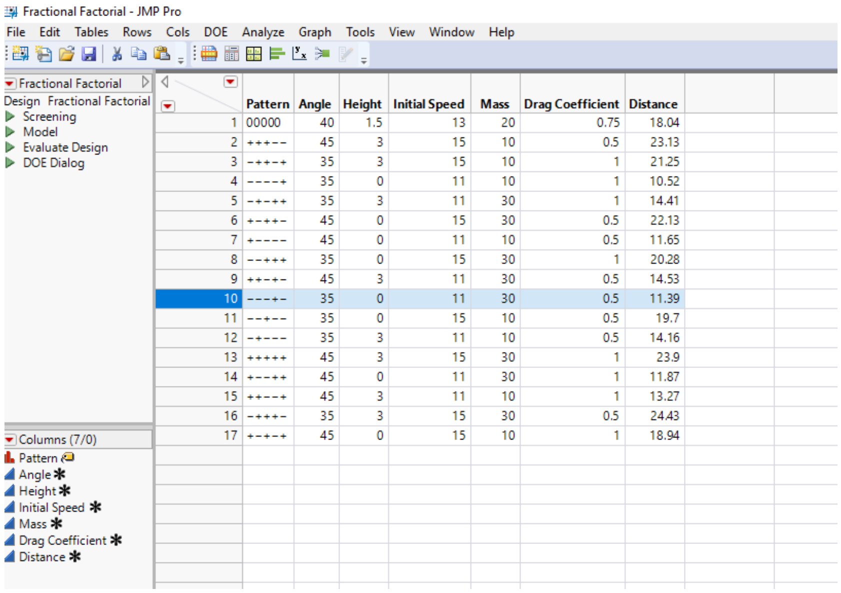
	Pattern	Angle	Height	Initial Speed	Mass	Drag Coefficient
1	00000	40	1.5	13	20	0.75
2	+++−−	45	3	15	10	0.5
3	−++−+	35	3	15	10	1
4	−−−−+	35	0	11	10	1
5	−+−++	35	3	11	30	1
6	+−++−	45	0	15	30	0.5
7	+−−−−	45	0	11	10	0.5
8	−−+++	35	0	15	30	1
9	++−+−	45	3	11	30	0.5
10	−−−+−	35	0	11	30	0.5
11	−−+−−	35	0	15	10	0.5
12	−+−−−	35	3	11	10	0.5
13	+++++	45	3	15	30	1
14	+−−++	45	0	11	30	1
15	++−−+	45	3	11	10	1
16	−+++−	35	3	15	30	0.5
17	+−+−+	45	0	15	10	1

c. In the Screening Design platform after you created the table, open the **Aliasing of Effects** report and discuss the aliasing that exists (if any) involving two-way interactions and main effects. What are the potential problems that may occur in using this resolution V design (the Fractional Factorial notes part 1 discuss this in some depth)?



There are no aliasing of effects in the design setup. Resolution V designs will have no aliasing on the main effects or two-factor interactions of the data.

d. Now use the simulator to perform the 17 experimental trials in the order they were created in the JMP data table (the order is randomized). In order to get full credit for this problem, include a screenshot of the data table with the 17 experimental trial settings and the simulator launch distances recorded for each trial.



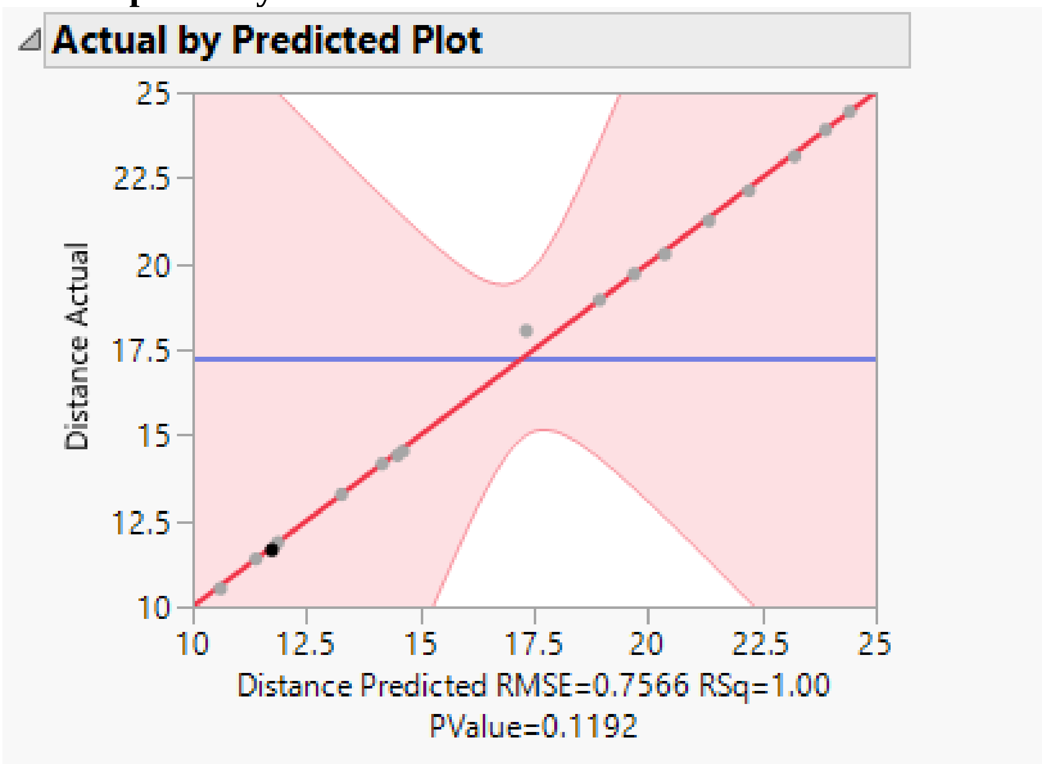
	Pattern	Angle	Height	Initial Speed	Mass	Drag Coefficient	Distance
1	00000	40	1.5	13	20	0.75	18.04
2	+++--	45	3	15	10	0.5	23.13
3	-+++-+	35	3	15	10	1	21.25
4	----+	35	0	11	10	1	10.52
5	-+-++	35	3	11	30	1	14.41
6	+-++-	45	0	15	30	0.5	22.13
7	+----	45	0	11	10	0.5	11.65
8	--+++	35	0	15	30	1	20.28
9	++-+-	45	3	11	30	0.5	14.53
10	---+-	35	0	11	30	0.5	11.39
11	--+--	35	0	15	10	0.5	19.7
12	-+---	35	3	11	10	0.5	14.16
13	+++++	45	3	15	30	1	23.9
14	+--++	45	0	11	30	1	11.87
15	++--+	45	3	11	10	1	13.27
16	-+++-	35	3	15	30	0.5	24.43
17	+-+-+	45	0	15	10	1	18.94

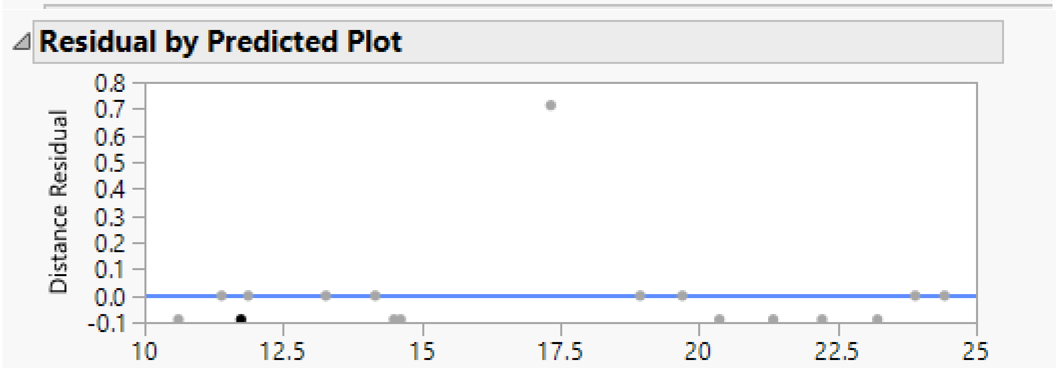
2. (10 points) Using the data from problem 1 we are ready to analyze the experimental results using JMP. Make certain you have saved the data table. You will have to reuse it in part 2 of this assignment.

   a. First we must examine the fit of the model. When you created the design table, JMP saved a **Model** script to the data table, so just click on the script button (upper left hand corner of the data table) and run the model. Your model should have main effects and two-way interactions. Next click on **Run** in the Fit Model launch window to open the Report window. **Do not** attempt to simplify the model at this point.

      A good way to spot lack of fit in the model is a **Residual by Predicted plot** and **Actual by Predicted plot**. Recall that Residuals are just differences between the actual observations and the model predicted value for those observations. If the model fits well, meaning no lack of fit, then the residual plot is a random scatter plot. If lack of fit exists (the model is not adequate) then a pattern will appear in the residual plot or one or more points appear to be distant from the distribution of residuals – the model does not fit that such point(s). To generate a residual plot in the Fit Model report window, click on the main report menu (red arrow at the top), then from the **Row Diagnostics** submenu select **Plot Residual by Predicted**.

Using the **Residual Plot** and the **Actual by Predicted** plot in the Fit Model report window, do you detect any evidence of lack of fit for the specified model (e.g., does the center point fit with the pattern of the factorial points in the two plots)? Explain your answer in terms of the Plots; **include a copy of the two plots in your homework document**.



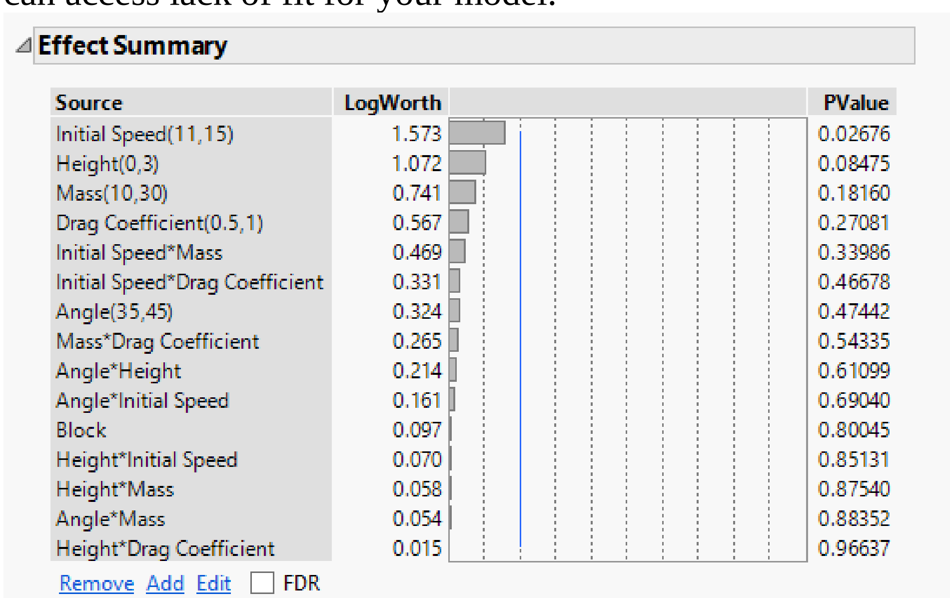


There appears to be no lack of fit. The points fit the predicted curve, meaning that the simulation used showed clear trends on how the parameters affect the overall travel distance. The R squared value is at exactly 1.00, meaning

that the fit is great and JMP has identified the factors and how they affect the overall travel distance.

b. The experiment has been run on a simulator with no experimental error – launches made at the same settings always result in the same distance. Therefore, replication is of no value for this experiment. Since there is no experimental error there is no actual basis for hypothesis testing and p-values. So, at this point do not attempt to simplify the fitted model. We will proceed using the full default model; i.e., all main effects and two-way interaction terms.

c. Please show a screen capture of your model so we know what terms are in your model and also include screen captures of the Actual by Predicted and Residual by Predicted plot so that we can access lack of fit for your model.



Effect Summary

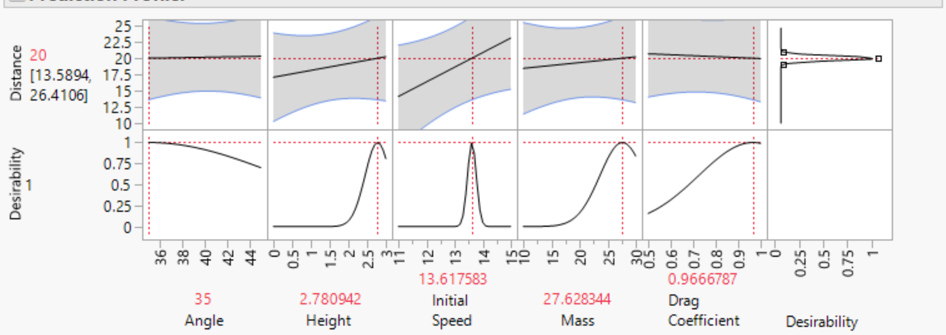
Source	LogWorth	PValue
Initial Speed(11,15)	1.573	0.02676
Height(0,3)	1.072	0.08475
Mass(10,30)	0.741	0.18160
Drag Coefficient(0.5,1)	0.567	0.27081
Initial Speed*Mass	0.469	0.33986
Initial Speed*Drag Coefficient	0.331	0.46678
Angle(35,45)	0.324	0.47442
Mass*Drag Coefficient	0.265	0.54335
Angle*Height	0.214	0.61099
Angle*Initial Speed	0.161	0.69040
Block	0.097	0.80045
Height*Initial Speed	0.070	0.85131
Height*Mass	0.058	0.87540
Angle*Mass	0.054	0.88352
Height*Drag Coefficient	0.015	0.96637

Remove Add Edit FDR

3. (10 pts) Once you have fit the model in problem 2, we will use it to determine settings of the experimental factors in order to hit specified targets. Go ahead and use your model even if you find evidence of lack of fit. We will use the **Prediction Profiler** and **Desirability Functions** to find settings to hit specified targets (the **Response Goal** is **Match Target**). The Two Level Factorial notes show how to perform these analyses in JMP).

a. In the Prediction Profiler report menu select the **Desirability Functions** option (**Optimize and Desirability**). Next, click in the **Desirability** Profile window that appears as the last column to the right in the Profiler. In the **Response Goal** window change the response goal to **Match Target** and set the target (middle value) to 20. Now select the **Maximize and Remember** option from the Profiler report menu to find the settings that will produce the target distance. Include a screenshot (or use Selection Tool) to show the Prediction Profiler configuration and the suggested factor settings to hit a target of 20 meters.

Prediction Profiler



Remembered Settings

Setting	Angle	Height	Initial Speed	Mass	Drag Coefficient	Block	Distance	Distance Lower CI	Distance Upper CI	Desirability
Optimal	40	1.5	13	20	0.75		17.267083	14.931406	19.602761	0.000000
Optimal	35	0	11	10	1		10.549583	1.2585392	19.840628	.
Optimal	35	2.780942	13.617583	27.628344	0.9666787		20	13.589369	26.410631	1.000000

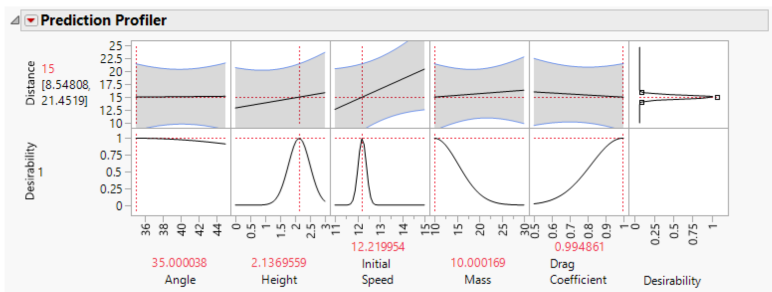
Differences

Setting A	- Setting B	Distance
Optimal	_Optimal_	-6.7175
Optimal	_Optimal_	2.7329167
Optimal	_Optimal_	9.4504167

b. Now use the simulator to see how you close you can come to the target for those specified settings of the factor determined in part a. Please, note that some factors can only be changed in integer values, so you will have to round up or down to the nearest integer value; it is permissible to try rounding up or down in these values to see if it improves accuracy. How close was the actual distance to the predicted distance of 20? If the actual distance is not close to 20 (be sure to report the actual distance in the write up) then can you think of reasons why it is not close? You may not be close to 20 in some cases and that is alright as long as you did the analyses correctly to this point. Again report your results whether close or not to the target.

Inputting the values into the simulation yielded a distance of 21.15 meters. The error we see is probably mostly due to not being able to input the exact number into the simulator.

c. Repeat parts a and b, but this time in the Prediction Profiler specify a target distance of **15** meters. Include a screenshot of the Profiler with the suggested factor settings. Try a launch with the simulator and the settings (rounding if necessary). How close to the target did you come? Report the results. Are the results in parts b and c similar in terms of accuracy? Explain. If the accuracy results are not the same, then can you speculate as to why they are different – no right or wrong answer here.



Remembered Settings

	Setting	Angle	Height	Initial Speed	Mass	Drag Coefficient	Block	Distance	Distance Lower CI	Distance Upper CI	Desirability
○	_Optimal_	40	1.5	13	20	0.75		17.267083	14.931406	19.602761	0.000000
○	_Optimal_	35	0	11	10	1		10.549583	1.2585392	19.840628	.
○	_Optimal_	35	2.780942	13.617583	27.628344	0.9666787		20	13.589369	26.410631	1.000000
○	_Optimal_	35.000038	2.1369559	12.219954	10.000169	0.994861		15	8.5480846	21.451915	1.000000

Differences

Setting A	- Setting B	Distance
Optimal	_Optimal_	-6.7175
Optimal	_Optimal_	2.7329167
Optimal	_Optimal_	9.4504167
Optimal	_Optimal_	-2.267083
Optimal	_Optimal_	4.4504167
Optimal	_Optimal_	-5

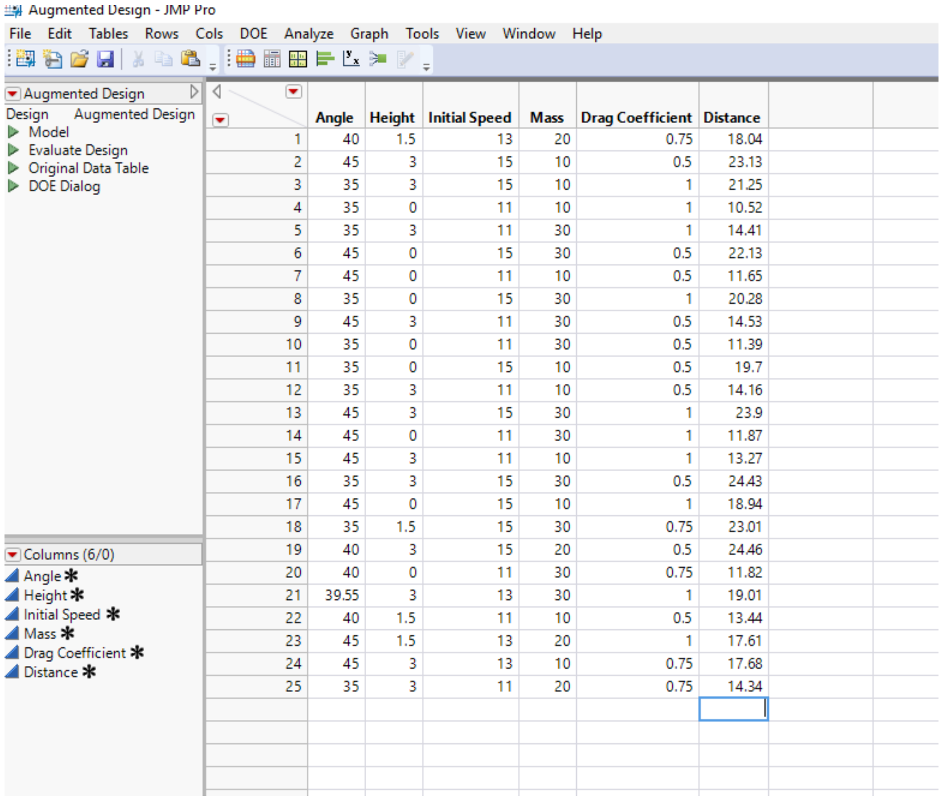
With finding the solution for 15 meters, the result is closer with a range of 14.48. This is mostly due to the fact that the solution it printed were all numbers very close to the values that could be inputted into the simulation program!! It is also because the majority of the data was centered around 15, allowing the program to pull more from direct data in that range.

Part 2 – Augmenting an Existing Design to Estimate More Experimental Effects.

This is a continuation of Part 1. You will need to reuse your JMP data table created for part 1. The design you used in part 1 was only two level so you cannot estimate quadratic terms (requires at least 3 levels) even if curvature seems to be present in the relationship to launch distance. In this section you will augment your original design in order to estimate squared (quadratic) terms in a model. Before you do part 2, I suggest you review (if you have not already done so) the **Video #22** on augmenting designs and the **Fractional Factorial Part 2 notes pages 25 – 35**. You should also read the instructor notes on Incomplete Blocks and watch **video #22** on Incomplete Block Designs and review **video #9** on Blocking Designs for One Factor.

4. (10 pts.) Open up the data table you created in part 1, with the table opened, go to the DOE menu and select the **Augment Design** option. In the dialog window that appears enter your five experimental factor columns into **X** and your response Distance column into **Y**.

    a. When the Augment Design window appears, do not select the **Group new runs into separate blocks**, since this is a simulator there can be no response shift between the original and the augmented trials. Now select the **Augment** option. Once you click on the **Augment** button, a **Model** window opens. In this window click on the **RSM** button to create a model with all two-way interactions and quadratic terms in your five factors. Below the Model window you will see a **Design Generation** node with a total number of runs specified (this includes the runs from your original design). Click on **Make Design** to create the augmented design. Finally, click on the **Make Table** button to create the new design table. Since Angle, Height, and Speed can only be changed in integer increments you will have to round off to the nearest integer the settings for those factors in your augmented data table; it does not matter if you round up or down at this point. Please include a screen shot of your augmented design after you have rounded off values.



	Angle	Height	Initial Speed	Mass	Drag Coefficient	Distance
1	40	1.5	13	20	0.75	18.04
2	45	3	15	10	0.5	23.13
3	35	3	15	10	1	21.25
4	35	0	11	10	1	10.52
5	35	3	11	30	1	14.41
6	45	0	15	30	0.5	22.13
7	45	0	11	10	0.5	11.65
8	35	0	15	30	1	20.28
9	45	3	11	30	0.5	14.53
10	35	0	11	30	0.5	11.39
11	35	0	15	10	0.5	19.7
12	35	3	11	10	0.5	14.16
13	45	3	15	30	1	23.9
14	45	0	11	30	1	11.87
15	45	3	11	10	1	13.27
16	35	3	15	30	0.5	24.43
17	45	0	15	10	1	18.94
18	35	1.5	15	30	0.75	23.01
19	40	3	15	20	0.5	24.46
20	40	0	11	30	0.75	11.82
21	39.55	3	13	30	1	19.01
22	40	1.5	11	10	0.5	13.44
23	45	1.5	13	20	1	17.61
24	45	3	13	10	0.75	17.68
25	35	3	11	20	0.75	14.34

b. Once you have created the augmented design table, go the simulator and perform the additional trials in your augmented design table. Now, run the Model script that JMP saved to your data table when you created the augmented design (see pane in upper left hand side of the data table window). Be certain to include a screenshot of the Parameter Estimates table for your fitted model.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	18.038277	0.059209	304.65	<.0001*
Angle(35,45)	0.2026146	0.020654	9.81	0.0006*
Height(0,3)	1.4139385	0.020115	70.29	<.0001*
Initial Speed(11,15)	4.5150757	0.01948	231.78	<.0001*
Mass(10,30)	0.6551997	0.019483	33.63	<.0001*
Drag Coefficient(0.5,1)	-0.416564	0.020714	-20.11	<.0001*
Angle*Height	-0.129578	0.021068	-6.15	0.0035*
Angle*Initial Speed	0.09155	0.021106	4.34	0.0123*
Angle*Mass	0.0307105	0.021205	1.45	0.2211
Height*Initial Speed	0.0519685	0.020716	2.51	0.0662
Height*Mass	0.0397954	0.020718	1.92	0.1272
Height*Drag Coefficient	-0.013012	0.021109	-0.62	0.5710
Initial Speed*Mass	0.3210571	0.020908	15.36	0.0001*
Initial Speed*Drag Coefficient	-0.21845	0.021106	-10.35	0.0005*
Mass*Drag Coefficient	0.1615633	0.02121	7.62	0.0016*
Angle*Angle	-0.212507	0.052234	-4.07	0.0152*
Height*Height	-0.565442	0.061532	-9.19	0.0008*
Initial Speed*Initial Speed	0.2723667	0.064333	4.23	0.0133*
Mass*Mass	-0.331692	0.061532	-5.39	0.0057*
Angle*Drag Coefficient	-0.015579	0.021352	-0.73	0.5061
Drag Coefficient*Drag Coefficient	0.0215839	0.05203	0.41	0.6995

Effect Tests

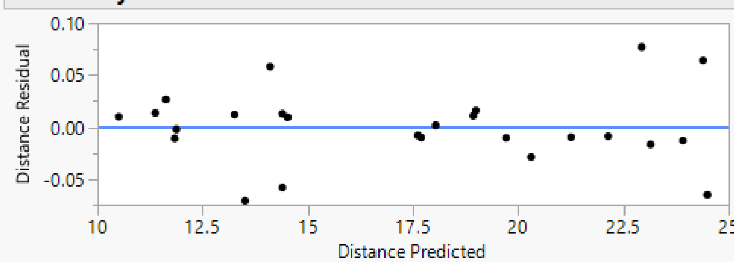
c. For the model with or without the block term, does there appear to be any lack of fit? Use the Actual by Predicted and Predicted by Residual plots to evaluate lack of fit? Be sure to include a screen shot of the Plots.

Effect Summary

Source	LogWorth	PValue
Initial Speed(11,15)	8.682	0.00000
Height(0,3)	6.610	0.00000
Mass(10,30)	5.331	0.00000
Drag Coefficient(0.5,1)	4.443	0.00004
Initial Speed*Mass	3.979	0.00010
Initial Speed*Drag Coefficient	3.308	0.00049
Angle(35,45)	3.218	0.00061
Height*Height	3.109	0.00078
Mass*Drag Coefficient	2.797	0.00159
Angle*Height	2.450	0.00354
Mass*Mass	2.242	0.00573
Angle*Initial Speed	1.911	0.01228
Initial Speed*Initial Speed	1.875	0.01333
Angle*Angle	1.817	0.01524
Height*Initial Speed	1.179	0.06615
Height*Mass	0.896	0.12716
Angle*Mass	0.655	0.22111
Angle*Drag Coefficient	0.296	0.50605
Height*Drag Coefficient	0.243	0.57096
Drag Coefficient*Drag Coefficient	0.155	0.69954

Remove Add Edit FDR

Residual by Predicted Plot

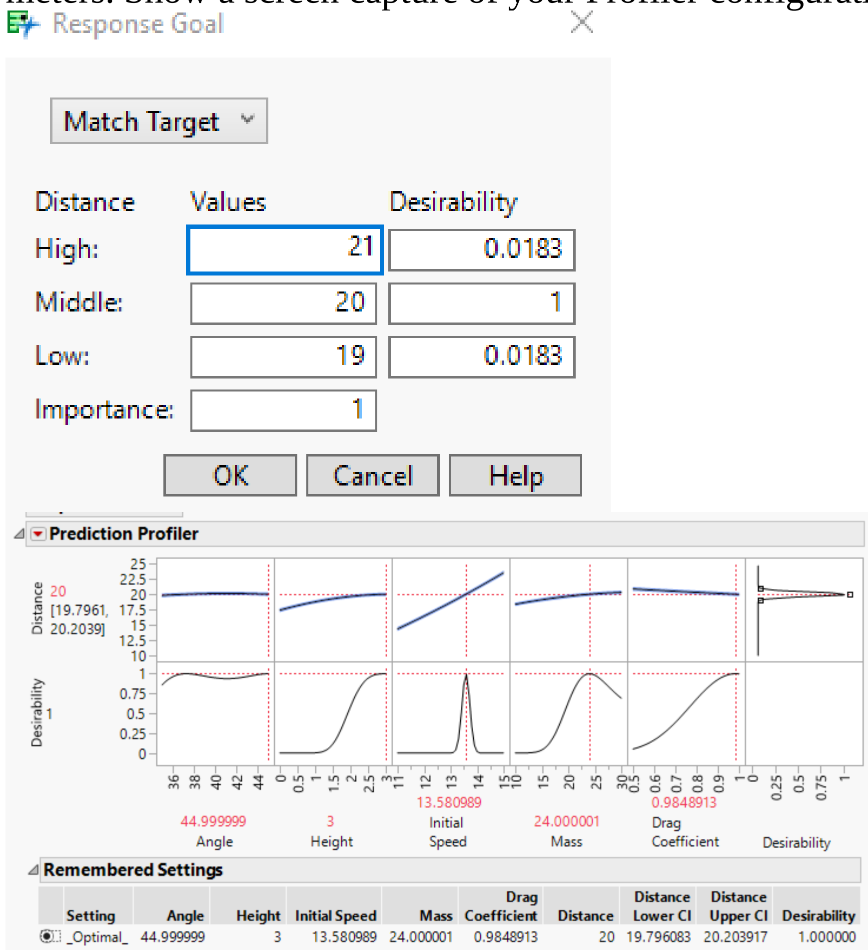


There does not appear to be any lack of fit! Again, the Rsq value is exactly 1.00, meaning JMP calculates that there is a direct correlation between the factors and the response of distance travelled.

d. At this point **do not** attempt to simplify the model beyond possibly removing the block term. Later in the course we will discuss strategies for model simplification that do not require the use of p-values.

5. (10 pts) Once you have fit the model in problem 4, we will use it to determine settings of the five factors to hit specified targets with the simulator. Go ahead and use your model even if you find evidence of lack of fit. We will use the **Prediction Profiler** and **Desirability Functions** to find settings to hit specified targets (the **Response Goal** is **Match Target**). The optimization is performed in exactly the same manner as you did in part 1.

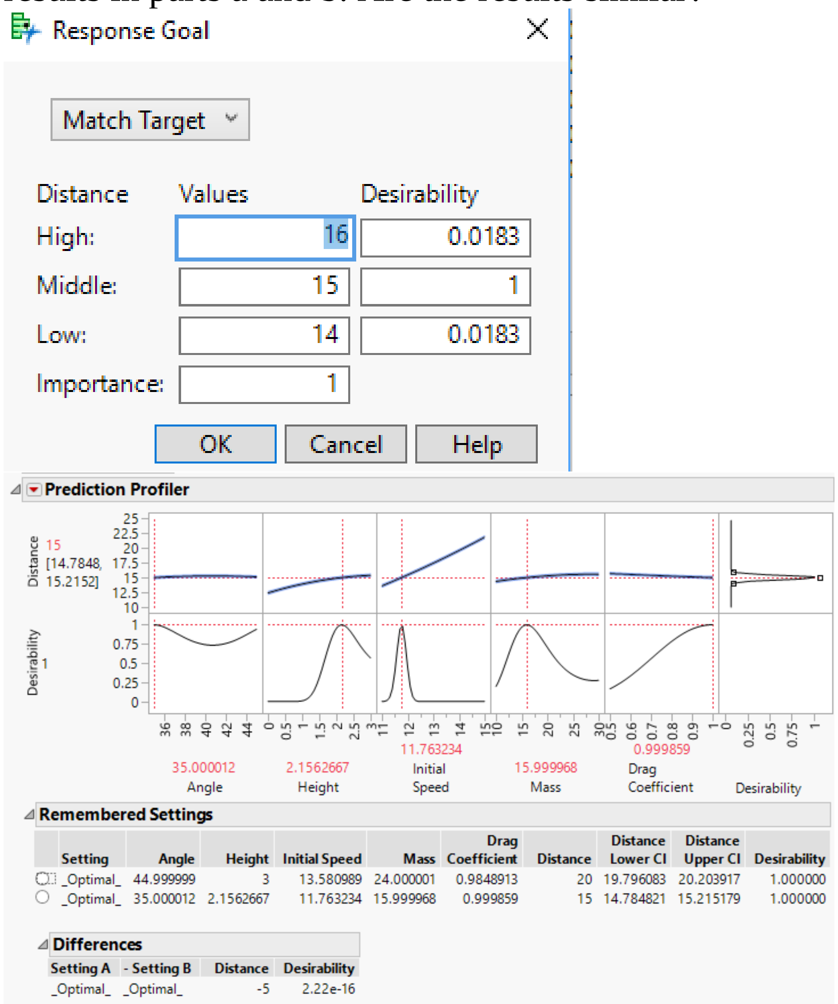
a. Using the Prediction Profiler and Desirability Functions determine the settings of the five factors to hit a target of 20 meters. Show a screen capture of your Profiler configuration.



b. Next, use the simulator to try the settings determined in part a to hit the 20 meter target. As in part 1, you will have to round off for settings of Angle, Height, and Initial Speed to integer values. How close did you come to the target? Explain.
The speed wanted was exactly in between the ineger values, so ranges of 21.01 and 18.99 we produced with the speed around that speed wanted. This tell me that the JMP software did a very good job in finding a solution to hit 20 meters, if able to be inputted into the software.

c. Repeat parts a and b, but this time in the Prediction Profiler specify a target distance of 15 meters. Include a screenshot of the Profiler with the suggested factor settings. Finally run the simulator at these suggested settings to hit a target of 15 meters.

How close did you come to the target? Compare the accuracy results in parts a and b. Are the results similar?



Setting	Angle	Height	Initial Speed	Mass	Drag Coefficient	Distance	Distance Lower CI	Distance Upper CI	Desirability
Optimal	44.999999	3	13.580989	24.000001	0.9848913	20	19.796083	20.203917	1.000000
Optimal	35.000012	2.1562667	11.763234	15.999968	0.999859	15	14.784821	15.215179	1.000000

Setting A	- Setting B	Distance	Desirability
Optimal	_Optimal_	-5	2.22e-16

With the inputted value, which were vey close to what could be entered in the simulation, yielded a result of 15.09 meters, just 9 centimeters away from what was wanted! Well within the CI. It is mostly more accurate due to the wanted input being closer to what could actually be entered.

6. (5 pts) Compare your results using the fractional factorial design in part 1 to the results using the augmented design in part 2. Which design provided better accuracy in terms of predicting shot distance for your targets? Explain.
Overall, the augmented design provided better results. This is because it took in what data it could use more of to help with predicting better what distance you wanted to hit.