Problem 1

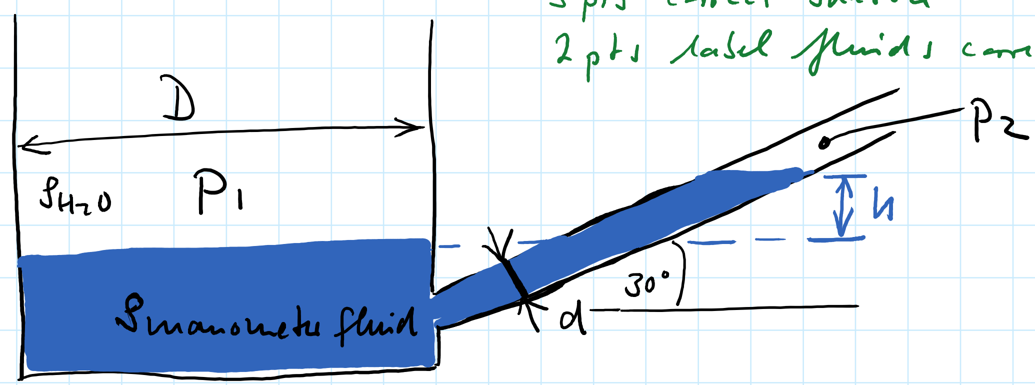
Tuesday, March 8, 2016 6:44 PM

(20 points)

3 pts correct sketch
2 pts label fluids correctly

(a)

5



(b) left leg = right leg 5

$$p_1 + \rho_{H_2O}\, g h = \rho_{manometer}\, g h + p_2$$

$$\Delta p = (p_1 - p_2) = (\rho_{manometer} - \rho_{H_2O})\, g h$$

$$= \rho_{H_2O}\,(S.G._{manometer} - 1)\, g h$$

5

w/ $S.G. \equiv \dfrac{\rho}{\rho_{H_2O}}$

(for pressure difference, the areas do not matter, only the difference in manometer fluid elevation)

(c) $\Delta p = (p_1 - p_2) = 1000\,\frac{kg}{m^3}\,(2.95 - 1)\, 10\,\frac{m}{s^2} \cdot 0.1\,m$ 5

$$= 1950\,\frac{kg\,m}{s^2}\,\frac{1}{m^2} = 1950\ Pa$$

- correct substitution of values
- correct algebra
- correct reduction / conversion of units.

Problem 2

(40 points)

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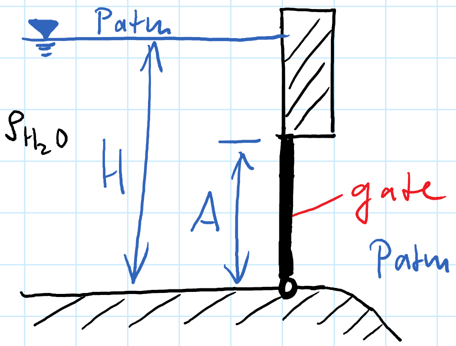
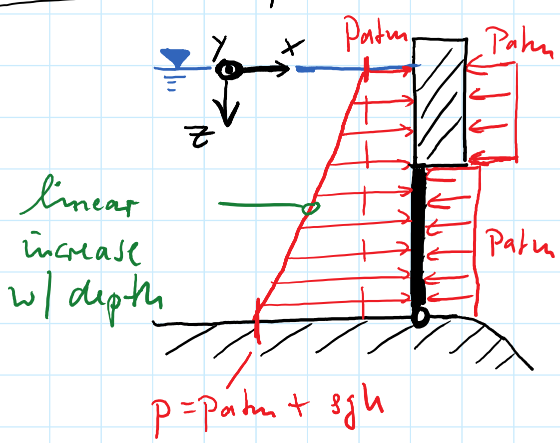


Width: W
(into the drawing plane)

(a) Sketch the pressure distribution p(z)



10

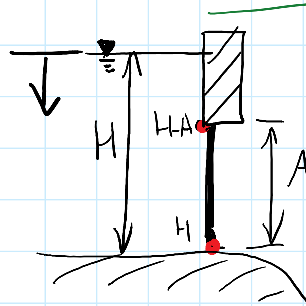
- Patm on right side – 2 pts.
- Patm on left side – 2 pts.
- $p(z) = p_{atm} + \rho g z$ linear increase w/ depth. – 6 pts.

(b) magnitude of force on gate

4 $\boxed{p = p_{atm} + \rho g h}$

4 $\boxed{F = -\iint_A p\, dA}$

Patm also acts on the right side of the gate ∴ use p_{gage} only! 2



$$F = -\iint_A p\, dA = -\iint_A \rho g h\, dA = W \int_{H-A}^{H} \rho g z\, dz =$$

p gage only!

$$= W \rho g \left[\frac{z^2}{2}\right]_{H-A}^{H}$$

$$= \frac{1}{2}\rho g W \left(H^2 - (H-A)^2\right) \quad *$$

$$= \frac{1}{2}\rho g W \left(\cancel{H^2} - (\cancel{H^2} - 2HA + A^2)\right)$$

$$= \frac{1}{2}\rho g W \left(2HA - A^2\right)$$

10

- $dA = W dz$

$$= \tfrac{1}{2} \rho g \bar{w} \left(\cancel{H^2} - \left(\cancel{H^2} - 2HA + A^2 \right) \right)$$

10

- $dA = \bar{w}\, dz$
- correct integration limits
- correct integration
- correct substitution and algebra

$$= \frac{1}{2} \rho g \bar{w} \left(2HA - A^2 \right)$$

(Check: For $H = A$:
$F = \frac{1}{2} \rho g \bar{w} H^2$ (✓))

(c) line of action of the force

10

$$z' F_R = \iint_A z\, p\, dA$$

$$= \bar{w} \int_{H-A}^{H} z\, \rho g z\, dz = \rho g \bar{w} \left[\frac{z^3}{3} \right]_{H-A}^{H} =$$

$$= \frac{1}{3} \rho g \bar{w} \left(H^3 - (H-A)^3 \right) \quad *$$

$$= \frac{1}{3} \rho g \bar{w} \left(\cancel{H^3} - \left(\cancel{H^3} - 3H^2 A + 3HA^2 - A^3 \right) \right)$$

$$= \frac{1}{3} \rho g \bar{w} \left(+3H^2 A - 3HA^2 + A^3 \right)$$

$$= \frac{1}{3} \rho g \bar{w} A \left(+3H^2 - 3HA + A^2 \right)$$

$$\therefore \quad z' = \frac{1}{F_R} \iint_A z\, p\, dA = \frac{\frac{1}{3} \cancel{\rho g} \cancel{\bar{w}} \cancel{A} \left(3H^2 - 3HA + A^2 \right)}{\frac{1}{2} \cancel{\rho g} \cancel{\bar{w}} \cancel{A} \left(2H - A \right)} =$$

$$= \frac{2}{3} \frac{\left(3H^2 - 3HA + A^2 \right)}{\left(2H - A \right)}$$

Check for $H = A$: $\therefore \quad z' = \dfrac{2}{3} \dfrac{+\cancel{3H^2} - \cancel{3H^2} + H^2}{2H - \cancel{H}\ H} = \dfrac{2}{3} H$ (✓)

- correct equation $z' F_R = \iint_A z\, p\, dA$, $p = \rho g z$
- correct rearrangement for $z' = \frac{1}{F_R} \iint \ldots$
- correct integration limits
- correct integration
- correct substitution and algebra

Problem 3

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(40 points)

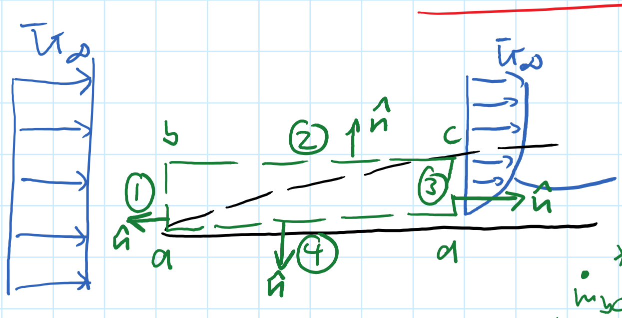
(a) Governing equation

$$\frac{Dm}{Dt} = \frac{\partial}{\partial t}\iiint_{C.V.} \rho \, d\forall + \iint_{C.S.} \rho (\vec{v}\cdot\hat{n})\, dA = \phi$$

steady state $\frac{\partial}{\partial t} = \phi$

$$\therefore \iint_{C.S.} \rho(\vec{v}\cdot\hat{n})\, dA = \phi$$

10



for $0 \le y \le \delta$:

$$\frac{u}{U_\infty} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$\dot{m}_{bc}$ * 5

(b)

$$\iint_{C.S.①} \rho(\vec{v}\cdot\hat{n})\,dA + \iint_{C.S.②} \rho(\vec{v}\cdot\hat{n})\,dA + \iint_{C.S.③} \rho(\vec{v}\cdot\hat{n})\,dA + \iint_{C.S.④} \rho(\vec{v}\cdot\hat{n})\,dA = \phi$$

C.S.①: $= -U_\infty$; C.S.②: $= \dot{m}_{bc}$; C.S.③: $= U_\infty \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$; C.S.④: $= \phi$ since $\vec{v} \perp \hat{n}$ (wall!)

5

ab:

$$-\iint_{C.S.①} \rho U_\infty \, dA = -\int_0^W \int_0^\delta \rho U_\infty \, dy\, dz = -\rho U_\infty W \delta$$

5

cd:

$$\iint_{C.S.③} \rho U_\infty \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) dA = \int_0^W \int_0^\delta \rho U_\infty \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) dy\, dz$$

$$= W \rho U_\infty \int_0^\delta \sin\left(\frac{\pi}{2\delta} y\right) dy$$

$$= \rho U_\infty W \left[-\frac{2\delta}{\pi} \cos\left(\frac{\pi}{2\delta} y\right)\right]_0^\delta$$

$$= \rho U_\infty W \left[-\frac{2\delta}{\pi} \underbrace{\cos\frac{\pi}{2}}_{\phi} + \frac{2\delta}{\pi} \underbrace{\cos\phi}_{1}\right]$$

$$= \rho U_\infty \frac{2\delta}{\pi} W$$

$$= \rho U_\infty W \delta \frac{2}{\pi}$$

10

$$= \rho\, \overline{U}_\infty \overline{W}\, \delta\, \frac{2}{\pi}$$

(c) * or here

$$\dot{m}_{bc} = +\rho\, \overline{U}_\infty \overline{W}\, \delta - \rho\, \overline{U}\, \overline{W}\, \delta\, \frac{2}{\pi} = \rho\, \overline{U}_\infty \overline{W}\, \delta \left(1 - \frac{2}{\pi}\right)$$

$$= \left(\iint_{ab} - \iint_{cd} \right) \qquad = \rho\, \overline{U}_\infty \overline{W}\, \delta\, \frac{\pi - 2}{\pi} \qquad 5$$