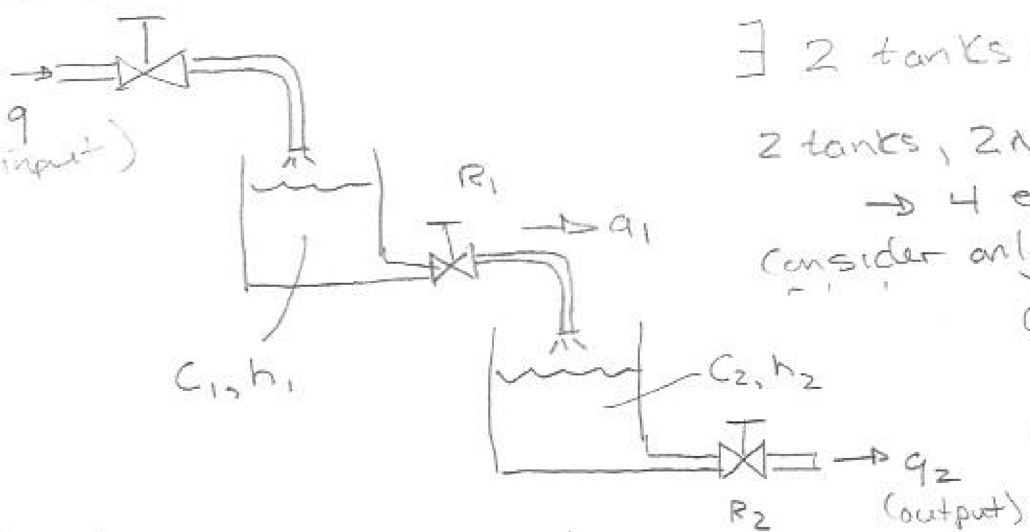
PROBLEM SET 7 SOLUTIONS

Problem 1 : B-7-4



∃ 2 tanks ⇒ ∴ 2 EOM

2 tanks, 2 valves
→ 4 equations
consider only : h_1, h_2, q_1, q_2

$$C\frac{dh}{dt} = q_{in} - q_{out}$$

$$qR = h_{1st} - h_{2nd}$$

Tanks

① $C_1 \dfrac{dh_1}{dt} = q - q_1$

② $C_2 \dfrac{dh_2}{dt} = q_1 - q_2$

valves

$q_1 R_1 = (h_1 - 0) \Rightarrow h_1 = q_1 R_1$

$\dfrac{dh_1}{dt} = \dfrac{dq_1}{dt} R_1$ ③

$q_2 R_2 = (h_2 - 0)$

$\Rightarrow h_2 = q_2 R_2$

$\dfrac{dh_2}{dt} = \dfrac{dq_2}{dt} R_2$ ④

① + ③ : $C_1\left[R_1 \dfrac{dq_1}{dt}\right] = q - q_1 \Rightarrow [C_1R_1s + 1]Q_1(s) = Q(s)$

② + ④ : $C_2\left[R_2 \dfrac{dq_2}{dt}\right] = q_1 - q_2 \qquad [-1]Q_1(s) + [C_2R_2s + 1]Q_2(s) = 0$

$$G(s) = \frac{Q_2(s)}{Q(s)} \qquad Q_2(s) = \frac{\begin{vmatrix} [C_1R_1s+1] & Q(s) \\ [-1] & 0 \end{vmatrix}}{\begin{vmatrix} [C_1R_1s+1] & 0 \\ [-1] & [C_2R_2s+1] \end{vmatrix}} = \frac{Q(s)}{[C_1R_1s+1][C_2R_2s+1]+1}$$

$$\therefore \boxed{G(s) = \frac{Q_2(s)}{Q(s)} = \frac{1}{[R_1C_1s+1][R_2C_2s+1]}}$$

Problem 2 : 8-7-17

$z = x^2 + 2xy + 5y^2$

$10 \le x \le 12 \Rightarrow x_0 = 11$

$4 \le y \le 6 \Rightarrow y_0 = 5$

$$z \approx z\Big|_{(x_0,y_0)} + \frac{\partial z}{\partial x}\Big|_{(x_0,y_0)}(x - x_0) + \frac{\partial z}{\partial y}\Big|_{(x_0,y_0)}(y - y_0)$$

$$= \left[(11)^2 + 2(11)(5) + 5(5)^2\right] + (2x + 2y)\Big|_{(x_0,y_0)}(x - 11)$$

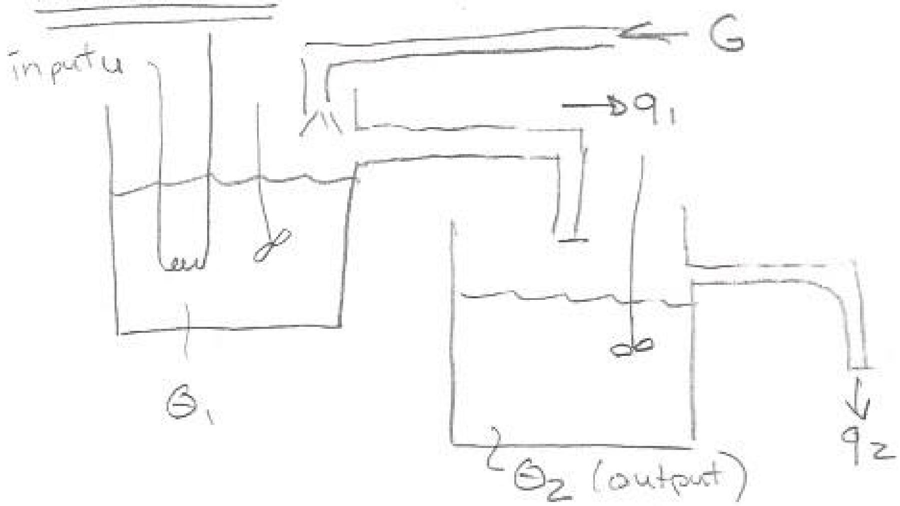
$$+ (2x + 10y)\Big|_{(x_0,y_0)}(y - 5)$$

$$= 356 + \left[2(11) + 2(5)\right](x - 11) + \left[2(11) + 10(5)\right](y - 5)$$

$$= 356 + 32(x - 11) + 72(y - 5)$$

$$\boxed{z = 356 + 32x + 72y}$$

Problem 3 : B-7-20



consider only: $\Theta_1, \Theta_2, q_1, q_2$

input : u

Compare heat flow to fluid problem

$$C\frac{d\Theta}{dt} = \text{heat}_{in} - h_{out}$$

heat flow $q_i = G \cdot c \cdot \Theta_i$

$$C_1\frac{d\Theta_1}{dt} = u - q_1 \qquad q_1 = Gc\Theta_1$$

$$C_2\frac{d\Theta_2}{dt} = q_1 - q_2 \qquad q_2 = Gc\Theta_2$$

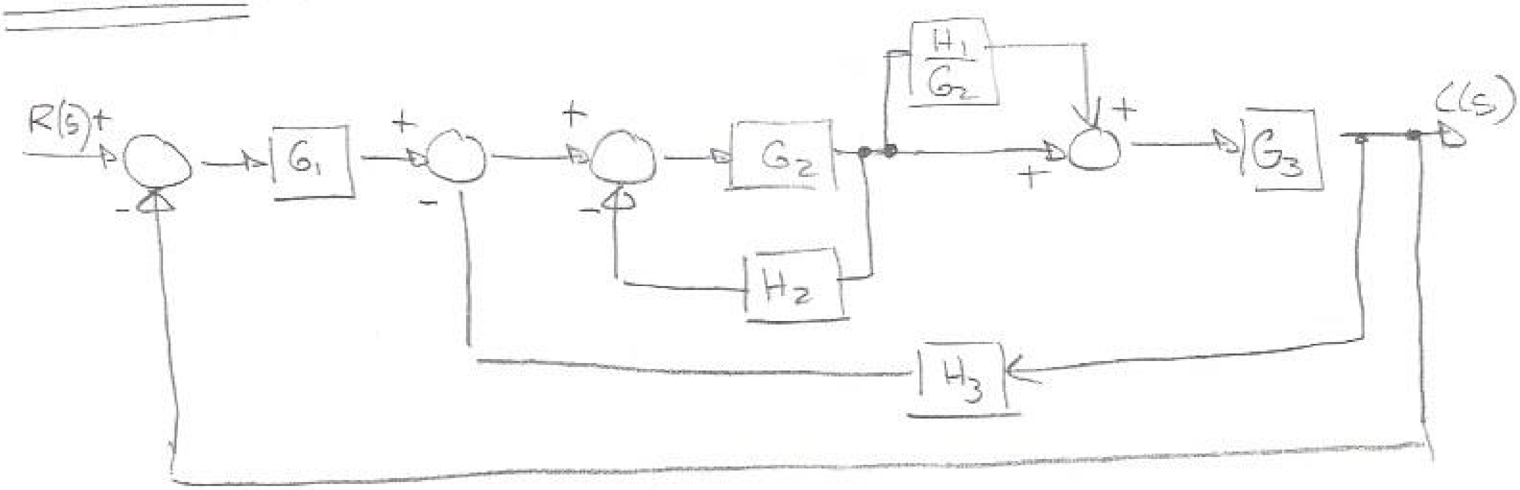
$$C_1\frac{d\Theta_1}{dt} = u - Gc\Theta_1 \Rightarrow [C_1s + Gc]\,\Theta_1(s) + [0]\,\Theta_2(s) = U(s)$$

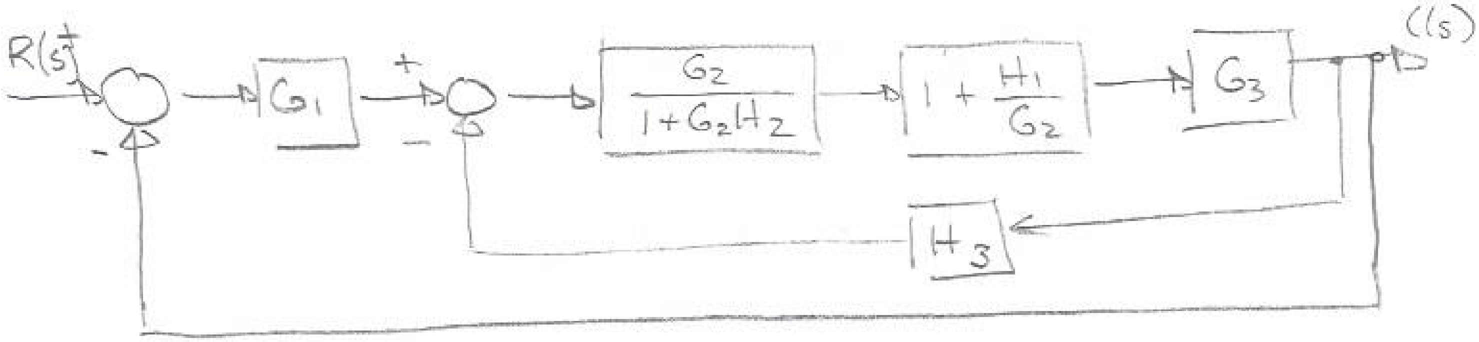
$$C_2\frac{d\Theta_2}{dt} = Gc\Theta_1 - Gc\Theta_2 \Rightarrow [-Gc]\,\Theta_1(s) + [C_2s + Gc]\,\Theta_2(s) = 0$$

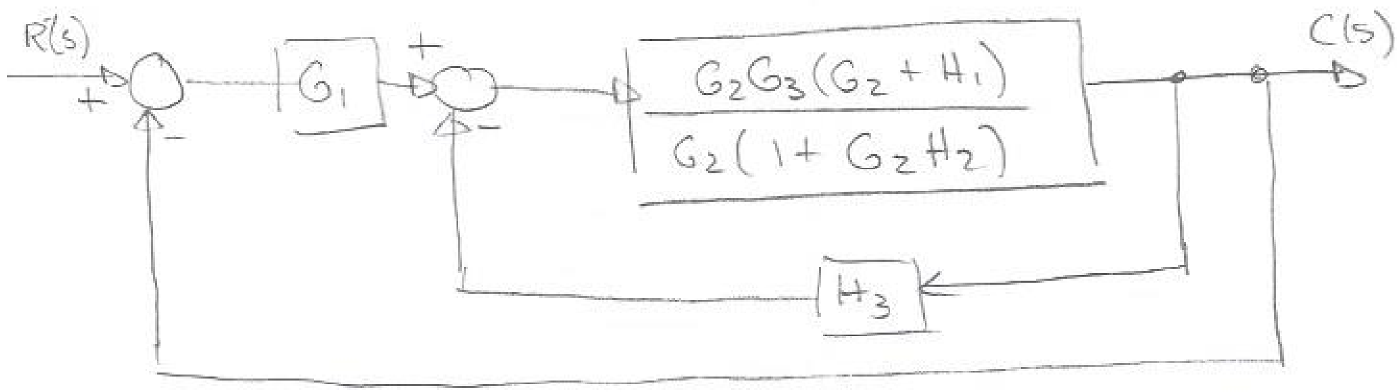
$$G(s) = \frac{\Theta_2(s)}{U(s)} \Rightarrow \Theta_2(s) = \frac{\begin{vmatrix} [C_1s+Gc] & U(s) \\ [-Gc] & 0 \end{vmatrix}}{\begin{vmatrix} [C_1s+Gc] & [0] \\ [-Gc] & [C_2s+Gc] \end{vmatrix}}$$

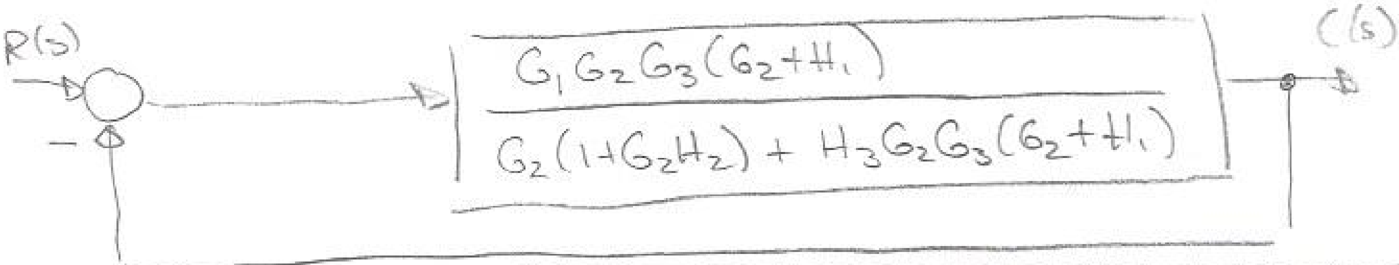
$$\Rightarrow \boxed{G(s) = \frac{\Theta_2(s)}{U(s)} = \frac{G \cdot c}{[C_1s+Gc][C_2s+Gc]}}$$

Problem 4 : B-10-2



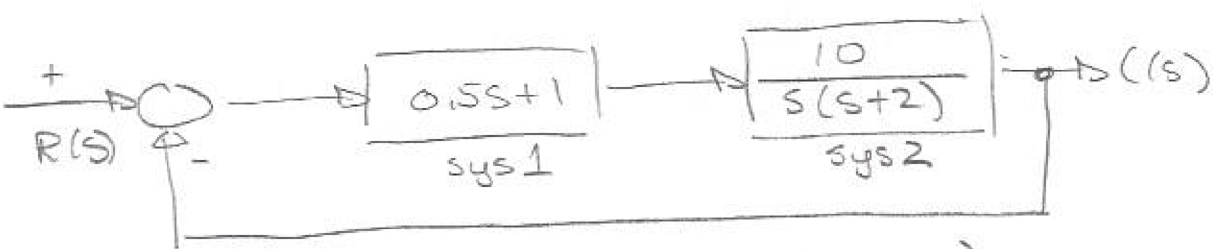






$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 \cancel{G_2} G_3 (G_2+H_1)}{\cancel{G_2}(1+G_2H_2)+H_3\cancel{G_2}G_3(G_2+H_1)+G_1\cancel{G_2}G_3(G_2+H_1)}}$$

Problem 5: B-10-3

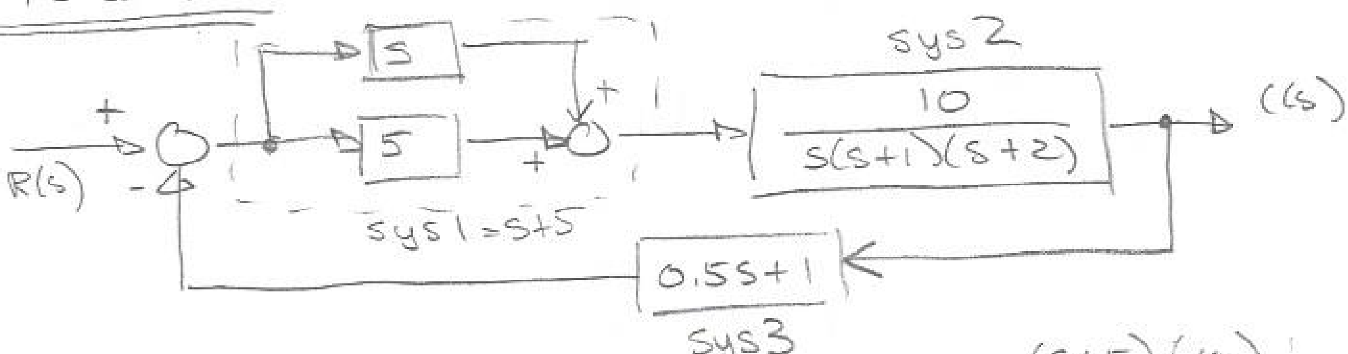


$$sysg \Rightarrow sys1 * sys2 = \frac{10(0.5s+1)}{s(s+2)}$$

$$sys = \frac{C(s)}{R(s)} = \frac{10(0.5s+1)}{s(s+2)+10(0.5s+1)} = \boxed{\frac{5s+10}{s^2+7s+10} = sys}$$

Refer to M-file and results

Problem 6: B-10-4



$$sys\_forward \Rightarrow sys1 * sys2 = \frac{(s+5)(10)}{s(s+1)(s+2)}$$

$$sys\_CL = \frac{C(s)}{R(s)} = \frac{10(s+5)}{s(s+1)(s+2)+10(s+5)(0.5s+1)}$$

$$\boxed{sys\_CL = \frac{10s+5}{s^3+8s^2+37s+50}}$$

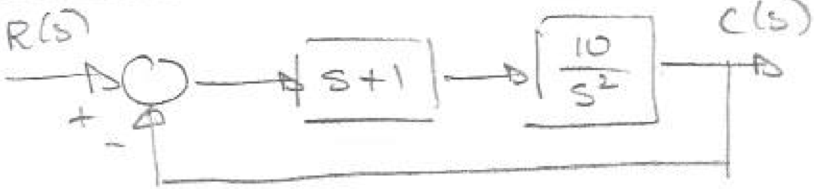
Refer to M-file and results

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Problem Set 7 Solutions
% Problem 5 - From dynamic system of Ogata B-10-3
% Problem 6 - From dynamic system of Ogata B-10-4
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Problem 5: Ogata B-10-3
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear

% Define sys1 and sys2
num_sys1 = [0.5 1];
den_sys1 = 1;
num_sys2 = 10;
den_sys2 = [1 2 0];

sys1 = tf(num_sys1,den_sys1);
sys2 = tf(num_sys2,den_sys2);

% Define forward loop TF sysg and closed-loop TF sys
sysg = series(sys1,sys2);
sys = feedback(sysg,[1])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Problem 5: Ogata B-10-3
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear

% Define sys1, sys2, and sys
num_sys1 = [1 5];
den_sys1 = 1;
num_sys2 = 10;
den_sys2 = [1 3 2 0];
num_sys3 = [0.5 1];
den_sys3 = 1;

sys1 = tf(num_sys1,den_sys1);
sys2 = tf(num_sys2,den_sys2);
sys3 = tf(num_sys3,den_sys3);

% Define forward loop TF sysg and closed-loop TF sys
sys_forward = series(sys1,sys2)
sys_CL = feedback(sys_forward,sys3)
```

7/11


```
>> PS7_Solutions_P5_P6_2018

sys =

     5 s + 10
  --------------
  s^2 + 7 s + 10

Continuous-time transfer function.


sys_forward =

      10 s + 50
  -----------------
  s^3 + 3 s^2 + 2 s

Continuous-time transfer function.


sys_CL =

         10 s + 50
  -----------------------
  s^3 + 8 s^2 + 37 s + 50

Continuous-time transfer function.

>>
```

Problem 7 : B-10-8



$$\frac{C(s)}{R(s)} = \frac{10(s+1)}{s^2 + 10(s+1)} = \frac{10s + 10}{s^2 + 10s + 1}$$

$$C(s) = \frac{1}{s}\left[\frac{10s+10}{s^2+10s+10}\right] = \frac{A}{s} + \frac{B}{s-r_1} + \frac{C}{s-r_2}$$

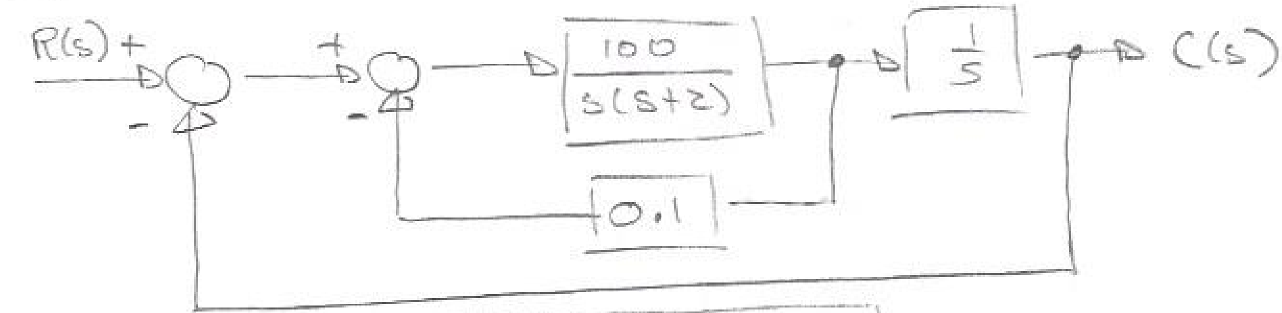
$$r_{1,2} = \frac{-10 \pm \sqrt{100-40}}{2} = \frac{-10 \pm 7.746}{2}$$

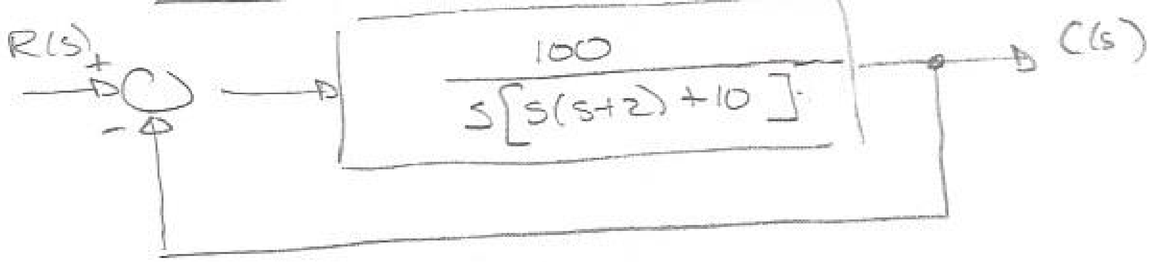
$$= -1.127 \,,\, -8.8730$$

$$= \frac{1}{s} + \frac{0.1455}{s+1.127} - \frac{1.1455}{s+8.873}$$

$$\boxed{c(t) = 1(t) + 0.1455\left[e^{-1.127t} - e^{-8.873t}\right]}$$

Problem 8 : B-10-10





$$\frac{C(s)}{R(s)} = \frac{100}{s[s(s+2)+10]+100}$$

$$= \frac{100}{s^3+2s^2+10s+100}$$

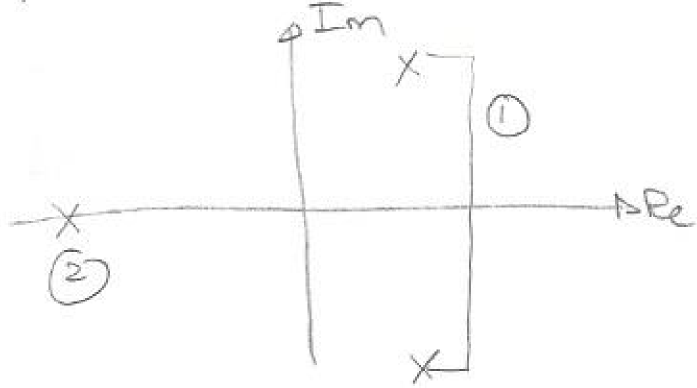
Check poles of C.E. $\Rightarrow$ $s^3+2s^2+10s+100$

MATLAB

```
>> roots([1 2 10 100])

ans =
     1.2907+4.4901i
     1.2907-4.4901i
    -4.5815
```

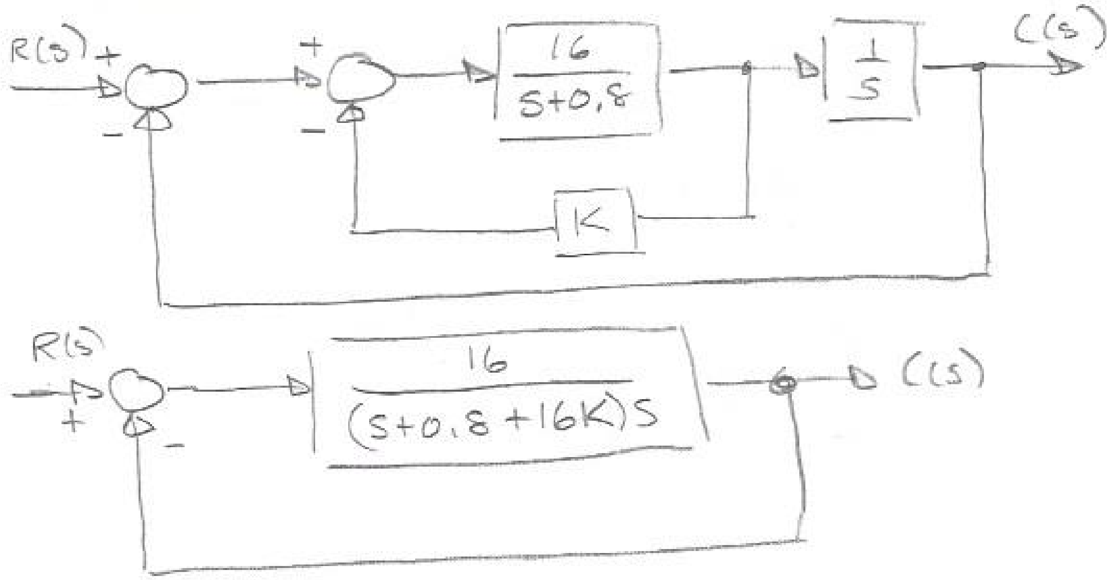
$\left.\begin{array}{l}1.2907+4.4901i\\1.2907-4.4901i\end{array}\right]$ ①

-4.5815 ②



$\Rightarrow$ $\boxed{\text{unstable unit step response}}$

Problem 9 : B-10-11



$$\frac{C(s)}{R(s)} = \frac{16}{s(s+0.8+16K)+16} = \frac{16}{s^2+s(16K+0.8)+16}$$

desired damping ratio $\Rightarrow \zeta_{des} = 0.5$

$$\frac{K}{\frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1} \Rightarrow \frac{C(s)}{R(s)} = \frac{1}{\frac{s^2}{16}+s\left(\frac{16K+0.8}{16}\right)+1}$$

$$\omega_n = \sqrt{16} = 4$$

$$\frac{2\zeta}{\omega_n} = \frac{16K+0.8}{16} \Rightarrow K = \frac{1}{16}\left[\frac{2\zeta}{\omega_n}(16)-0.8\right]$$

$$= \frac{1}{16}\left[\frac{2(0.5)(16)}{4}-0.8\right]$$

$$\boxed{K = 0.2}$$

Problem 9 (cont'd)

From Fig 8-27, $\zeta = 0.5$ $\Big\}$ $\boxed{M_p \approx 17\%}$

$$t_s = \frac{4}{\zeta \omega_n} = \frac{4}{(0.5)(4)} \Rightarrow \boxed{t_s = 2s}$$

From Fig 8-24, $\zeta = 0.5$ $\Big\}$ $t_{chart} = \omega_n t$

$$t_{r,chart} \approx 2.4 \Rightarrow t_r = \frac{t_{r,chart}}{\omega_n} = \frac{2.4}{4} \Rightarrow \boxed{t_r \approx 0.6s}$$

$$t_{p,chart} \approx 3.6 \Rightarrow t_p = \frac{t_{p,chart}}{\omega_n} = \frac{3.6}{4} \Rightarrow \boxed{t_p \approx 0.9s}$$