ME 608 Fluid Dynamics
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Problem 13.1

Questions about the film "Fundamentals of Boundary Layers" (National Committee for Fluid Mechanics Films, http://techtv.mit.edu/collections/ifluids/videos/32603-fundamentals-of-boundary-layers). The questions can be answered by watching the film, with the assistance of the relevant sections in your text book (Chapter 9, Part A).

(a) In the hydrogen-bubble visualizations of flow over a flat plate, which boundary condition is shown to exist at the surface of the plate?
Answer: the hydrogen bubbles visualize the no-slip boundary condition

(b) In the narrow region adjacent to a flat plate, which two forces are said to be of equal importance?
Answer: The shear stress and the inertial forces (i.e., the local Reynolds number , very clos to the wall, is of order one)

(c) How is the boundary layer thickness δ defined in the movie?
Answer: It is defined as the wall-normal location where the velocity has reached 95% of its free stream value.

(d) In a laminar boundary layer flow, how does the wall shear stress change along the plate?
Answer: the shear stress decreases along the plate (as the region of lower momentum diffuses outward from the plate).

(e) In a laminar boundary layer flow, how does the boundary layer thickness grow with downstream location on the plate (given a relation in terms of a non-dimensional parameter?)
Answer: The boundary layer thickness grows as $\delta \sim Re_x^{-1/2}$, as one over the square root of the local Reynolds number

(f) What happens to the *boundary layer thickness* and the *wall shear stress* in a favorable pressure gradient boundary layer (for example when the flow is subjected to a contraction)?
Answer: The boundary layer thickness decreases and the wall shear stress increases.

(g) What happens to the *boundary layer thickness* and the *wall shear stress* in an unfavorable (adverse) pressure gradient boundary layer (for example when the flow encounters a divergent channel)?
Answer: The boundary layer thickness increases and the wall shear stress decreases.

(h) In a divergent channel (a.k.a. diffuser) with a slightly larger angle, what happens when the flow slows down so much that the local velocity gradient becomes zero?
Answer: The flow will separate from the wall, and near the wall there will be local reversal of flow (backflow, recirculating flow).

(i) Compared to laminar boundary layer, will a turbulence boundary layer promote or delay flow separation?
Answer: A turbulent boundary layer will delay flow separation compared to a laminar one

(j) Which has the larger wall shear stress (=drag), the laminar or turbulent boundary layer?
Answer: The turbulent boundary layer has larger shear stress than the laminar boundary layer.

13.2

Thursday, May 5, 2016 7:12 PM

Problem 13.2

For flow around a sphere the boundary layer becomes turbulent around $Re_D \approx 2.5 \times 10^5$. Find the speeds at which

(a) an American golf ball ($D = 1.68$ *inch*),
(b) a British golf ball ($D = 41$ *mm*), and
(c) a soccer ball ($D = 220$ *mm*)

develop turbulent boundary layers. Assume standard atmospheric conditions.

(d) At typical speeds that a golf ball travels at, is the Reynolds number sufficiently high to develop a turbulent boundary layer?
(e) Why would you want to create a turbulent boundary layer on a golf ball?
(f) With your calculated results, and insight into turbulent boundary layers, explain the role of dimples on a golf ball.

$$Re_D = \frac{\rho U D}{\mu} = \frac{UD}{\nu}$$

w/ transition to turbulence @ $Re_D \cong 2.5 \times 10^5$

STP: 0°C, 101.325 kPa: $\rho = 1.29 \ \mathrm{kg/m^3}$, $\mu = 1.72 \times 10^{-5} \ \frac{Ns}{m^2}$

$$U = Re_D \frac{\mu}{\rho D}$$

(a) American golf ball
$D = 1.68'' = 42.67\,mm$
$U = 78.1 \ \frac{m}{s}$

(b) British golf ball
$D = 41\,mm$
$U = 81.3 \ \frac{m}{s}$

(c) Soccer ball
$D = 220\,mm$
$U = 15.2 \ \frac{m}{s}$

(Note: These speeds will be higher if $T = 20°C$ was used, since viscosity of air increases with temperature)

(d) typical golf ball speeds (Google – results from Trackman Pro):

w/ Driver: 165 mph $= 73.75 \ \frac{m}{s}$ $\quad Re_D = 236{,}016$
w/ 8-iron: 115 mph $= 51.40 \ \frac{m}{s}$ $\quad Re_D = 164{,}496$

→ likely too low a Reynolds number to have a turbulent BL

(e) The majority of drag on a bluff body like a sphere is pressure drag, due to a low-pressure region of separated flow behind the sphere. A turbulent BL has more energy and can remain attached further downstream against the adverse pressure gradient on the back of the sphere, separating later and thereby reducing the low-pressure region and the resulting pressure drag.

(f) dimples "trip" the boundary layer, forcing a transition to a turbulent BL. Due to reasons stated under (e), this reduces drag.

Problem 13.3

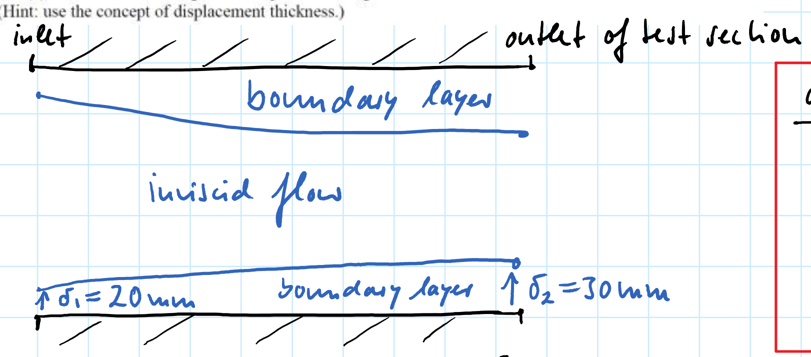
A wind tunnel in university laboratory has a test section of 45 cm square and 90 cm long. With a nominal air speed of $U_1 = 25\ m/s$ at the test section inlet, turbulent boundary layers grow on all four (top, bottom, and side) walls of the tunnel. The boundary-layer thickness is $\delta_1 = 20\ mm$ at the inlet and $\delta_2 = 30\ mm$ at the outlet of the test section. The boundary-layer velocity profiles are of power-law form, with $u/U_\infty = (y/\delta)^{1/7}$.

(a) Find the freestream velocity, U_2, at the exit from the wind-tunnel test section.

(b) Determine the change in static pressure along the test section.

(Hint: use the concept of displacement thickness.)

→ these are the dimensions of the UNH-ME student wind tunnel.



displacement thickness

$$\boxed{\delta^* \equiv \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy \quad \text{w/} \quad \frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}}$$

see lecture for physical meaning of δ^*

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right) dy = \int_0^\delta \left(1 - \frac{1}{\delta^{\frac{1}{7}}} y^{\frac{1}{7}}\right) dy$$

$$= \left[y - \frac{1}{\delta^{\frac{1}{7}}} \frac{7}{8} y^{\frac{8}{7}} \right]_0^\delta = \delta - \frac{7}{8}\delta = \frac{\delta}{8}$$

$$\therefore \quad \delta_1^* = \frac{\delta_1}{8} = \frac{20\ mm}{8} = 2.5\ mm \Rightarrow \text{equivalent flow area } A_1 = (h - 2\delta_1^*)^2 = 0.1980\ m^2$$

$$\delta_2^* = \frac{\delta_2}{8} = \frac{30\ mm}{8} = 3.75\ mm \Rightarrow \text{ " " " " } A_2 = (h - 2\delta_2^*)^2 = 0.1958\ m^2$$

(a) Conservation of mass between (1) and (2): $(\rho U A)_1 = (\rho U A)_2$

or: $$U_2 = U_1 \frac{A_1}{A_2} = 25\ \frac{m}{s}\ \frac{0.1980}{0.1958} = 25.28\ \frac{m}{s}$$

(b) Change in pressure from inviscid flow in center of test section, with Bernoulli equation: $p_1 + \frac{1}{2}\rho U_1^2 + \cancel{\rho g z_1} = p_2 + \frac{1}{2}\rho U_2^2 + \cancel{\rho g z_2}$

$$\Delta p = (p_2 - p_1) = \frac{1}{2}\rho\left(U_1^2 - U_2^2\right) = \frac{1}{2}\ 1.21 \frac{kg}{m^3}\left(25^2 - 25.28^2\right)\frac{m^2}{s^2} =$$

$$= -8.62\ Pa$$

13.4

Thursday, May 5, 2016 7:12 PM

Problem 13.4

You are asked by your college crew to estimate the skin friction drag on their eight-seat racing shell. The ~~hull of~~ the shell may be approximated as half a circular cylinder with 457 mm diameter and 7.32 m length. The speed of the shell through the water is 6.71 m/s.

(a) Estimate the location of transition from laminar to turbulent flow in the boundary layer on the hull of the shell.

(b) Calculate the thickness of the turbulent boundary layer at the rear of the hull. (using boundary layer thickness for turbulent flow, here you can assume that the entire boundary layer is turbulent)

(c) Determine the total skin friction drag force on the hull under the given conditions. (note: you can use the empirical relation by Schlichting)

(d) What average power required from the rowers does this correspond to?

@ 20°C: $\rho = 998\ \mathrm{kg/m^3}$

$\mu = 1.01 \times 10^{-3}\ \frac{Ns}{m^2}$

$\nu = \mu/\rho = 1.01 \times 10^{-6}\ \frac{m^2}{s}$

(a) transition at $Re_{transition} \simeq 5 \times 10^5$ $\therefore$ $x_t = \frac{Re_t\, \nu}{u} = \frac{5 \times 10^5 \cdot 1.01 \times 10^{-6} \frac{m^2}{s}}{6.71 \frac{m}{s}} = 0.0754$

(b) boundary layer thickness:

turbulent BL: $\frac{\delta}{x} = 0.382\ Re_x^{-1/5}$

$\therefore$ $\delta = L \cdot 0.382\ Re_L^{-1/5} = 0.0812\,m$ (or ~ 8 cm)

with $Re_L = \frac{L \cdot u}{\nu} = \frac{7.32 m \cdot 6.71 \frac{m}{s}}{1.01 \times 10^{-6} \frac{m^2}{s}} = 4.85 \times 10^7$

(c) Schlichting: For $Re_L < 10^9$: $c_D = \frac{0.455}{(\log Re)^{2.58}} = 0.002360$

drag force: $\boxed{F_D = c_D A \frac{1}{2} \rho u^2}$ wetted area $A = \frac{\pi \cdot D}{2} \cdot L = 5.255\, m^2$

$\therefore$ $F_D = c_D A \frac{1}{2} \rho u^2 = 0.002360 \cdot 5.255 \cdot \frac{1}{2} 998 \cdot 6.71^2 = 279\ N$

(d) Power $P = F_D \cdot u = 279 N \cdot 6.71 \frac{m}{s} = 1869\ W$ $\left(\frac{Nm}{s} = \frac{J}{s} = W\right)$

13.5

Thursday, May 5, 2016 7:12 PM

Problem 13.5

An airplane with an effective lift area of 25 m^2 is fitted with airfoils of NACA 23012 section (Fig. 9.23). The maximum flap setting that can be used at takeoff corresponds to configuration 2 in Fig. 9.23. Determine the maximum gross mass possible for the airplane if its takeoff speed is 150 km/hr at sea level (neglect added lift due to ground effect). Find the minimum takeoff speed required for this gross mass if the airplane is instead taking off from Denver (elevation approximately 1.6 km).

$= C_{L,max}$

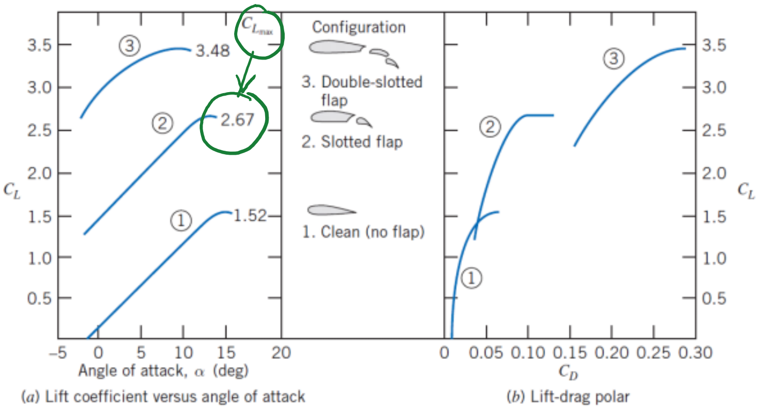


Fig. 9.23 Effect of flaps on aerodynamic characteristics of NACA 23012 airfoil section. (Data from Abbott and von Doenhoff [21].)

$\rho_{air} = 1.23 \ kg/m^3$

definition of lift coefficient $C_L \equiv \dfrac{F_L}{A_{planform} \frac{1}{2} \rho V^2}$

at take-off: $\boxed{F_L = mg}$

For maximum mass, use $C_{L,max}$ of configuration 2, $C_{L,max} = 2.67$.

$$\therefore m_{max} = \frac{F_L}{g} = \frac{1}{g} C_{L,max} A \tfrac{1}{2} \rho V^2 = \frac{1}{9.81} 2.67 \cdot 25 \cdot \tfrac{1}{2} 1.23 \cdot \left(\frac{150}{3.6}\right)^2 = 7260 \ kg$$

(Usually there is a margin of safety w.r.t. stall angle and/or mass)

Denver, $z = 1.6$ km Table A.3: $\dfrac{\rho}{\rho_0} = 0.855$

For the same mass, the lift force remains the same.

$$F_{L,0} = C_L A \tfrac{1}{2} \rho_0 V_0^2 = F_{L,Denver} = C_L A \tfrac{1}{2} \rho_{Denver} V_{Denver}^2$$

$$\therefore V_{Denver} = V_0 \left(\frac{\rho_0}{\rho_{Denver}}\right)^{\frac{1}{2}} = 150 \frac{km}{h} \left(\frac{1}{0.855}\right)^{\frac{1}{2}} = 162 \frac{km}{h}$$

→ take-off velocity must be increased by ~ 8%