12.1

Monday, April 30, 2018 9:14 PM

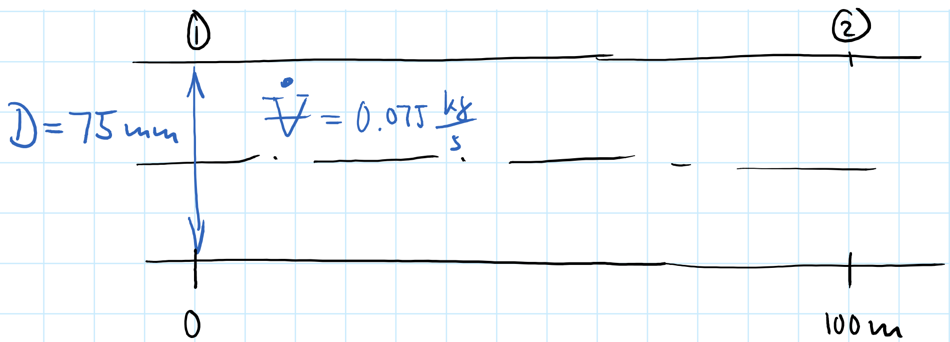
Problem 12.1

A smooth, 75-mm-diameter pipe carries water (65 deg C) horizontally. When the mass flow rate is 0.075 kg/s, the pressure drop is measured to be 7.5 Pa per 100 m of pipe.

(a) Based on these measurements, what is the friction factor? (from Bernoulli and definition of friction factor)?

(b) What is the Reynolds number? Does this Reynolds number generally indicate laminar or turbulent flow?

(c) Is the flow actually laminar or turbulent? Check with the Colebrook equation and Moody chart.



$$\frac{\Delta p}{L} = \frac{7.5\ Pa}{100\ m}$$

Bernoulli Equ w/ head loss: $\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{\Delta p}{\rho g} = H_{L,f}$

w/ $\boxed{H_{L,f} \equiv f \frac{L}{D} \frac{V_{avg}^2}{2g}}$

∴ need V_{avg}: $V_{avg} = \frac{\dot{V}}{A_{c.s.}} = \frac{\dot{m}/\rho}{\frac{\pi}{4} D^2} = \frac{4\dot{m}}{\pi \rho D^2}$

∴ $f = \frac{D}{L} \frac{2g}{V_{avg}^2} H_{L,f} = \frac{D}{L} \frac{2\cancel{g}}{V_{avg}^2} \frac{\Delta p}{\rho \cancel{g}} = \frac{D}{L} \frac{2 \pi^2 \rho^{\cancel{2}} D^4}{\dot{m}^2 4^2} \cdot \frac{\Delta p}{\cancel{\rho}}$

$$= \frac{D^5}{L} \frac{\pi^2 \rho \Delta p}{\dot{m}^2 \cdot 8}$$

$$= \frac{(0.075)^2}{100} \frac{\pi^2\ 980 \cdot 7.5}{(0.075)^2 \cdot 8}$$

$$= 0.03825$$

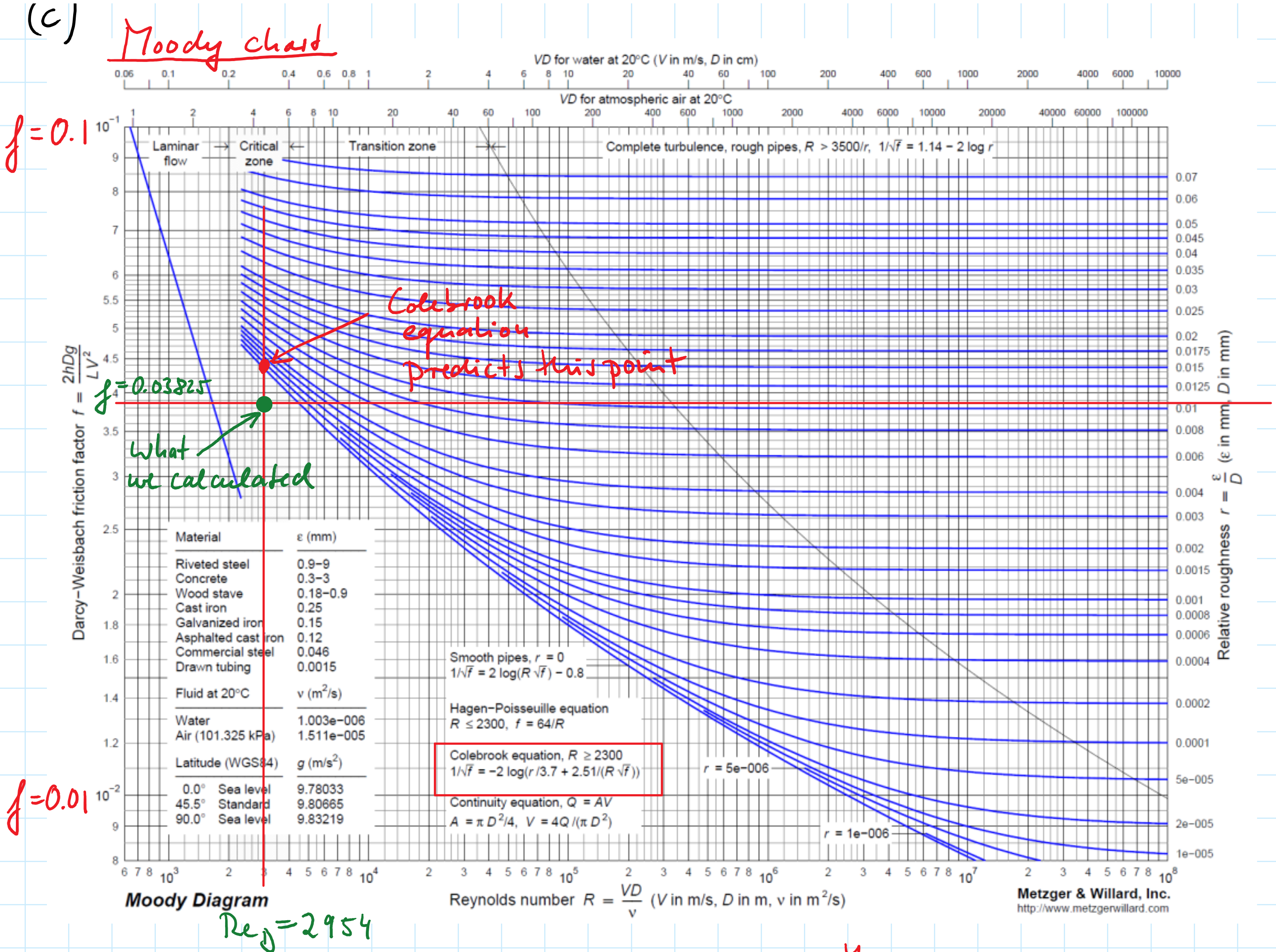
units check: $\frac{\cancel{m^5}}{\cancel{m}} \frac{\cancel{kg}}{\cancel{m^3}} \frac{\cancel{kg}\,\cancel{m}}{\cancel{s^2}} \frac{1}{\cancel{m^2}} \Big/ \frac{\cancel{kg^2}}{\cancel{s^2}}$ all cancel ✓

(b) $Re_D = \frac{\rho V_{avg} D}{\mu} = \frac{\cancel{\rho}\ \dot{m}\ \cancel{D}\ 4}{\mu\ \cancel{\rho}\ D^{\cancel{2}} \pi} = \frac{4\dot{m}}{\pi \mu D} = \frac{4 \cdot 0.075 \frac{kg}{s}}{\pi\ 4.31 \times 10^{-4} \frac{Ns}{m^2} \cdot 0.075\ m}$

$$= 2{,}954$$

⇒ The Reynolds number is above the critical Re# of 2,300, therefore it should be turbulent flow.

(c)



<u>Colebrook eqn:</u> $$f = \left\{ -2.0 \log\left(\frac{\overset{=0,\ \text{smooth}}{\cancel{e/D}}}{3.7} + \frac{2.51}{Re_D f^{1/2}} \right) \right\}^{-2}$$

$$\therefore\ f = 0.04372$$

- It seems that the flow is still in a transitional state. For a friction factor of 0.03825, a Reynolds # of ~4500 would be required for the point to fall on the line for turbulent flow in a smooth pipe.
- One could also argue that it is turbulent flow to within measurement accuracy.

12.2

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Problem 12.2

A water tank (open to the atmosphere) contains water to a depth of 5 m. A 25-mm-diameter hole is punched in the bottom.

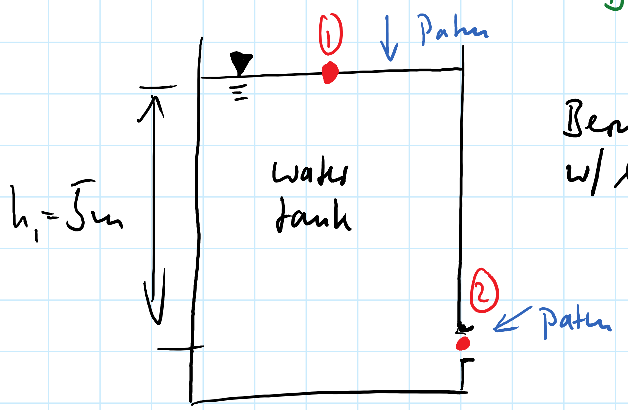
(a) Modeling the hole as square-edged, estimate the flow rate (liters/s) exiting the tank.

(b) If you were to stick a short section of pipe into the hole, by how much would the flow rate change?

(c) If instead you were to machine the inside of the hole to give it a rounded edge (r=5 mm), by how much would the flow rate change?

minor losses

$$\left.\begin{array}{l} K_L = 0.5 \\ K_L = 0.78 \\ \frac{r}{D} = \frac{5}{25} = 0.2 \Rightarrow K_L = 0.04 \end{array}\right\} \text{Table 8.2}$$



Bernoulli w/ losses: $\cancel{\frac{V_1^2}{2g}} + \cancel{\frac{P_1}{\rho g}} + h_1 - \left(\frac{V_2^2}{2g} + \cancel{\frac{P_2}{\rho g}} + \cancel{h_2}\right) = H_L$

(if we neglected losses: $H_L = 0$

$\therefore V_2 = \sqrt{2gh_1}$
$= 4.862 \frac{m}{s}$)

⇒ We are investigating the effects of three different types of outflow losses (= minor losses)

$$h_1 - \frac{V_2^2}{2g} = H_{L,m} = K_L \frac{V_{avg}^2}{2g} = K_L \frac{V_2^2}{2g} \quad \therefore \quad \boxed{V_2 = \sqrt{\frac{2gh_1}{1+K_L}}}$$

(a) square-edged hole: $\boxed{K_L = 0.5}$ (Table 8.2)

$$V_2 = \sqrt{\frac{2gh_1}{1+0.5}} = \left(\frac{4}{3} 9.81 \cdot 5\right)^{\frac{1}{2}} = 8.09 \frac{m}{s}$$

$$\dot{V} = V_2 \cdot A = V_2 \frac{\pi}{4} D^2 = 8.09 \frac{\pi}{4} 0.025^2 = 3.970 \frac{liters}{s}$$

(b) short pipe: $\boxed{K_L = 0.78}$

$$V_2 = \left(\frac{2gh_1}{1+0.78}\right)^{\frac{1}{2}} = 7.42 \frac{m}{s} \; ; \quad \dot{V} = V_2 \cdot A = 3.644 \frac{liters}{s}$$

(c) rounded edges: $r = 5mm \Rightarrow \frac{r}{D} = \frac{5}{25} = 0.2 \Rightarrow \boxed{K_L = 0.04}$

$$V_2 = \left(\frac{2gh_1}{1+0.04}\right)^{\frac{1}{2}} = 9.71 \frac{m}{s} \; ; \quad \dot{V} = V_2 A = 4.767 \frac{liters}{s}$$

Problem 12.3

You recently bought a house and want to improve the flow rate of water on your top floor. The poor flow rate is due to three reasons: The city water pressure at the water meter is poor ($p = 200kPa$ gage); the piping has a small diameter ($D = 1.27cm$) and has been crudded up, increasing its roughness ($\frac{e}{D} = 0.05$); and the top floor of the house is 15 m higher than the water meter. You are considering two options to improve the flow rate: Option 1 is replacing all the piping after the water meter with new smooth piping with a diameter of 1.9 cm; and option 2 is installing a booster pump while keeping the original pipes. The booster pump has an outlet pressure of 300 kPa. Which option would be more effective? Neglect minor losses.

$\longrightarrow \frac{e}{D} = 0$

existing piping

$\rho = 998 \frac{kg}{m^3}$

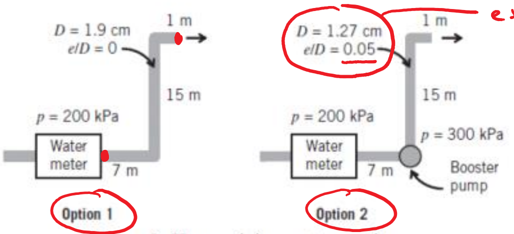
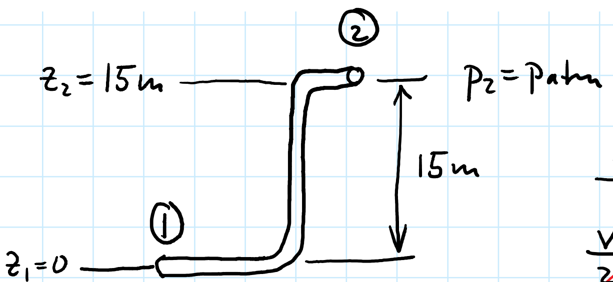


Figure 2: House piping system options.



P_1 (gage) $= 200kPa$

($P_{1,total} = P_1 + P_{atm}$)

total length $L = 7 + 15 + 1 = 23m$

Energy (Bernoulli) equation:

$$\cancel{\frac{V_1^2}{2g}} + \cancel{\frac{P_1}{\rho g}} + \cancel{z_1} - \left(\cancel{\frac{V_2^2}{2g}} + \cancel{\frac{P_2}{\rho g}} + z_2\right) = H_L - H_{pump}$$

(constant D pipe: $V_1 = V_2$)

$\Rightarrow$ only considering friction losses

$$H_L = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

- As-is condition: $D = 12.7\ mm$, $\frac{e}{D} = 0.05$

$$\frac{P_1}{\rho g} - z_2 = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

$$\boxed{V_{avg} = \left[\left(\frac{P_1}{\rho g} - z_2\right) \frac{D}{L} \frac{2g}{f}\right]^{\frac{1}{2}}} \quad (*)$$

$f(Re_D)$ is unknown, iterative solution:

Note: $[Pa] = \left[\frac{N}{m^2}\right] = \left[\frac{kg\,m}{s^2} \frac{1}{m^2}\right]$

Step 1: pick a value for f, e.g.: $f_1 = 0.07$

Step 2: calculate $V_{avg} = \left[\left(\frac{200\,000\,Pa}{998 \frac{kg}{m^3} 9.81 \frac{m}{s^2}} - 15m\right) \frac{0.0127m}{23\ m} \frac{2 \cdot 9.81 \frac{m}{s^2}}{0.07}\right]^{\frac{1}{2}} = 0.917 \frac{m}{s}$

Step 3: calculate $Re_D = \frac{\rho V_{avg} D}{\mu} = \frac{998 \cdot 0.894 \cdot 0.0127}{1.01 \times 10^{-3}} = 11{,}502$

Step 4: calculate new f_2 from Colebrook equ.

Step 4: calculate new f_2 from Colebrook equ.

$$\boxed{\frac{1}{f_2^{1/2}} = -2.0 \log\left[\frac{e/D}{3.7} + \frac{2.51}{Re_D\, f_1^{1/2}}\right]}$$

or: $f_2 = \left\{-2.0 \log\left[\frac{e/D}{3.7} + \frac{2.51}{Re_D\, f_1^{1/2}}\right]\right\}^{-2}$

now set $f_1 = f_2$, and repeat calculations until f converges

Result: $f = 0.073562$, $V_{avg} = 0.894\ \frac{m}{s}$, $Re_D = 11{,}220$

$$\dot{V} = V_{avg} \cdot A = V_{avg} \frac{\pi}{4} D^2 = 0.894 \frac{\pi}{4} 0.0127^2 = 0.113\ \frac{l}{s} = 1.80\ gpm$$

- Option 1: new pipes, larger $D = 0.019\ m$, $\frac{e}{D} = 0$

same equation for V_{avg} (*), same iteration for f

Result: $f = 0.022329$, $V_{avg} = 1.985\ \frac{m}{s}$, $Re_D = 37{,}267$

$$\dot{V} = V_{avg} \cdot A = 1.985 \frac{\pi}{4} 0.019^2 = 0.563\ \frac{l}{s} = 8.93\ gpm$$

- Option 2: old pipes ("as-is"), add booster pump (+300 kPa)

$$H_{pump} = -\frac{\Delta p}{\rho g}$$ (pump head is added as negative head loss)

$$\frac{p_1}{\rho g} - z_2 = f \frac{L}{D} \frac{V_{avg}^2}{2g} - \frac{\Delta p}{\rho g} \qquad (w/\ \Delta p = 300\ kPa)$$

$$\boxed{V_{avg} = \left[\left(\frac{p_1}{\rho g} - z_2 + \frac{\Delta p}{\rho g}\right)\frac{D}{L}\frac{2g}{f}\right]^{1/2}} \qquad (**)$$

using this equation for V_{avg} (**), and iterating for f

Result: $f = 0.072335$, $V_{avg} = 2.324\ \frac{m}{s}$, $Re_D = 29{,}168$

$$\dot{V} = V_{avg} \cdot A = 2.324 \frac{\pi}{4} 0.0127^2 = 0.294\ \frac{l}{s} = 4.67\ gpm$$

- **Option 2***: old pipes ("as-is"), add booster pump

If the problem is understood as 300 kPa being available for the last 16 m of the pipe, then the analysis is the same as for Option 1, with p_1 (gage) = 300 kPa and L = 16 m

$$\frac{p_1}{\rho g} - z_2 = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

$$\boxed{V_{avg} = \left[\left(\frac{p_1}{\rho g} - z_2 \right) \frac{D}{L} \frac{2g}{f} \right]^{\frac{1}{2}}} \quad (*)$$

using this equation for V_{avg} (*), and iterating for f as before

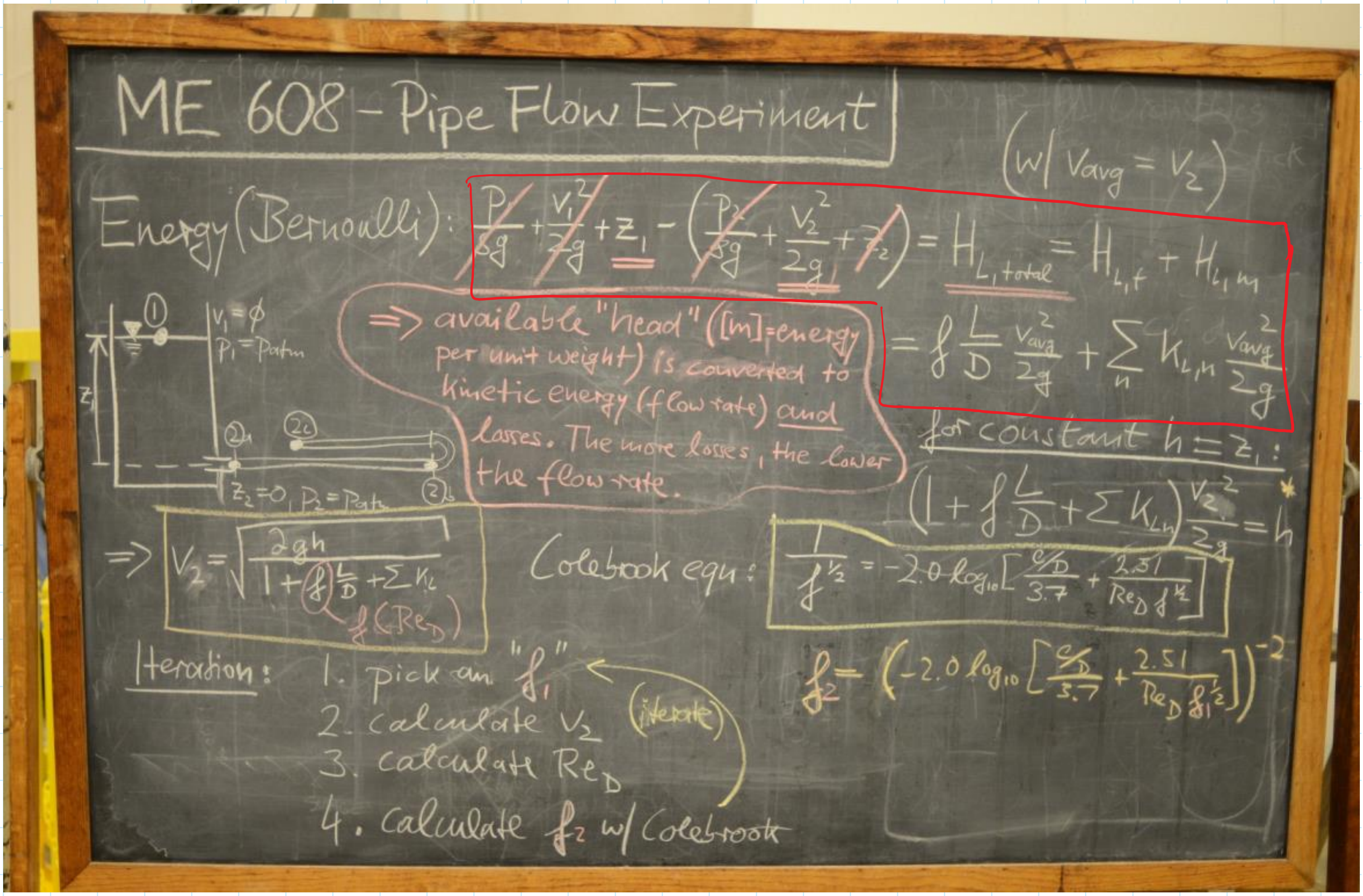
Result: $f = 0.072543$, $V_{avg} = 1.833 \ \frac{m}{s}$, $Re_D = 22{,}996$

$$\dot{V} = V_{avg} \cdot A = 1.833 \ \frac{\pi}{4} 0.0127^2 = 0.232 \ \frac{\ell}{s} = \underline{3.68 \text{ gpm}}$$

Conclusion: Get new pipes with larger D! (~ 2x flow rate)

12.4

Monday, April 30, 2018 2:28 AM



Results:

ME 608 Pipe Flow Experiment 5/6/2016

M. Wosnik Chase Lab

Water properties during experiment

Temperature	Density	Dyn.Viscosity	
[deg C]	[kg/m^3]	[Ns/m^2]	[deg C]
14	999	1.18E-03	

gravity:	9.81	[m/s^2]
pipe ID:	0.602	inch
pipe ID: D=	0.01529	m

Note: your measurements ranged from 15.07 to 15.79 mm
the published value for "average I.D. fo 1/2" pipe is 0.602"

Configuration 1: short pipe in reservoir

"head" to drive the flow (h=z_1)

	run 1	run 2	run 3	
head h:	0.5			[m]
mass:	7			[kg]
time a:	15.5			[s]
time b:				[s]
time c:				[s]
time d				[s]
time avg =	15.5			[s]
m_dot =	0.452			[kg/s]

experimentally measured flowrate

inviscid flow = "no losses" (just Bernoulli eqn.)

v_2 [m/s] =	3.132	[m/s]	
vol. flowrate =	0.000575	[m^3/s]	
	0.575	[liters/s]	
	9.129	[gpm]	
mass. flowrate =	0.575	[kg/s]	
difference =	27%	Bernoulli eqn w/out losses overpredicts flowrate	

calculation including minor losses, short pipe in reservoir

K_L :	0.78	<-- K_L for a short pipe entrance condition	
v_2 =	2.348	[m/s]	(the flange ID widened)
vol. flowrate =	0.000431	[m^3/s]	
	0.431	[liters/s]	
	6.843	[gpm]	
mass. flowrate =	0.431	[kg/s]	
difference =	-4.6%		

Configuration 2, minor and major losses: pipe attached

"head" to drive the flow (h=z_1)

	run 1	run 2	run 3	
head h:	0.5			[m]
mass:	7.3			[kg]
time a:	33.50			[s]
time b:				[s]
time c:				[s]
time avg =	33.5			[s]
m_dot =	0.218			[kg/s]

experimentally measured flowrate

inviscid flow = "no losses" (just Bernoulli eqn.)

mass. flowrate =	0.575	[kg/s]
difference =	164%	Bernoulli eqn w/out losses overpredicts flowrate

calculation including minor and major losses

pipe length:	3.048	m	
roughness:	0.0015	mm	<-- e for "drawn tubing"
e/D =	9.810E-05		not sure of actual e, could be higher

Step 1: pick f	0.028034	**iterate!** (note: iteration can be automated)
Step 2: v_avg=	1.154	
Step 3: Re_D=	14,907	check Re#: turbulent flow!
Step 4: calculate f	0.028034	

vol. flowrate =	0.000212	[m^3/s]
	0.212	[liters/s]
	3.363	[gpm]
mass. flowrate =	0.212	[kg/s]
difference =	-2.8%	

Problem 12.5

A large mass is supported by a piston of diameter $D = 10cm$ and length $L = 10cm$. The piston sits in a cylinder closed at the bottom, and the gap of $a = 0.02mm$ between the cylinder wall and piston is filled with SAE 10 oil at 20°C. The piston slowly sinks due to the mass, and oil is forced out at a rate of 0.4 liters/min. Considering $D \gg a$, the gap between the piston and the wall can be modeled as flow between infinite parallel plates.

(a) Is the flow laminar or turbulent?

(b) Derive the velocity profile in the gap from the Navier Stokes equations. (hint: compare fluid velocity to piston velocity to make some reasonable assumptions)

(c) Derive a relation for volumetric flow rate in the gap.

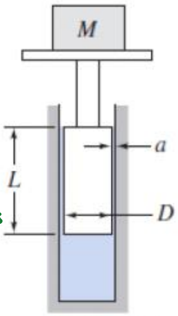
(d) For the given oil flow rate, what is the mass of the piston? (in [kg])

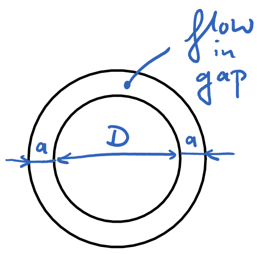
SAE 10W oil @ 20°C:

$\mu = 0.1 \frac{Ns}{m^2}$

$S.G. = 0.92$

$\therefore \rho = 0.92 \cdot \rho_{H_2O} = 0.92 \cdot 998 \frac{kg}{m^3}$





$D + 2a = 100mm + 2 \cdot 0.02mm = 100.04mm$

Figure 3: Sketch of piston with mass m, forcing oil flow through circumferential gap a.

(a) Is the flow laminar or turbulent? Check Re # based on gap width a

$$Re_a = \frac{\rho\, a\, V_{avg}}{\mu} \qquad \text{with: } V_{avg} = \frac{\dot{V}}{A} = \frac{\dot{V}}{\frac{\pi}{4}\left[(D+2a)^2 - D^2\right]} =$$

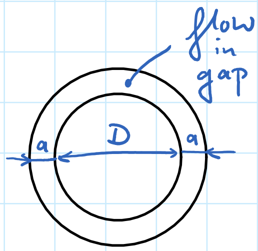
$$= \frac{0.4 \frac{l}{min} \frac{1\ min}{60\ s} \cdot \frac{1\ m^3}{1000\ l}}{\frac{\pi}{4}\left(100.04^2 - 100^2\right) mm^2\ \frac{1\ m^2}{10^6\ mm^2}}$$

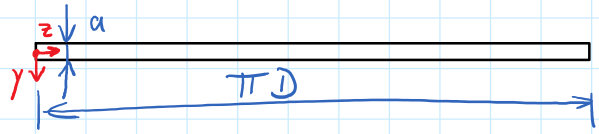
$$= 1.061 \frac{m}{s}$$

$$Re_a = \frac{\rho\, a\, V_{avg}}{\mu} = \frac{0.92 \cdot 998 \cdot 0.02 \cdot \frac{1}{1000}\ 1.061}{0.1} = 0.195$$

laminar flow!

(b)





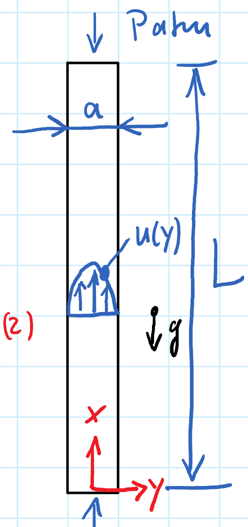
Note: $\frac{a}{D/2} = \frac{0.02}{100/2} = \frac{4}{10\,000}$ ⟹ negligible curvature, can be modeled as flow in

very long slender gap
(= between "infinite plates")

Piston velocity: $\underline{\underline{v_{piston}}} = \dfrac{\dot{V}}{A_{piston}} = \dfrac{0.4 \frac{l}{min} \cdot \frac{1\,min}{60\,s} \cdot \frac{1\,m^3}{1000\,l}}{\frac{\pi}{4}(0.1\,m)^2} = \underline{\underline{0.000849 \frac{m}{s}}}$

$\therefore \dfrac{V_{avg}}{v_{piston}} = 1250 \Rightarrow$ The piston moves so slowly that it can be assumed to be stationary!

velocity profile in gap:



$p = \dfrac{m \cdot g}{A_{piston}} + p_{atm}$

$\Delta p = \dfrac{m \cdot g}{A_{piston}}$

$L \gg a$: fully developed flow (1)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = 0$$

incompressible (oil): $\nabla \cdot \vec{V} = 0$ (2)

steady state $\dfrac{\partial}{\partial t} = 0$ (3)

between "infinite plates", 2-D

$\dfrac{\partial}{\partial z} = 0, \; w = 0$ (4)

continuity: $\nabla \cdot \vec{V} = \cancel{\dfrac{\partial u}{\partial x}}^{(1)} + \dfrac{\partial v}{\partial y} + \cancel{\dfrac{\partial w}{\partial z}}^{(4)} = 0 \Leftrightarrow \dfrac{\partial v}{\partial y} = 0$ or: $v = const.$

w/ $v(y = \frac{a}{2}) = 0 \therefore \boxed{v = 0}$ (5)

x-momentum (Navier-Stokes) for incompressible flow, $\mu = const$:

$$\rho\left(\cancel{\frac{\partial u}{\partial t}}^{(3)} + u\cancel{\frac{\partial u}{\partial x}}^{(1)} + \cancel{v}^{(5)}\frac{\partial u}{\partial y} + \cancel{w}^{(4)}\cancel{\frac{\partial u}{\partial z}}^{(4)}\right) = -\frac{\partial p}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{(1)} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{(4)}\right)$$

$$\left(\frac{\partial p}{\partial x}\right) = \mu \frac{\partial^2 u}{\partial y^2} = const.$$

$$\frac{du}{dy} = \frac{const}{\mu} y + C_1$$

$$u(y) = \frac{const}{2\mu} y^2 + C_1 y + C_2$$

boundary conditions:

$$u\left(\tfrac{a}{2}\right) = 0 = \frac{const.}{2\mu}\frac{a^2}{4} + C_1\frac{a}{2} + C_2$$

$$u\left(-\tfrac{a}{2}\right) = 0 = \frac{const.}{2\mu}\frac{a^2}{4} - C_1\frac{a}{2} + C_2$$

$$u(-\tfrac{a}{2}) = 0 = \frac{\text{const.}}{2\mu}\frac{a^2}{4} - c_1\frac{a}{2} + c_2$$

$$\therefore\ c_1 = 0\ ,\quad c_2 = -\frac{\text{const}}{2\mu}\frac{a^2}{4}$$

$$\boxed{u(y) = -\frac{\text{const.}}{2\mu}\left(\frac{a^2}{4} - y^2\right) = -\frac{dp/dx\, a^2}{8\mu}\left(1 - \left(\frac{2y}{a}\right)^2\right)}$$

(c) volumetric flow rate in gap: integrate velocity profile over cross-section

$$\dot{V} = \iint_A u\,dA = \pi D \int_{-a/2}^{a/2} u(y)\,dy$$

$$= \pi D \int_{-a/2}^{a/2} -\frac{dp/dx}{2\mu}\left(\frac{a^2}{4} - y^2\right)dy$$

$$= -\pi D\,\frac{dp/dx}{2\mu}\left[\frac{a^2}{4}y - \frac{1}{3}y^3\right]_{-a/2}^{a/2}$$

$$= -\pi D\,\frac{dp/dx}{2\mu}\left[\frac{a^3}{8} - \frac{1}{3}\frac{a^3}{8} - \left(-\frac{a^3}{8} + \frac{1}{3}\frac{a^3}{8}\right)\right]$$

$$= -\pi D\,\frac{dp/dx}{2\mu}\frac{1}{8}\left[2a^3 - \frac{2}{3}a^3\right]$$

$$= -\pi D\,\frac{dp/dx}{2\mu}\frac{1}{8}\frac{4}{3}a^3 = -\frac{\pi D}{12\mu}\frac{dp}{dx}a^3$$

with: $-\frac{dp}{dx} = \frac{\Delta p}{L}$ $\quad\therefore\quad$ $\dot{V} = \frac{\pi D}{12\mu}\frac{\Delta p}{L}a^3$ $\quad$ or: $\Delta p = \frac{12\mu\dot{V}L}{\pi D\, a^3}$

(d) mass of the piston: $\quad \Delta p = \frac{m\cdot g}{A_{piston}}$

or: $$m = \frac{A_{piston}}{g}\Delta p = \frac{\cancel{\frac{\pi}{4}}D^{\cancel{2}}}{g}\frac{\cancel{12}^{3}\mu\dot{V}L}{\cancel{\pi D}\,a^3} = \frac{3D\mu\dot{V}L}{g\,a^3}$$

$$= \frac{3\cdot 0.1\,\text{m}\cdot 0.1\,\frac{\text{Ns}}{\text{m}^2}\cdot 0.4\,\frac{\ell}{\text{min}}\cdot\frac{1\,\text{min}}{60\,\text{s}}\,\frac{1\,\text{m}^3}{1000\,\ell}\cdot 0.1\,\text{m}}{9.81\,\frac{\text{m}}{\text{s}^2}\cdot(0.02\times10^{-3}\,\text{m})^3}$$

$$= 254{,}842\ \text{kg} \simeq 255\ \text{tons}$$ (a large mass, indeed)

$$\left(\Delta p = 318{,}309{,}886\ \text{N/m}^2 = 3{,}183\ \text{bar}\right)$$