3.1

Friday, February 16, 2018 10:00 AM

Problem 3.1

A U-tube manometer is used to measure the pressure difference in air (as shown in Figure 1). The height differences observed are $h_1 = 30cm$, $h_2 = 20cm$, $h_3 = 10cm$.

(a) What is the pressure difference $(p_1 - p_2)$ if the manometer liquid ρ_B is water, and the temperature is 20°C? What is $(p_1 - p_2)$ if the temperature is 50°C?

(b) What is the pressure difference $(p_1 - p_2)$ if the manometer liquid ρ_B is mercury, and the temperature is 20°C?

(c) Now the working fluid (ρ_A) is water, and the manometer fluid ρ_B is Bromoethylbenzene (Meriam blue). What is the pressure difference $(p_1 - p_2)$ if the temperature is 20°C?

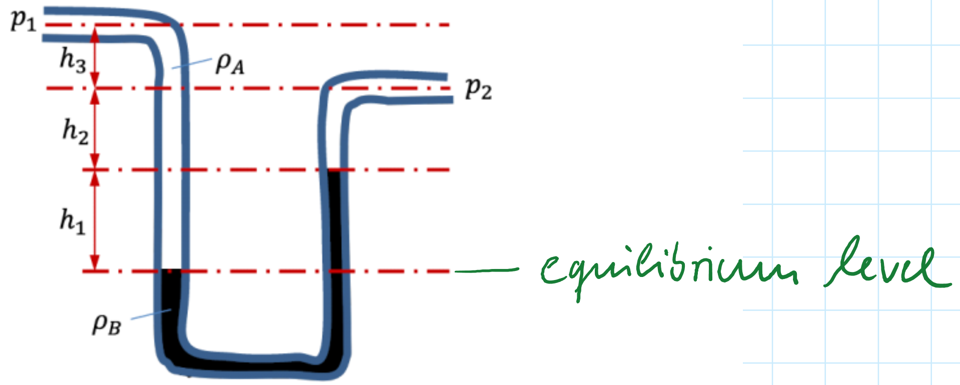


Figure 1: Sketch of a U-tube manometer, where h denotes the liquid height difference between different level in the two legs of the manometer.

Before we consider specific fluids, let's derive the general equation for this type of manometer:

Add up pressures and fluid weights for each leg going down to the "equilibrium level":

$$\underbrace{p_1 + \rho_A g h_3 + \cancel{\rho_A g h_2} + \rho_A g h_1}_{\text{left leg}} = \underbrace{\rho_B g h_1 + \cancel{\rho_A g h_2} + p_2}_{\text{right leg}}$$

$$\boxed{\Delta p = p_1 - p_2 = (\rho_B - \rho_A) g h_1 - \rho_A g h_3}$$

(a) $\rho_A = \rho_{air} = \begin{cases} 1.21 \ \frac{kg}{m^3} \ @ \ 20°C \\ 1.09 \ '' \ @ \ 50°C \end{cases}$ Table A.10 Fox & McDonald 8e

$\rho_B = \rho_{H_2O} = \begin{cases} 998 \ \frac{kg}{m^3} \ @ \ 20°C \\ 988 \ '' \ @ \ 50°C \end{cases}$ Table A.8 Fox & McDonald 8e

@ 20°C: $p_1 - p_2 = (998 - 1.21) \frac{kg}{m^3} \, 9.81 \frac{m}{s^2} \cdot 0.30 \, m - 1.21 \frac{kg}{m^3} \, 9.81 \frac{m}{s^2} \cdot 0.10 \, m \quad [= \frac{N}{m^2}]$

$= 2937 - 3.56 \qquad - 1.19 \qquad Pa$

$= \underline{\underline{2932 \text{ Pa}}}$

@ 50°: $p_1 - p_2 = (988 - 1.09)\, 9.81 \cdot 0.30 \quad - 1.09 \cdot 9.81 \cdot 0.10$
$= 2908 - 3.21 \qquad - 1.07$
$= \underline{\underline{2903 \text{ Pa}}}$

Change from $\Delta p(20°C)$ to $\Delta p(50°C)$:

$$\frac{\Delta p(20°C) - \Delta p(50°C)}{\Delta p(20°C)} = \frac{2932 - 2903}{2932} = 0.0099$$
$$= \underline{\underline{1.0\%}}$$

Discussion:
- Of the three terms in the equation, the term with water density dominates (to be quantified in next HW problem)
- The temperature increase from 20° to 50°C reduces the pressure difference by ~1%.

(b) Mercury Hg: $S.G._{Hg}(20°C) = 13.55$ (Table A.1)

$\therefore \rho_{Hg} = S.G. \, \rho_{H_2O} = 13.55 \cdot 998 \frac{kg}{m^3} = 13{,}523 \frac{kg}{m^3}$

$\rho_B = \rho_{Hg} = 13523 \frac{kg}{m^3}$ @ 20°C

$\underline{\underline{p_1 - p_2}} = (13523 - 1.21)\, 9.81 \cdot 0.30 - 1.21 \cdot 9.81 \cdot 0.100$
$= 39{,}798 - 3.56 \qquad - 1.19$
$= \underline{\underline{39{,}793 \text{ Pa}}}$

(c) $\rho_A = \rho_{H_2O} = 998$ @ 20°C

Table A.1 in text book

$S.G._B = S.G._{\text{Merian Blue}} = \underline{\underline{1.75}} = \frac{\rho_{\text{Merian Blue}}}{\rho_{H_2O}}$ $\therefore \underline{\underline{\rho_{M.B.} = S.G. \, \rho_{H_2O}}}$

$\underline{\underline{p_1 - p_2}} = (\rho_{M.B.} - \rho_{H_2O})\, g h_1 - \rho_{H_2O}\, g h_3$

$$= (S.G. - 1)\rho_{H_2O}\, g h_1 - \rho_{H_2O}\, g h_3$$

$$= \rho_{H_2O}\, g \left[(S.G. - 1) h_1 - h_3\right]$$

$$= 998 \cdot 9.81 \left[(1.75 - 1) \cdot 0.30 - 0.10\right]$$

$$= 5{,}140 - 2{,}937 - 979$$

$$= 1{,}224 \text{ Pa}$$

Problem 3.2
Repeat the previous problem (U-tube manometer), but now use the "simple" manometer equation:

$$\Delta P = p_1 - p_2 \approx g h_1 \rho_{manometer\ liquid}$$

Comment on the differences (errors) observed for (a), (b) and (c).

Instead of using:

$$\Delta p = p_1 - p_2 = (\rho_B - \rho_A) g h_1 - \rho_A g h_3 \quad \text{(c.f. HW 2.3)}$$

we are now using:

$$\Delta p = p_1 - p_2 = \rho_B g h_1$$

Answers 3.1

(a) @ 20°C: $p_1 - p_2 = \rho_{H_2O} g h_1 = 998 \cdot 9.81 \cdot 0.30 = 2{,}937\ Pa$ 2932 Pa (+0.16%)

@ 50°C: $p_1 - p_2 = \rho_{H_2O} g h_1 = 988 \cdot 9.81 \cdot 0.30 = 2{,}908\ Pa$ 2903 Pa (+0.15%)

(b) @ 20°C: $p_1 - p_2 = \rho_{Hg} g h_1 = 13523 \cdot 9.81 \cdot 0.30 = 39{,}798\ Pa$ 39,793 Pa (+0.012%)

(c) @ 20°C: $p_1 - p_2 = S.G._{M.B.} \rho_{H_2O} g h_1 = 1.75 \cdot 998 \cdot 9.81 \cdot 0.30 = 5{,}140\ Pa$ 1,224 Pa (+320%)

Note: The error becomes larger, the closer the density of the manometer is to the density of the working fluid.

The density of the working fluid should only be neglected if there are at least two orders of magnitude difference in densities!

For (c), a very large error (320%!) is incurred if the simple manometer equation is used!

3.3

Friday, February 16, 2018 10:00 AM

Problem 3.3

The inclined manometer shown in Figure 2 contains water (shaded portion of tubes) at 20°C. The inclination angle $\theta = 15°$. The diameter of the reservoir (pressure p_1) is D = 10cm and the diameter of the tube (pressure p_2) is d = 1cm. The reservoir is open to atmosphere such that $p_1 = p_{atm}$.

(a) For $p_2\ (gage) = -300\ Pa$: Find h and ΔL, where ΔL is the change in L from its position when $p_2 = p_1$.

(b) For $p_2\ (gage) = 300\ Pa$: Find h and ΔL, where ΔL is the change in L from its position when $p_2 = p_1$.

(c) For $p_2\ (gage) = -500\ Pa$: Find h, L, and the change in water level of the reservoir h_1 and the tube h_2. (See Example 3.2).

(d) Did you need to know the diameter of the reservoir and tube to answer (a) and (b)? Explain your answer.

Careful: ΔL vs. L !

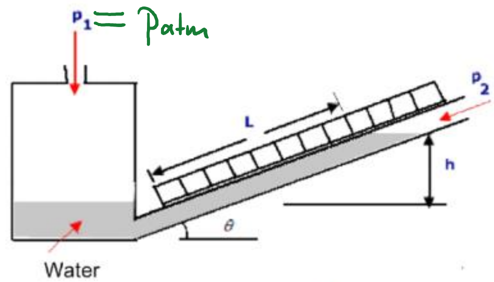
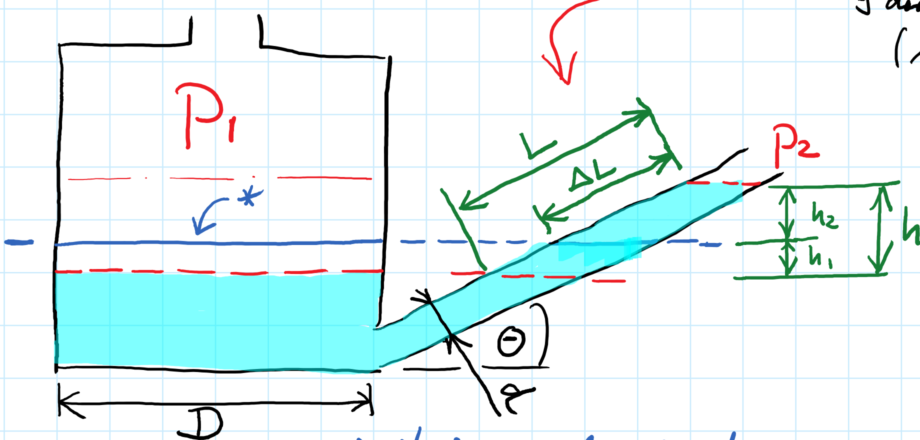


Figure 2: Sketch of an inclined manometer, where h denotes the liquid height difference between the two legs of the manometer, L denotes the fluid length over the measurement scale, and θ denotes the angle of the manometer tube with and the horizontal.

$\rho_{H_2O}(20°C) = 998\ \frac{kg}{m^3}$

$\rho_{air}(20°C) = 1.21\ \frac{kg}{m^3}$

(Appendix A)



equilibrium level for $p_1 = p_2 = p_{atm}$ *

equilibrium level for $p_1 = p_2 = p_{atm}$

new fluid levels for $p_1 > p_2$

manometer geometry: $h = L \sin\theta$ (length as drawn)

$h_2 = \Delta L \sin\theta$ (change in L from position when $p_1 = p_2$)

(a) left leg = right leg (go down to new "equilibrium level", below which there is only manometer liquid)

$$p_1 + \rho_{air} g (h_1 + h_2) = \rho_{H_2O} g (h_1 + h_2) + p_2$$

$$\therefore \boxed{p_1 - p_2 = (\rho_{H_2O} - \underbrace{\rho_{air}}_{\text{can neglect, since } \rho_{air} \ll \rho_{H_2O}}) g (h_1 + h_2)} \quad (*)$$

w/ $\Delta p = p_1 - p_2 =$
$= p_{atm} - (p_{atm} - 300\,Pa)$
$= 300\ Pa$

$\Rightarrow$ given a pressure difference, this equation tells us

what the total elevation difference $h = (h_1 + h_2)$ is.
To find how much the elevation in each part of the manometer changed, $\Delta L = \frac{h_2}{\sin\theta}$, we need to consider geometry and conservation of mass.

$$\frac{\pi}{4} D^2 h_1 = \frac{\pi}{4} d^2 \Delta L \quad \therefore \quad h_1 = \Delta L \frac{d^2}{D^2}$$

$$h_2 = \Delta L \sin\theta$$

substitute into (*):

$$p_1 - p_2 = (\rho_{H_2O} - \rho_{air})\, g\, \Delta L \left(\frac{d^2}{D^2} + \sin\theta\right) \quad (**)$$

$$\therefore (*) \quad h = (h_1 + h_2) = \frac{p_1 - p_2}{(\rho_{H_2O} - \rho_{air})\, g} = \frac{\cancel{p_{atm}} - (\cancel{p_{atm}} - 300\,Pa)}{(998 - 1.21)\frac{kg}{m^3}\, 9.81\frac{m}{s^2}} = 0.0307\,m$$

$$\Delta L = \frac{p_1 - p_2}{(\rho_{H_2O} - \rho_{air})\, g} \frac{1}{\frac{d^2}{D^2} + \sin\theta} = h \frac{1}{\frac{d^2}{D^2} + \sin\theta} = 0.1141\,m$$

(b) $\Delta p = p_1 - p_2 = p_{atm} - (p_{atm} + 300\,Pa) = -300\,Pa$

Same analysis as above, negative results

$$h = (h_1 + h_2) = -0.0307\,m$$

$$\Delta L = h \frac{1}{\left(\frac{d}{D}\right)^2 + \sin\theta} = -0.1141\,m$$

(c) $h = (h_1 + h_2) = \frac{p_1 - p_2}{(\rho_{H_2O} - \rho_{air})\, g} = \frac{p_{atm} - (p_{atm} - 500\,Pa)}{(998 - 1.21) \cdot 9.81} = 0.0511\,m$

$$\Delta L = h \frac{1}{\left(\frac{d}{D}\right)^2 + \sin\theta} = 0.1902\,m$$

$$L = \frac{h}{\sin\theta} = 0.1976\,m$$

$$h_2 = \Delta L \cdot \sin\theta = 0.0492\,m$$

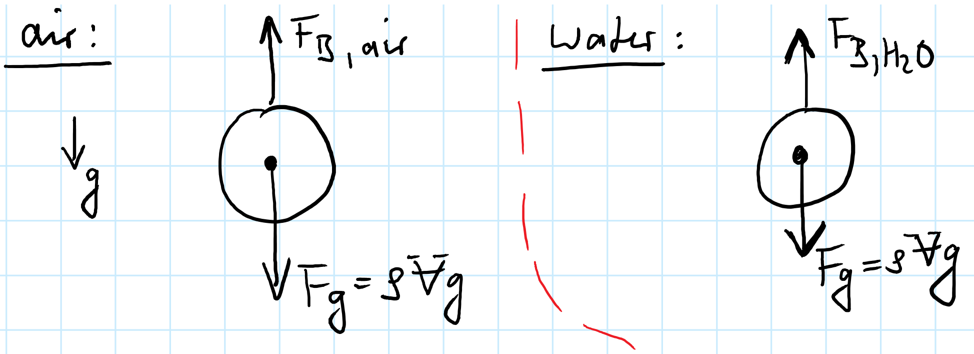
$$h_1 = h - h_2 = 0.0019\,m$$

(d) To find h, we do not need to know the geometry.

To find ΔL, we need to know the ratio of the diameters, but not the diameters themselves.

Problem 3.4

You have a piece of metal of unknown origin and type. Provided you have the ability to measure the weight of the piece of metal both in air and in water (W_{air}, W_{H_2O}), and know the density of air and water (ρ_{air}, ρ_{H_2O}), determine the density and the specific gravity of the metal.
Hint: draw a free body diagram for each fluid, and use Archimedes' principle.
Do not make any simplifying assumptions regarding density ratios -- until the end, when you should comment on the magnitude of each of the terms that may arise in your solution.



w/ $F_{B,air} = -\int p_{air}\, dA = \rho_{air}\forall g$ $\qquad$ $F_{B,H_2O} = -\int p_{H_2O}\, dA = \rho_{H_2O}\forall g$

The resultant force is what is measured as weight W:

$W_{air} = \rho\forall g - \rho_{air}\forall g = (\rho - \rho_{air})\forall g$ $\qquad$ $W_{H_2O} = (\rho - \rho_{H_2O})\forall g$

combine:

$$\frac{W_{air}}{\rho - \rho_{air}} = \frac{W_{H_2O}}{\rho - \rho_{H_2O}}$$

$$(\rho - \rho_{air}) W_{H_2O} = (\rho - \rho_{H_2O}) W_{air}$$

$$\rho\,(W_{H_2O} - W_{air}) = \rho_{air} W_{H_2O} - \rho_{H_2O} W_{air}$$

$$\therefore \boxed{\rho = \frac{\rho_{air} W_{H_2O} - \rho_{H_2O} W_{air}}{W_{H_2O} - W_{air}}}$$ density of metal

$$\boxed{S.G. = \frac{\rho}{\rho_{H_2O}} = \frac{\frac{\rho_{air}}{\rho_{H_2O}} W_{H_2O} - W_{air}}{W_{H_2O} - W_{air}}}$$ specific gravity

small

Problem 3.5

(a) You may have seen this "party trick": you partially fill a glass with water, place an index card on top of the glass, and then turn the glass upside down while holding the card in place. You can then remove your hand from the card and the card remains in place, holding the water in the glass. Explain how this works.

(b) An inverted cylinder with diameter D=0.1m is partially filed with water and held in place as shown in the figure below. A force of 20 N is needed to pull the plate from the cylinder. The plate is not fastened to the cylinder and has negligible mass. What is the air pressure in the cylinder?

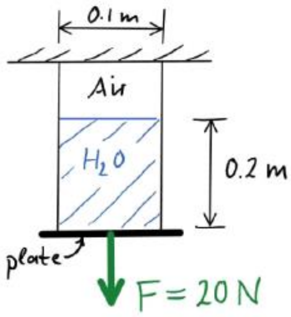
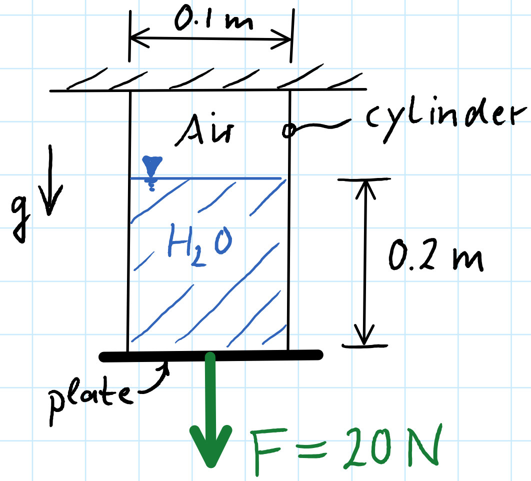


Figure 3: Vertically oriented cylinder partially filled with water.

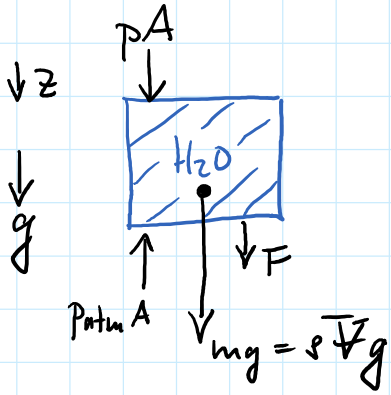


do (b) first:

$A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.1)^2 = 7.854 \times 10^{-3}\ m$, $h = 0.2\ m$

$p_{atm} = 101{,}325\ Pa$, $F = 20\ N$, $\rho_{H_2O} = 1000\ \frac{kg}{m^2}$

FBD:



assumptions:
- fluids at rest (statics)
- plate mass is negligible
- neglect weight of air

$$\Sigma F_z = 0 = pA + \rho V g + F - p_{atm} A$$

$$\therefore \; p = p_{atm} - \rho g \frac{V}{A} - \frac{F}{A} \quad \text{with } \frac{V}{A} = h$$

$$= p_{atm} - \rho g h - \frac{F}{A}$$

$$= 101{,}325 - 1{,}962 - 2{,}546$$

$$= 96{,}817\ Pa$$

Now we can answer (a):

It is the same problem as under (b), now with $F = 0$

- If the water + card are held in place only by air pressure, then the pressure has to be less than:

$$P \leq P_{atm} - \rho g h = 101{,}325 - 1{,}962 = 99{,}363\,\text{Pa}$$

(small decrease)

How can the pressure in the air space be lowered, if it initially was P_{atm}?

→ card deforms a bit, air volume increases a bit, pressure must decrease (Boyle's law)

→ a small amount of water might leak out and then the card re-seals at the rim → air volume increases.

- a bit of surface tension may also come into play...

→ a small difference in air pressure is needed to hold the water in place.