ME 608 Fluid Dynamics
M. Wosnik

Problem 2.1
A piston is held at the top inside a long, vertical, stationary tube (see Figure 1). The piston diameter D_p is 98 mm, its length (height) H is 80 mm, and the tube inner diameter D_t is 100 mm. The piston is made from aluminum ($S.G. = 2.7$). The tube inner wall is lined with SAE 10W oil at 20°C. The piston is then let go and can fall freely through the tube, except for resistance of the oil between the piston and the tube wall. Assuming that the gap between the piston and the tube is completely filled with oil, and that the velocity profile within the oil is linear, what is the terminal velocity of the piston?

(a) Sketch the velocity field in the oil next to the piston (o.k. to enlarge gap for drawing), and all the relevant dimensions for the analysis.
(b) Derive the algebraic form of the solution for the piston terminal velocity.
(c) Substitute the given values and calculate the value of the terminal velocity.

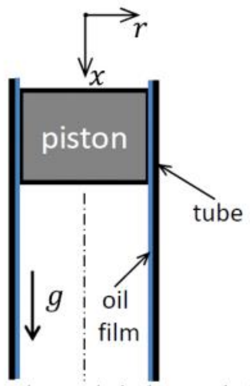


Figure 1. Piston in a vertical tube coated with a film of oil.

$D_p = 98\,mm = 0.098\,m$

$H = 80\,mm = 0.08\,m$

$D_t = 102\,mm = 0.102\,m$

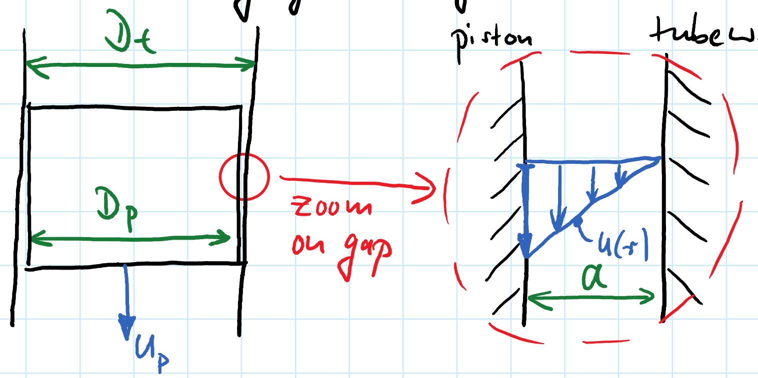
$A_p = \pi D_p H$ (area of piston in contact with oil)

- Appendix A.2: SAE 10W oil @ 20°C, $\underline{\underline{\mu = 0.1 \frac{Ns}{m^2}}}$
- mass of piston: $\underline{\underline{\rho_{AL}}} = S.G. \, \rho_{H_2O} = 2.7 \cdot 1{,}000 \frac{kg}{m^3} = \underline{\underline{2{,}700 \frac{kg}{m^3}}}$

$$\underline{\underline{m_p}} = \rho_{AL} V_p = \rho_{AL} \frac{\pi}{4} D_p^2 \cdot H = 2{,}700 \frac{kg}{\cancel{m^3}} \frac{\pi}{4} (0.098\,\cancel{m})^2 \, 0.08\,\cancel{m} =$$

$$= \underline{\underline{1.629\,kg}}$$

(a) Sketch the velocity field of the oil next to the piston:



gap size

$\underline{\underline{a}} = \frac{1}{2}(D_t - D_p)$

$= \frac{1}{2}(102 - 98)$

$= 2\,mm = \underline{\underline{0.002\,m}}$

Assume: • steady state

- symmetry (gap of equal size around piston)
- laminar flow (linear velocity profile)
- Newtonian fluid (= linear stress & rate of strain relationship)

(b) "Terminal velocity" will occur when the force of gravity and the viscous forces due to the fluid in the gap (oil) are in equilibrium.

$$\Sigma F = 0 = m_p g + \tau_w A_p$$

$$\text{w/ } \tau_w = \mu \left.\frac{du}{dr}\right|_{wall} = \mu \underbrace{\frac{\Delta u}{\Delta r}}_{\text{o.k. for linear profile}} = \mu \frac{0 - u_p}{\frac{1}{2}(D_t - D_p)} = \frac{-\mu u_p}{a}$$

$$m_p g + \pi D_p H \left(-\frac{\mu u_p}{a}\right) = 0$$

$$\therefore \boxed{\underline{\underline{u_p}} = \frac{a\, m_p g}{\pi D_p H \mu} = \frac{(D_t - D_p) m_p g}{2\pi D_p H \mu} = \frac{(D_t - D_p)\rho_{AL} \cancel{\frac{\pi}{4}} D_p^{\cancel{2}} \cancel{H} g}{2 \cancel{\pi} \cancel{H} \mu \cancel{D_p}} = \underline{\underline{\frac{\rho_{AL}\, g\, D_p (D_t - D_p)}{8\mu}}}}$$

(c) Substitute values $\quad \underline{\underline{u_p}} = \underline{\underline{6.48 \frac{m}{s}}}$

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Problem 2.2
A belt of width W moves with velocity U_o over a pool of liquid, with length L in contact with the liquid (see Figure 2). For example, this could be an application where cooling or coating is required. The depth of the liquid pool below the belt is h, and the fluid properties are known (ρ_l, μ_l). To a good approximation, a linear velocity profile is established in the liquid below the belt. What is the force exerted on the belt by the liquid? What is the power required to drive the belt?

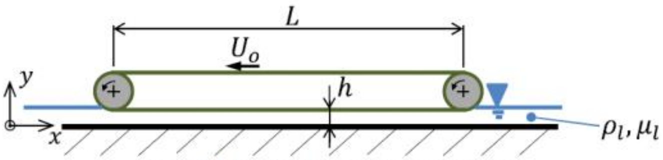


Figure 2. Moving belt over a pool of liquid.

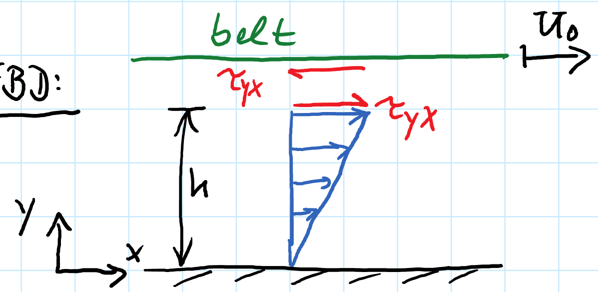
Given: width W, length L, velocity U_o, depth of liquid pool h

fluid properties ρ_l, μ_l

Assumptions:
- Laminar flow
- Newtonian fluid
- Steady state

(a) force exerted on belt by liquid

FBD:



$$F_{belt} = \tau_{yx}\big|_{y=h} \cdot A = \mu \frac{du}{dy}\bigg|_{y=h} LW$$

w/ $\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{U_o}{h}$

$$\therefore \boxed{F_{belt} = \mu U_o \frac{LW}{h}}$$

(b) power required to drive belt

power = rate of work (energy)

$$\boxed{P = \frac{dW}{dt}} \quad \text{with} \quad W = \int_0^L F_x \, dx = \mu U_o \frac{LW}{h} x \Big|_0^L$$

$\boxed{P = \frac{\quad}{dt}}$ with $W = \int_0 F_x \, dx = \mu U_0 \frac{\quad}{h} x \Big|_0$

$\therefore \boxed{P} = \frac{dW}{dt} = \mu \bar{U}_0 \frac{L\bar{W}}{h} \frac{dx}{dt} = \boxed{\mu \bar{U}_0^2 \frac{L\bar{W}}{h}}$

2.3

Friday, February 9, 2018 10:00 AM

Problem 2.3

Problem 2.3 is a simple velocity field identification problem from the text book (problem 2.1).

2.1 For the velocity fields given below, determine:

a. whether the flow field is one-, two-, or three-dimensional, and why.

b. whether the flow is steady or unsteady, and why.

(The quantities a and b are constants.)

(1) $\vec{V} = [(ax+t)e^{by}]\hat{i}$ (2) $\vec{V} = (ax-by)\hat{i}$

(3) $\vec{V} = ax\hat{i} + [e^{bx}]\hat{j}$ (4) $\vec{V} = ax\hat{i} + bx^2\hat{j} + ax\hat{k}$

(5) $\vec{V} = ax\hat{i} + [e^{bt}]\hat{j}$ (6) $\vec{V} = ax\hat{i} + bx^2\hat{j} + ay\hat{k}$

(7) $\vec{V} = ax\hat{i} + [e^{bt}]\hat{j} + ay\hat{k}$ (8) $\vec{V} = ax\hat{i} + [e^{by}]\hat{j} + az\hat{k}$

Generally, $\vec{V} = \vec{V}(x, y, z, t)$

- If $\vec{V} = \vec{V}(t) \rightarrow$ unsteady
  $\vec{V} \neq \vec{V}(t) \rightarrow$ steady
- If $\vec{V} = \vec{V}(x)$ or $\vec{V}(y)$ or $\vec{V}(z) \rightarrow$ 1-D
  If $\vec{V} = \vec{V}(x,y)$ or $\vec{V}(y,z)$ or $\vec{V}(x,z) \rightarrow$ 2-D
  If $\vec{V} = \vec{V}(x,y,z) \longrightarrow$ 3-D

	(a)	(b)
(1)	2-D,	unsteady
(2)	2-D,	steady
(3)	1-D,	steady
(4)	1-D,	steady
(5)	1-D,	unsteady
(6)	2-D,	steady
(7)	2-D,	unsteady
(8)	3-D,	steady

Problem 2.4

Given is a velocity field $\vec{v}(x,y) = \frac{A}{x}\,\hat{\imath} + \frac{Ay}{x^2}\,\hat{\jmath}$, with $A = 1\ [m^2/s]$.
Determine the equation for a streamline that passes through a point $(x,y) = (3,4)$.
Calculate the time required for a fluid particle to move from $x_1 = 2m$ to $x_2 = 4m$.
State all assumptions and governing equations.

(a) Streamline that passes through $(x_0, y_0) = (3,4)$

definition of streamline in 2-D $\boxed{\frac{v}{u} = \frac{dy}{dx}}$

with $u = \frac{A}{x}$, $v = \frac{Ay}{x^2}$

$$\therefore \quad \frac{v}{u} = \frac{A\frac{y}{x^2}}{\frac{A}{x}} = \frac{y}{x} = \frac{dy}{dx} \iff \frac{dy}{y} = \frac{dx}{x}$$

integrate $\ln(y) = \ln(x) + \text{const.}$

$$e^{\ln y} = e^{\ln x + \text{const}} = e^{\ln x} e^{\text{const}} \qquad c = e^{\text{const}}$$

$$\therefore \quad y = x\,c$$

for $(x_0, y_0) = (3,4)$: $4 = 3\,c \;=\; c = \frac{4}{3}$

$\therefore$ equation for streamline: $\boxed{y = \frac{4}{3}x}$

(b) particle path equation

$$u(x,y,t) = \frac{dx_p}{dt}$$

$$v(x,y,t) = \frac{dy_p}{dt}$$

$$\therefore \quad \frac{A}{x} = \frac{dx_p}{dt} \implies \int_{x_1=2}^{x_2=4} x\,dx = \int_0^t A\,dt$$

$$\left.\frac{x^2}{2}\right|_2^4 = At$$

$$t = \frac{1}{A}\left(\frac{4^2}{2} - \frac{2^2}{2}\right) = \frac{6\,m^2}{A} = 6\,s$$

Problem 2.5
Consider the velocity field given by: $\vec{v} = A(1 + Bt)\hat{\imath} + Cty\,\hat{\jmath}$
where $A = 1\ m/s$, $B = 1\ s^{-1}$, $C = 1\ s^{-2}$, and x and y are measured in meters.

(a) What are the u and v components of the velocity?
(b) Find the pathline (particle path) $x_p(t)$ and $y_p(t)$ (see Example problem 2.1)
(c) Plot the pathline traced out by a particle that passes through the point (1,1) at time $t = 0$.

(a) $u = A(1+Bt)$; $v = Cty$

(b) $u(t) = \frac{dx_p}{dt} = A(1+Bt) \Leftrightarrow \int_{x_0}^{x} dx = \int_0^t A(1+Bt)\,dt$

$$x - x_0 = At + B\frac{t^2}{2}$$

$$\boxed{x = At + B\frac{t^2}{2} + x_0}$$

$v(t) = \frac{dy_p}{dt} = Cty \Leftrightarrow \int_{y_0}^{y} \frac{1}{y}\,dy = \int_0^t Ct\,dt$

$$\ln\left(\frac{y}{y_0}\right) = \frac{Ct^2}{2}$$

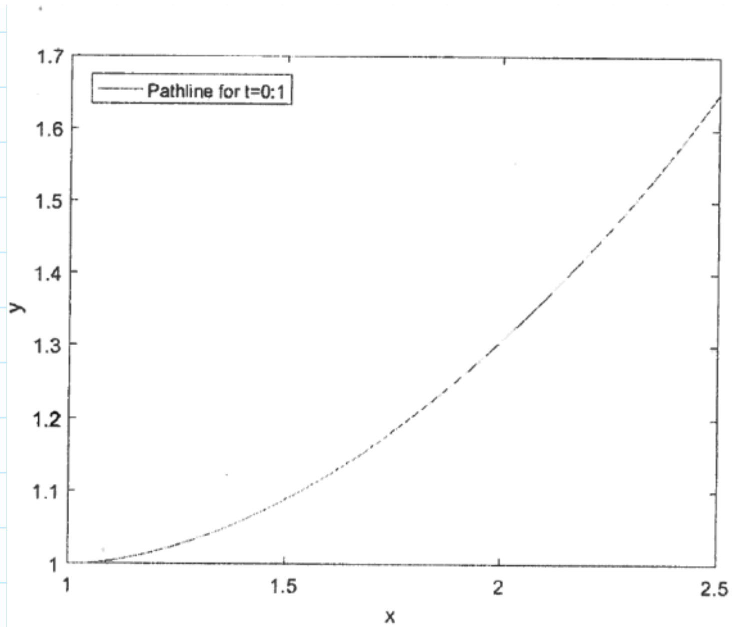
$$\boxed{y = y_0\, e^{\frac{Ct^2}{2}}}$$

(c) set $(x_0, y_0) = (1, 1)$

to plot use Matlab: set $t = 0{:}0.1{:}1$

input functions above for $\underline{x}$ and $\underline{y}$

use figure, plot(x,y)


Pathline for t=0:1
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
y
1
1.5
2
2.5
x