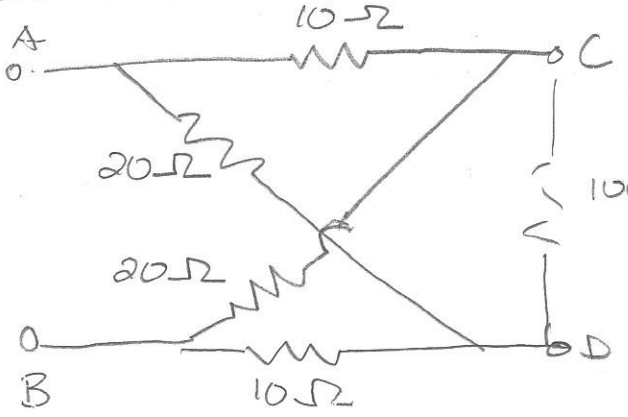
PROBLEM SET 4

SOLUTIONS

Problem 1 : B-6-2



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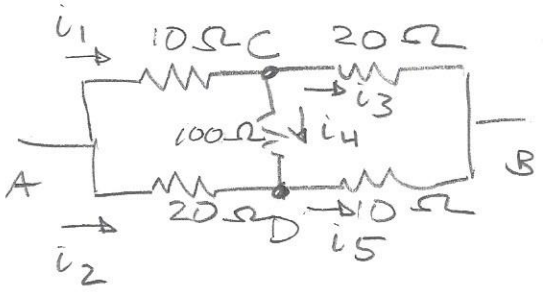


$$e_{AB} = R_{AB} \cdot (i_1 + i_2)$$

@ C: $i_1 = i_3 + i_4 \Rightarrow i_3 = i_1 - i_4$

@ D: $i_5 = i_4 + i_2$

} variables: i_1, i_2, i_4 ∴ need 3 equations

Path ACB: $e_{AC} + e_{CB} = e_{AB}$

$$10 i_1 + 20 i_3 = e_{AB}$$

$$10 i_1 + 20(i_1 - i_4) = e_{AB} \quad (1)$$

Path ADB: $e_{AD} + e_{DB} = e_{AB}$

$$20 i_2 + 10 i_5 = e_{AB}$$

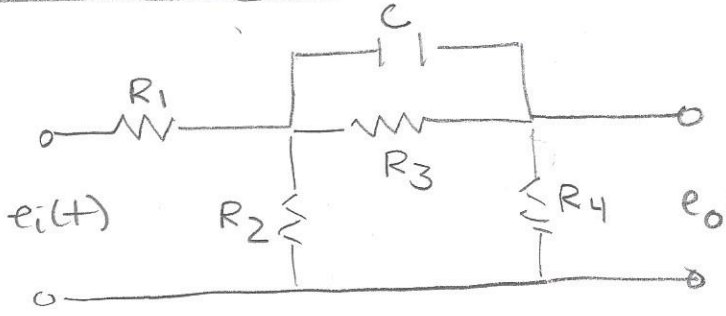
$$20 i_2 + 10(i_4 + i_2) = e_{AB} \quad (2)$$

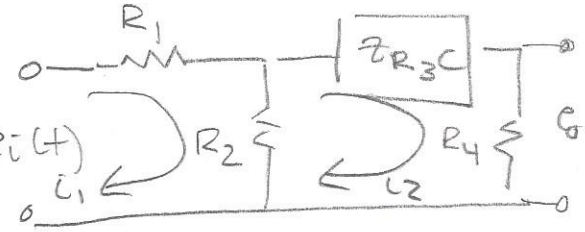
Path ACD = Path AD $\Rightarrow e_{AC} + e_{CD} = e_{AD}$

$$10 i_1 + 100 i_4 = 20 i_2 \quad (3)$$

$$e_{AB} = R_{AB}(i_1 + i_2) \Rightarrow R_{AB} = \frac{e_{AB}}{i_1 + i_2} \Rightarrow \boxed{R_{AB} = 14.8\,\Omega}$$

Problem 2. : B-6-7





$e_o(t) = e_{R_4}(t)$

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{R_4 I_2(s)}{E_i(s)}$$

$$Z_{RC}(s) = \frac{\frac{1}{Cs} \cdot R_3}{\frac{1}{Cs} + R_3} = \frac{R_3}{R_3 Cs + 1}$$

Loop i_1: $E_i(s) - R_1 I_1(s) - R_2\left[I_1(s) - I_2(s)\right] = 0$ ①

Loop i_2: $-Z_{R_3C}(s) I_2(s) - R_4 I_2(s) - R_2\left[I_2(s) - I_1(s)\right] = 0$ ②

① $\left[R_1 + R_2\right] I_1(s) + \left[-R_2\right] I_2(s) = E_i(s)$

$\left[-R_2\right] I_1(s) + \left[Z_{R_3C}(s) + R_4 + R_2\right] I_2(s) = 0$

$$\therefore I_2(s) = \frac{\begin{vmatrix} R_1+R_2 & E_i(s) \\ -R_2 & 0 \end{vmatrix}}{\begin{vmatrix} R_1+R_2 & -R_2 \\ -R_2 & Z_{R_3C}(s)+R_4+R_2 \end{vmatrix}}$$

$$= \frac{R_2 E_i(s)}{(R_1+R_2)\left[Z_{R_3C}(s) + R_4 + R_2\right] - R_2^2}$$

$$= \frac{R_2(R_3Cs+1)\, E_i(s)}{(R_1+R_2)\left[R_3 + (R_4+R_2)(R_3Cs+1)\right] - R_2^2(R_3Cs+1)}$$

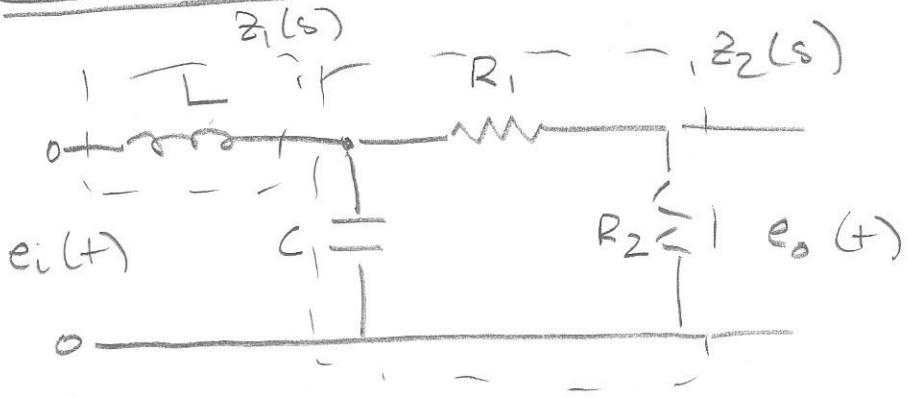
Problem 2: (cont'd)

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{R_4 I_2(s)}{E_i(s)}$$

$$= \frac{R_4}{E_i(s)} \cdot \frac{R_2 (R_3 Cs+1) E_i(s)}{(R_1+R_2)\left[R_3 + (R_4+R_2)(R_3 Cs+1)\right] - R_2^2 (R_3 Cs+1)}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4 (R_3 Cs+1)}{(R_1+R_2)\left[R_3 + R_4 (R_3 Cs+1)\right] + R_1 R_2 (R_3 Cs+1)}}$$

Problem 3: B-6-8

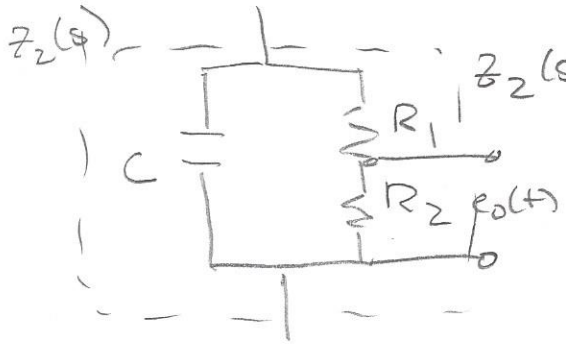


$$E_c(s) = E_{R_1}(s) + E_{R_2}(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{E_c(s)}{E_i(s)} \cdot \frac{E_o(s)}{E_c(s)}$$

$$\frac{E_c(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$Z_1(s) = Z_L(s) = Ls$$

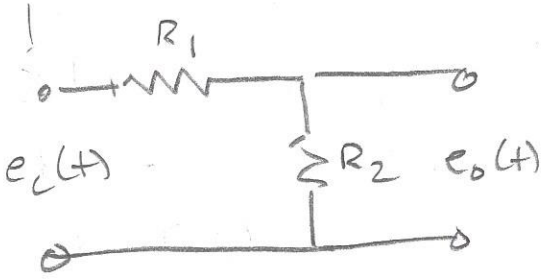


$$Z_2(s) = \frac{Z_C(s)\left[Z_{R_1}(s) + Z_{R_2}(s)\right]}{Z_C(s) + \left[Z_{R_1}(s) + Z_{R_2}(s)\right]}$$

$$= \frac{\frac{1}{Cs} \cdot (R_1 + R_2)}{\frac{1}{Cs} + R_1 + R_2} = \frac{R_1 + R_2}{(R_1+R_2)Cs + 1}$$

$$\frac{E_c(s)}{E_i(s)} = \frac{\frac{R_1+R_2}{(R_1+R_2)Cs+1}}{Ls + \frac{R_1+R_2}{(R_1+R_2)Cs+1}} = \frac{R_1+R_2}{Ls\left[(R_1+R_2)Cs+1\right] + R_1 + R_2}$$

Problem 3 : Cont'd



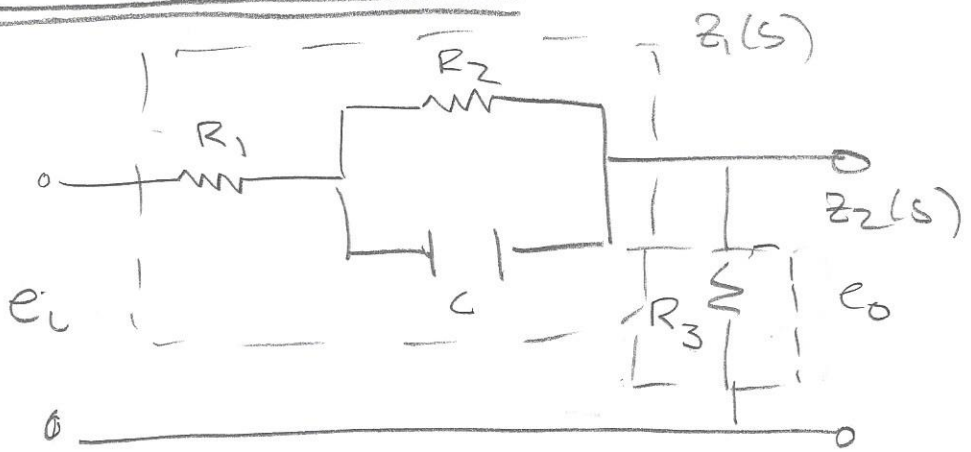
$$\frac{E_o(s)}{E_c(s)} = \frac{Z_{R_2}(s)}{Z_{R_1}(s)+Z_{R_2}(s)} = \frac{R_2}{R_1+R_2}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{E_c(s)}{E_i(s)} \cdot \frac{E_o(s)}{E_c(s)}$$

$$= \frac{\cancel{R_1+R_2}}{Ls\left[(R_1+R_2)Cs+1\right]+R_1+R_2} \cdot \frac{R_2}{\cancel{R_1+R_2}}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R_2}{Ls\left[(R_1+R_2)Cs+1\right]+R_1+R_2}}$$

Problem 4: B-6-11



$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$Z_2(s) = Z_{R_3}(s) = R_3$$

$$Z_1(s) = Z_{R_1}(s) + \frac{Z_{R_2}(s)\cdot Z_C(s)}{Z_{R_2}(s)+Z_C(s)}$$

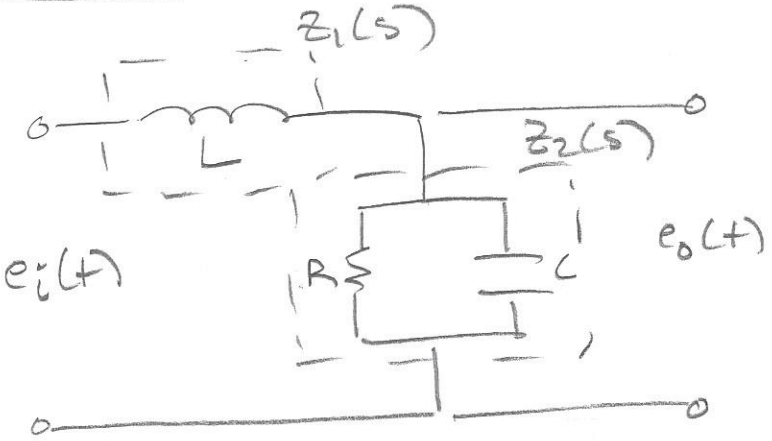
$$= R_1 + \frac{R_2 \frac{1}{Cs}}{R_2 + \frac{1}{Cs}}$$

$$= R_1 + \frac{R_2}{R_2(Cs+1)}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{R_3}{R_1 + \frac{R_2}{R_2(Cs+1)} + R_3}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R_3(R_2(Cs+1))}{(R_1+R_3)(R_2(Cs+1)) + R_2}}$$

Problem 5 : B-6-12



$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

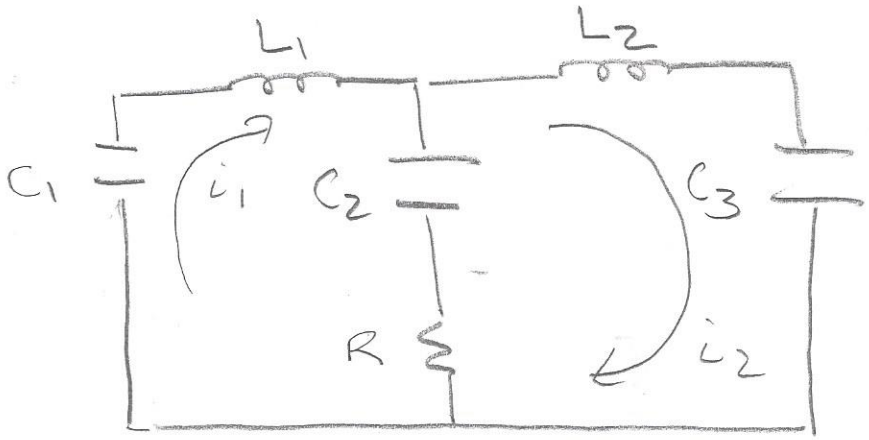
$$Z_1(s) = Z_L(s) = Ls$$

$$Z_2(s) = \frac{Z_C(s)\cdot Z_R(s)}{Z_C(s)+Z_R(s)} = \frac{\frac{1}{Cs}\cdot R}{\frac{1}{Cs}+R} = \frac{R}{RCs+1}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{R}{RCs+1}}{Ls+\frac{R}{RCs+1}}$$

$$\Rightarrow \boxed{\frac{E_o(s)}{E_i(s)} = \frac{R}{Ls\left[RCs+1\right]+R}}$$

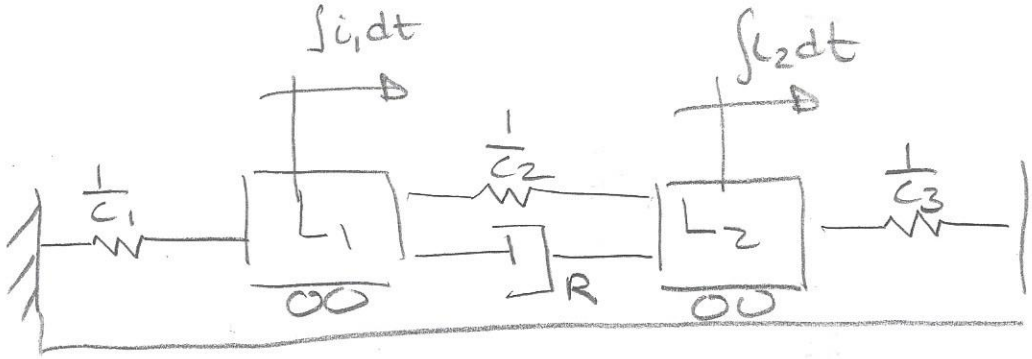
Problem 6: B-6-17



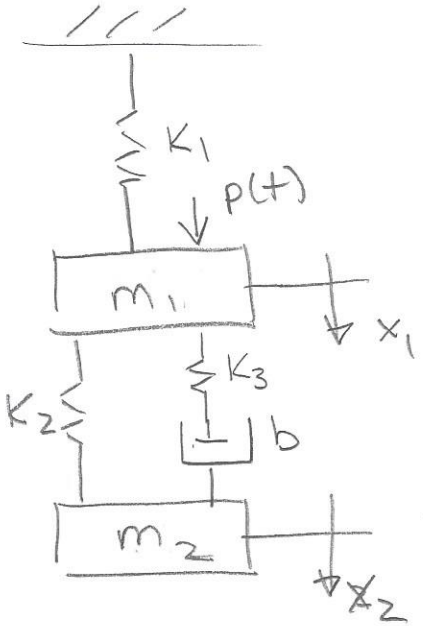
$C_1, L_1 \Rightarrow$ share common velocity i_1

$L_2, C_3 \Rightarrow$ share common velocity i_2

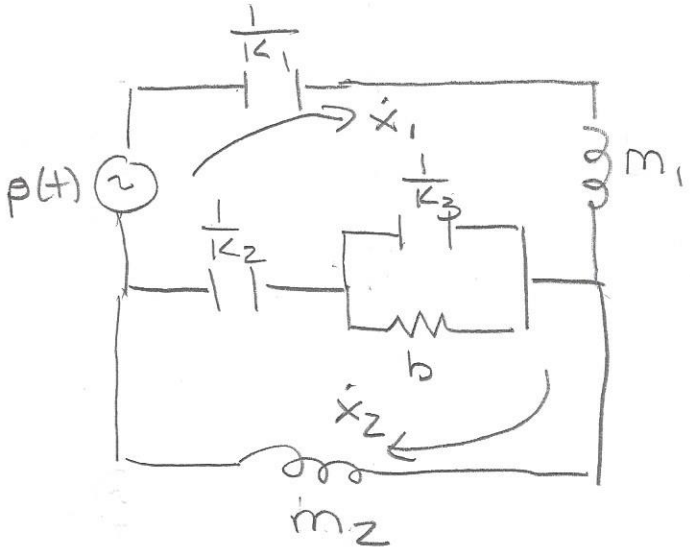
$C_2, R \Rightarrow$ relative velocity between $i_1 + i_2$



Problem 7: B-6-18



$K_2, (K_3 + b)$
$\Rightarrow$ share relative current between $\dot{x}_1$ and $\dot{x}_2$

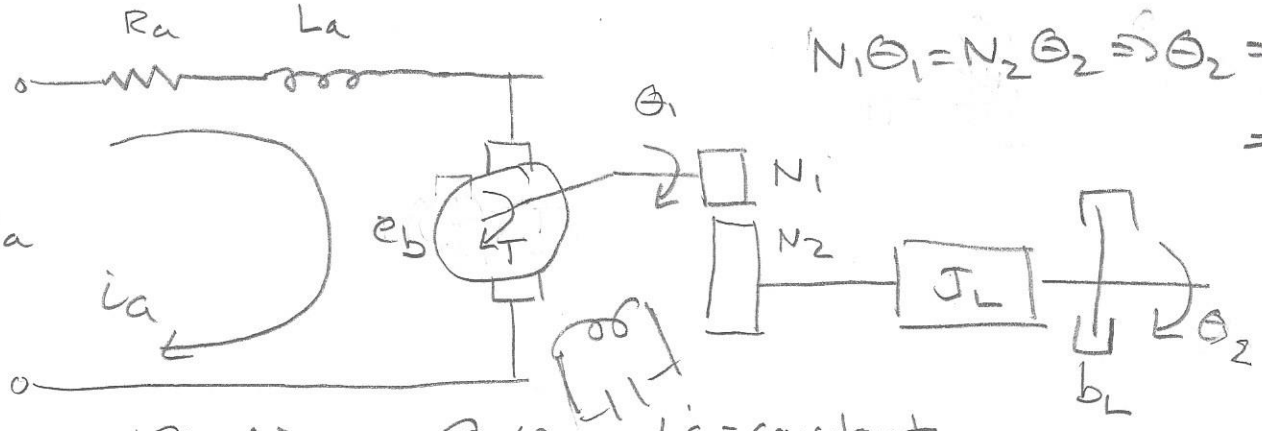


$K_1, m_1 \Rightarrow$ share common current $\dot{x}_1$, with voltage source $p(t)$

$m_2 \Rightarrow$ has current $\dot{x}_2$

K_3 and $b \Rightarrow$ undergo same voltage

Problem 8 : B-6-19



$$N_1\theta_1 = N_2\theta_2 \Rightarrow \theta_2 = \frac{N_1}{N_2}\theta_1 = n\theta_1$$

$$G(s) = \frac{\Theta_2(s)}{E_a(s)} = \frac{n\Theta_1(s)}{E_a(s)}$$

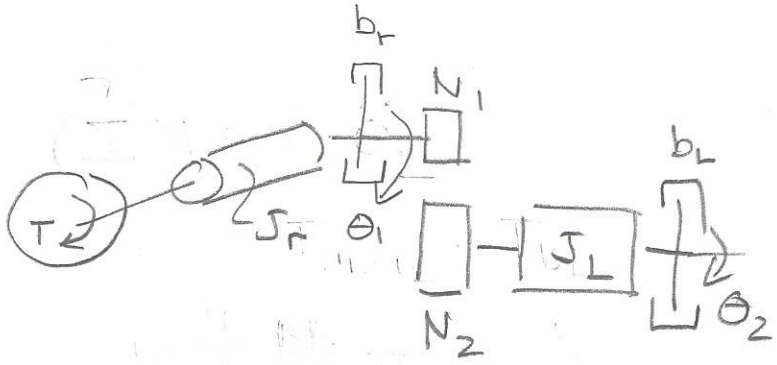
Electrical:

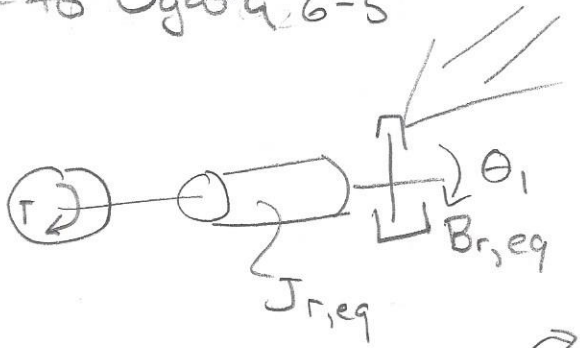
$$E_a(s) - R_a I_a(s) - L_a s I_a(s) - E_b(s) = 0$$

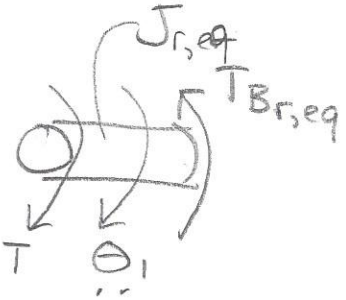
Mechanical

$$\begin{cases} J_{r,eq} = J_r + n^2 J_L \\ B_{r,eq} = b_r + n^2 b_L \end{cases}$$

→ Refer to Ogata 6-5







$$\sum M = J_{r,eq}\ddot{\theta}_1 = T - T_{B_{r,eq}} = T - B_{r,eq}\dot{\theta}_1$$

$$\Rightarrow J_{r,eq}\ddot{\theta}_1 + B_{r,eq}\dot{\theta}_1 = T = \left[J_{r,eq}s^2 + B_{r,eq}s\right]\Theta_1(s) = T(s)$$

Problem 8.7 (Cont'd)

Electromechanical

(1) $e_b = K_b \dot{\theta}_1 \Rightarrow \underline{E_b(s) = K_b s \, \Theta_1(s)}$

(2) $T = K \cdot i_a \Rightarrow \underline{T(s) = K I_a(s)}$

$$\Rightarrow [K_b s] \Theta_1(s) + [R_a + L_a s] I_a(s) = E_a(s)$$

$$\Rightarrow [J_{r,eq} s^2 + B_{r,eq} s] \Theta_1(s) + [-K] I_a(s) = 0$$

$$\Theta_1(s) = \frac{\begin{vmatrix} E_a(s) & R_a + L_a s \\ 0 & -K \end{vmatrix}}{\begin{vmatrix} K_b s & R_a + L_a s \\ [J_{r,eq} s^2 + B_{r,eq} s] & -K \end{vmatrix}}$$

$$= \frac{K E_a(s)}{[J_{r,eq} s^2 + B_{r,eq} s][R_a + L_a s] + K K_b s}$$

$$\therefore \frac{\Theta_2(s)}{E_a(s)} = \frac{n \Theta_1(s)}{E_a(s)} = \frac{Kn}{[J_{r,eq} s^2 + B_{r,eq} s][R_a + L_a s] + K K_b s}$$

$L_a = 0$

$$\therefore \boxed{\frac{\Theta_2(s)}{E_a(s)} = \frac{0.072}{s(0.1296 s + 1)}}$$