10.1

Thursday, April 12, 2018 10:45 AM

Problem 10.1

Questions about the film on Pressure Fields and Acceleration - Bernoulli Equation (National Committee for Fluid Mechanics Films, http://techtv.mit.edu/collections/ifluids/videos/32607-pressure-fields-fluid-acceleration). The questions can be answered by considering Euler's equation along a streamline, and its integral, the Bernoulli equation, as well as Euler's equation normal to a streamline (last two questions).

(a) In an inviscid flow, what causes the net force on a fluid particle in the direction of the streamline?
Answer: the difference in pressure on the faces of the fluid particle, i.e., the pressure gradient.

(b) Complete the following sentence, about the first experiment shown in the film:
The water tunnel experiment shows that for the flow through the contraction the water velocity *increase* is accompanied by a pressure … Answer: *decrease*.

(c) What is the fluid velocity at the "stagnation point" on the front face of a bluff body (here: a semi-infinite round body)?
Answer: the velocity at the stagnation point is zero.

(d) If there are no effects of elevation change (changes in potential energy) in an inviscid, incompressible flow, how does the stagnation pressure change through a gradual flow contraction or expansion (diffuser)?
Answer: The stagnation pressure does not change (or: it almost doesn't change, there is a small change due to viscous losses).

The following two questions pertain to the Euler equation normal to a streamline.

(e) When streamlines are curved, in which direction does the pressure increase w.r.t. the center of flow curvature?
Answer: pressure increases outward from the center of flow curvature.

(f) Consider a 2-D duct with a one-sided contraction: As the contraction begins, does the pressure on the side with the contraction initially increase or decrease? Argue from a point of flow curvature.
Answer: the pressure initially increases on the wall with the contraction, since the wall is located outward from the center of flow curvature (center of curvature is on the "flow side" of the wall).

10.2

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Problem 10.2
The water flow rate through the siphon is 5 liters/s, its temperature is 20°C, and the pipe diameter is 25 mm. Compute the maximum allowable height, h, so that the pressure at point A remains above the vapor pressure (cavitation!) of the water. Assume the flow is frictionless.

@ $T = 20°C$:

$\rho_{H_2O} = 998 \frac{kg}{m^3}$

$P_{atm} = 101.3\,kPa$

$P_v = 2.339$

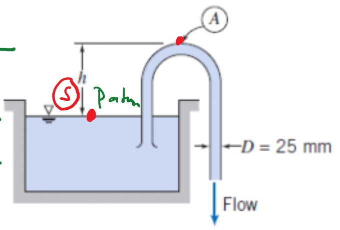
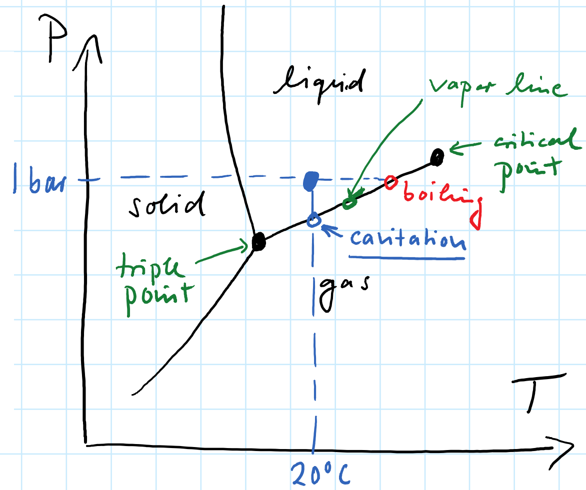


Figure 1: Sketch of tank and siphon.



Phase diagram for water

If the pressure @ A falls below vapor pressure, then water crosses the vapor line and will become water vapor, a.k.a. "cavitation"

What is the maximum allowable height h to avoid cavitation?

Assumptions: Inviscid, steady, incompressible flow along streamline

$\Rightarrow$ Bernoulli eqn: $\boxed{\frac{P}{\rho} + \frac{v^2}{2} + gz = \text{const.}}$

1-D continuity $vA = \dot{V} = \text{const.}$

avg. velocity in siphon: $V_A = \frac{\dot{V}}{A} = \frac{\dot{V}}{D^2 \frac{\pi}{4}} = \frac{4 \cdot 0.005 \frac{m^3}{s}}{(0.025)^2 m^2 \cdot \pi} = 10.186 \frac{m}{s}$

Write Bernoulli between water surface (S) and highest point (A):

$$\frac{\overbrace{P_S}^{P_{atm}}}{\rho} + \frac{\overbrace{V_S^2}^{\simeq 0}}{2} + g\underbrace{z_S}_{\emptyset} = \frac{\overbrace{P_A}^{P_v = \text{lowest permissible pressure}}}{\rho} + \frac{V_A^2}{2} + g\underbrace{z_A}_{=h}$$

$V_S << V_A \quad \therefore \frac{V_S^2}{V_A^2} \to \emptyset$

$$\therefore \quad \frac{P_{atm}}{\rho} = \frac{P_v}{\rho} + \frac{V_A^2}{2} + gh$$

$$\therefore \quad h = \frac{1}{\rho g}(p_{atm} - p_v) - \frac{V_A^2}{2g} = 4.82 \text{ m}$$

Note: This "siphon height" h can be significantly increased by increasing the pipe diameter (for the same flow rate).
E.g.: using a $d = 50$ mm siphon pipe reduces the pipe velocity to $2.55 \frac{m}{s}$ and increases h to 9.78 m

However, the hard limit for siphon height when pumping from a free surface at p_{atm} is $\frac{1}{\rho g}(p_{atm} - p_v)$, or ~ 10.1 m

10.3

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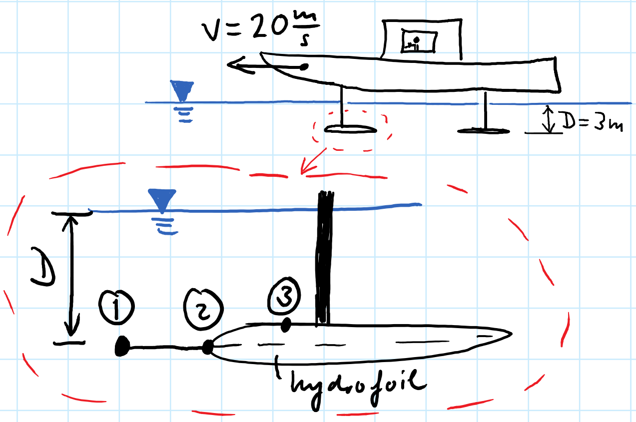
Problem 10.3

A speedboat on hydrofoils is moving at 20 m/s in a freshwater lake. Each hydrofoil is 3 m below the surface. Assuming, as an approximation, frictionless, incompressible flow, find the stagnation pressure (gage) at the front of each hydrofoil. At one point on a hydrofoil, the pressure is -75 kPa (gage). Calculate the speed of the water relative to the hydrofoil at this point and the absolute water speed.



Bernoulli between (1) and stagnation point (2):

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^{2\,\nearrow =0} \qquad p_1\,(\text{gage}) = \rho g D = 29.4\ \text{kPa}$$

$$\therefore\ p_2 = \rho g D + \frac{1}{2}\rho V_1^2 = 998\frac{kg}{m^3}\cdot 9.81\frac{m}{s^2}\cdot 3m + \frac{1}{2}\,998\frac{kg}{m^3}\cdot\left(20\frac{m}{s}\right)^2$$

$$= 2.2897\times 10^5\ \frac{N}{m^2} = 229\ \text{kPa}$$

Bernoulli between (1) and (3)

$$p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^2$$

$$\therefore\ V_3 = \sqrt{\frac{2}{\rho}(p_1 - p_3) + V_1^2} = 24.7\ \frac{m}{s}$$ speed relative to hydrofoil

absolute water speed @ (3) $V_{3,abs} = V_3 - V_1 = 4.7\ \frac{m}{s}$

Problem 10.4

The tank, of diameter D, has a round nozzle with diameter d, as shown in the figure below. At $t = 0$, the water level is at height h_0. Develop an expression for dimensionless water height, h/h_0, versus time as the tank drains.

(a) For $D/d = 10$ plot h/h_0 as a function of time with h_0 as a parameter for $0.1 \le h_0 \le 1\,m$ (use at least three values of h_0).

(b) For $h_0 = 1\,m$, , plot h/h_0 as a function of time with D/d as a parameter for $2 \le D/d \le 10$. (use at least three values of D/d).

(It is suggested that you do this in a spreadsheet, so it can easily be modified. This will be helpful, since this calculation will also be used in a course experiment.)

(c) Compare your result to the solution of HW problem 5.2 (integral method).

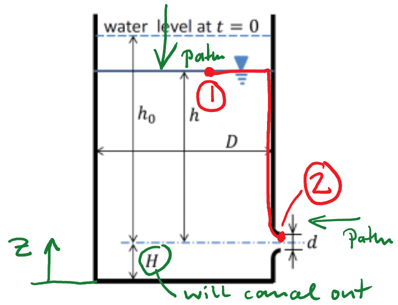


Figure 2: Sketch of tank with diameter D and hole/nozzle with diameter d.

Write Bernoulli equation along a streamline between a point on the surface and the nozzle.

Assumptions:
- inviscid, incompressible flow along a streamline
- quasi-steady: acceleration in tank can be neglected.
- $p_1 = p_2 = p_{atm}$

Continuity: $\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2 \quad \therefore \quad V_2 = \frac{A_1}{A_2} V_1 = \frac{D^2 \frac{\pi}{4}}{d^2 \frac{\pi}{4}} V_1 = \left(\frac{D}{d}\right)^2 V_1$

Bernoulli: $\frac{1}{2} V_1^2 + \cancel{\frac{p_1}{\rho}} + g z_1 = \frac{1}{2} V_2^2 + \cancel{\frac{p_2}{\rho}} + g z_2$

Note: If $D \gg d$, we often neglect V_1, and $V_2 = \sqrt{2gh}$. Here, however, $V_1 = \frac{dh}{dt}$ is what we are after...

$$\therefore \quad V_2^2 - V_1^2 = 2g(z_1 - z_2) = 2g\left((H+h) - H\right) = 2gh$$

w/ continuity: $V_1^2 \left(\left(\frac{D}{d}\right)^4 - 1\right) = 2gh$

$$V_1 = -\frac{dh}{dt} = \left[\frac{2gh}{\left(\frac{D}{d}\right)^4 - 1}\right]^{\frac{1}{2}}$$

Separate variables: $-h^{-\frac{1}{2}}\, dh = \left[\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}\right]^{\frac{1}{2}} dt$

integrate: $-2 h^{\frac{1}{2}} = \left[\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}\right]^{\frac{1}{2}} t + C_1$

initial condition: at $t = 0$, $h = h_0$: $-2 h_0^{\frac{1}{2}} = C_1$

$$\therefore \quad h^{\frac{1}{2}} = h_0^{\frac{1}{2}} - \frac{1}{2}\left[\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}\right]^{\frac{1}{2}} t$$

(square root of height varies linearly w/ time)

$$\therefore \quad h^{\frac{1}{2}} = h_0^{\frac{1}{2}} - \frac{1}{2}\left[\frac{2g}{\left(\frac{D}{d}\right)^4-1}\right]^{\frac{1}{2}} t$$

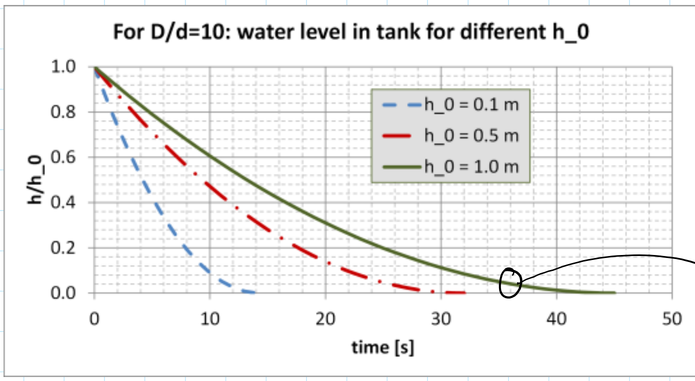
(square root of height varies linearly w/ time)

$$\boxed{\therefore \quad h(t) = \left\{ h_0^{\frac{1}{2}} - \left[\frac{g}{2\left(\left(\frac{D}{d}\right)^4-1\right)}\right]^{\frac{1}{2}} \cdot t \right\}^2}$$

or:
$$\frac{h(t)}{h_0} = \left\{ 1 - \left[\frac{g}{2h_0\left(\left(\frac{D}{d}\right)^4-1\right)}\right]^{\frac{1}{2}} \cdot t \right\}^2$$

(a) For $D/d = 10$, plot (h/h_0) vs. t for $0.1 \le h_0 \le 1.0\,\text{m}$

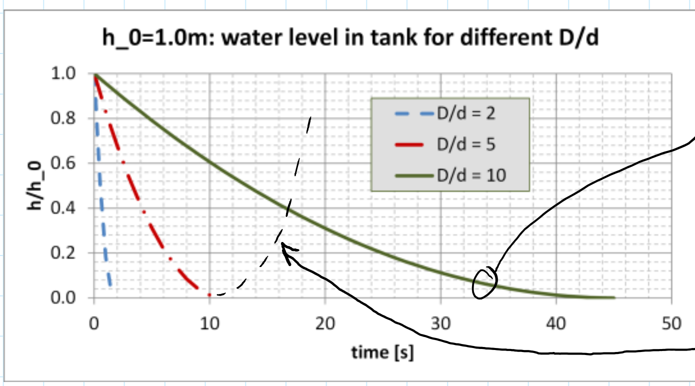
(b) For $h_0 = 1.0\,\text{m}$, plot (h/h_0) vs. t for $2 \le \frac{D}{d} \le 10$



Notes:

(1) $h_0 = 1.0\,\text{m}$, $\frac{D}{d} = 10$ should be the same in both plots

(2) the solution is a quadratic equation with a horizontal tangent (minimum) at $h = 0$. You need to "cut off" the plot when h reaches 0, since it will increase again after that for increasing t.



(c) For HW Problem 5.2, the average outflow velocity was given in terms of the Reynolds number $Re_d = \frac{V_2 d}{\nu}$ at the moment the valve at 2 was opened. Since the outflow velocity was given, we were able to use just conservation of mass

The result was:
$$\left.\frac{dz}{dt}\right|_{t=0} = V_2 \frac{\pi R^2}{WL}$$
where $2R = d$ and WL = area of water surface in tank

or: $V_1 WL = V_2 \pi R^2$

water surface
in tank

or: $v_1 \, W L = v_2 \, \pi R^2$

$$\boxed{v_1 A_1 = v_2 A_2}$$

clearly, this is equivalent to the continuity eqn. above

Now, we have: $v_1 = -\dfrac{dh}{dt} = \left[\dfrac{2gh}{\left(\frac{D}{d}\right)^4 - 1}\right]^{\frac{1}{2}}$ with: $2gh = v_2^2 - v_1^2$

$$\therefore \; v_1^2 = \frac{v_2^2 - v_1^2}{\left(\frac{D}{d}\right)^4 - 1} \quad \text{or:} \quad v_1^2\left[\left(\frac{D}{d}\right)^4 - 1\right] = v_2^2 - v_1^2$$

$$v_1 \left(\frac{D}{d}\right)^2 = v_2 \iff v_1 D^2 = v_2 d^2 \quad \text{or:} \quad \boxed{v_1 A_1 = v_2 A_2}$$

10.5

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Problem 10.5

A nozzle flowmeter is a device for measuring the flow rate in a pipe. The pipe diameter is D_1, the nozzle diameter is D_2. This particular nozzle is to be used to measure low-speed air flow for which compressibility may be neglected. During operation, the pressures upstream and downstream of the nozzle, p_1 and p_2, are recorded, as well as upstream temperature, T_1.

(a) Find the mass flow rate in terms of $\Delta p = p_2 - p_1$, and T_1, the gas constant for air, and device diameters D_1 and D_2. Assume the flow is frictionless.

(b) Will the actual flow be more or less than this predicted flow? Why?

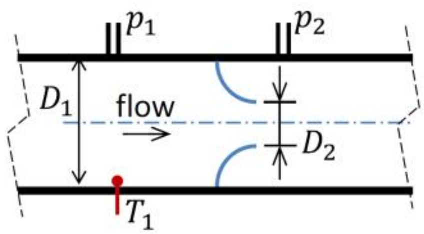
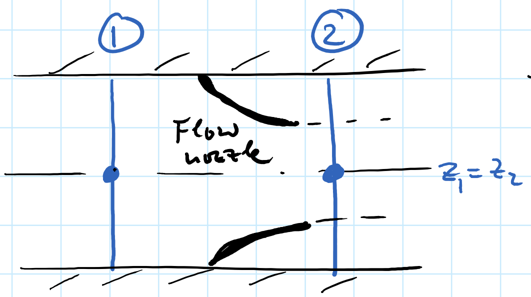


Figure 3: Sketch of nozzle flowmeter.

(a)



Assumptions:

- p_1, p_2 constant across cross-sections at locations 1 and 2
- $p = \rho R T$ (ideal gas)

continuity:

$$\dot m_1 = \dot m_2 = \dot m = \rho_1 A_1 V_1 = \frac{p_1}{RT_1} D_1^2 \frac{\pi}{4} V_1 \quad (*)$$

Note: This will be the answer, need to express V_1 in terms of $p_1 - p_2$

$$\cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 V_2$$

w/ $\rho_1 \simeq \rho_2$:
$$V_2 = \frac{A_1}{A_2} V_1$$

Bernoulli:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2}$$

$$(p_1 - p_2) = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$= \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}V_1\right)^2 - V_1^2\right] = \frac{1}{2}\rho V_1^2\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]$$

$$V_1 = \left(\frac{2(p_1 - p_2)}{\rho\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]}\right)^{\frac{1}{2}}$$

back into (*)

$$\dot m = \rho_1 A_1 V_1 = \rho D_1^2 \frac{\pi}{4}\left(\frac{2(p_1 - p_2)}{\rho\left[\left(D_1/D_2\right)^4 - 1\right]}\right)^{\frac{1}{2}} = D_1^2\frac{\pi}{4}\left[\frac{2\rho(p_1 - p_2)}{\left(D_1/D_2\right)^4 - 1}\right]^{\frac{1}{2}} =$$

$$= D_1^2 \frac{\pi}{4}\left[\frac{p_1 (p_1 - p_2)}{RT_1\left[\left(D_1/D_2\right)^4 - 1\right]}\right]^{\frac{1}{2}}$$

$$= D_1^2 \frac{\pi}{4} \left[\frac{p_1 (p_1 - p_2)}{RT_1 \left[\left(\frac{D_1}{D_2}\right)^4 - 1 \right]} \right]^{1/2}$$

for an ideal gas (air)

for any fluid

(b) Actual flow rate will be less than predicted flow rate due to friction losses.

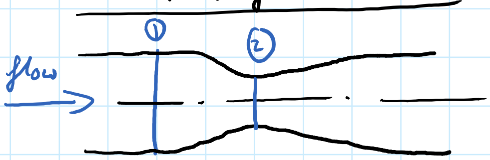
In practice this is handled by introducing a "flow meter coefficient", such that

$$C_m = \frac{\dot{m}_{actual}}{\dot{m}_{ideal}}$$

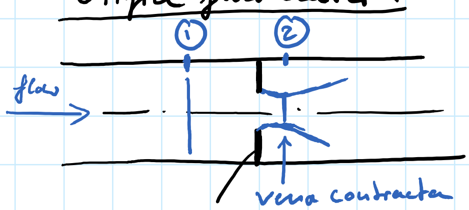
where $\dot{m}_{ideal}$ is what was calculated above.

⇒ The nozzle flow meter is one type in a class of variable area flow meters, where pressure measurements and the Bernoulli equation are used to measure flow rates. Other flow meters in this class:

Venturi flow meter:



Orifice flow meter:



(The flow nozzle is – sort of – a cross between the two.)