Functions, Nested Loops, and Recursion

MATH-151: Mathematical Algorithms in Matlab

September 11, 2023

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  - Furthermore, we would see `factorial(n)` in each loop of that sum, which is another function that replaces a set of code!
- Matlab has many built-in functions to perform commonly seen computations, but it also allows us to create our own functions!

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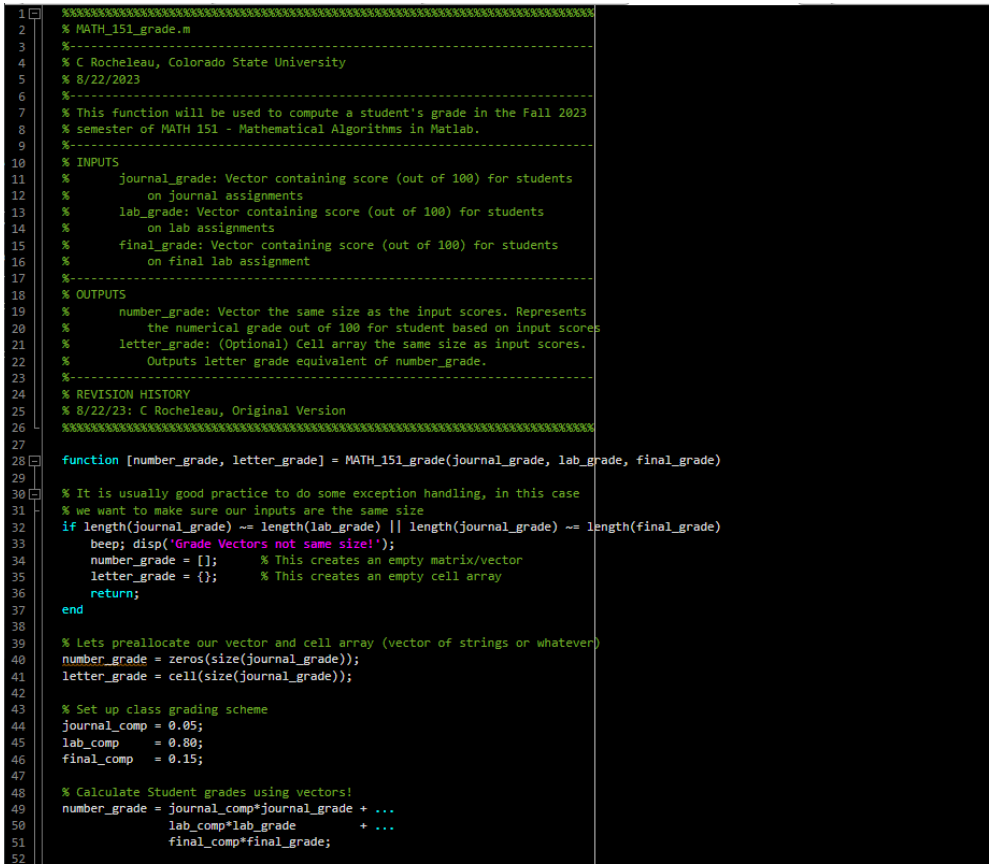
  - `input` is the input to the function, the code should be expected to run with only the given inputs.
- It is usually good commenting practice to include a preamble describing the function's use and format of the input and output variables.

Function Example

Here is an example script for a function to calculate your grade in this class



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% MATH_151_grade.m
%-------------------------------------------------------------------------
% C Rocheleau, Colorado State University
% 8/22/2023
%-------------------------------------------------------------------------
% This function will be used to compute a student's grade in the Fall 2023
% semester of MATH 151 - Mathematical Algorithms in Matlab.
%-------------------------------------------------------------------------
% INPUTS
%       journal_grade: Vector containing score (out of 100) for students
%           on journal assignments
%       lab_grade: Vector containing score (out of 100) for students
%           on lab assignments
%       final_grade: Vector containing score (out of 100) for students
%           on final lab assignment
%-------------------------------------------------------------------------
% OUTPUTS
%       number_grade: Vector the same size as the input scores. Represents
%           the numerical grade out of 100 for student based on input scores
%       letter_grade: (Optional) Cell array the same size as input scores.
%           Outputs letter grade equivalent of number_grade.
%-------------------------------------------------------------------------
% REVISION HISTORY
% 8/22/23: C Rocheleau, Original Version
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [number_grade, letter_grade] = MATH_151_grade(journal_grade, lab_grade, final_grade)

% It is usually good practice to do some exception handling, in this case
% we want to make sure our inputs are the same size
if length(journal_grade) ~= length(lab_grade) || length(journal_grade) ~= length(final_grade)
    beep; disp('Grade Vectors not same size!');
    number_grade = [];      % This creates an empty matrix/vector
    letter_grade = {};      % This creates an empty cell array
    return;
end

% Lets preallocate our vector and cell array (vector of strings or whatever)
number_grade = zeros(size(journal_grade));
letter_grade = cell(size(journal_grade));

% Set up class grading scheme
journal_comp = 0.05;
lab_comp     = 0.80;
final_comp   = 0.15;

% Calculate Student grades using vectors!
number_grade = journal_comp*journal_grade + ...
               lab_comp*lab_grade         + ...
               final_comp*final_grade;

```

Wait a second, what's going on with the inputs and outputs? Can you have more than one of each?

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  `[number_grade, letter_grade]`
- Similarly, to have multiple inputs we list them in the parentheses of the function call. Again, separated by commas

  `(journal_grade, lab_grade, final_grade)`

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- Inputs may be treated as optional in the function, or outputs may be optional when calling the function, for example

  `number_grade = MATH_151_grade(100,100,100)`

  will only output the number grade

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We'll see an example of this shortly, but while we're talking about functions inside functions...

Loops in Loops

- In many real-world cases, we deal with multivariable functions. In those cases, we don't just have a single `x` that we need to iterate across, we also have a `y` that varies as well.
- To calculate all combinations of values from `x` and `y` we need to use a **nested loop**
  - This means that we have a loop inside of a loop
  - We hold the `x` value constant and iterate across all the `y` values, then we move onto the next `x` value and repeat.

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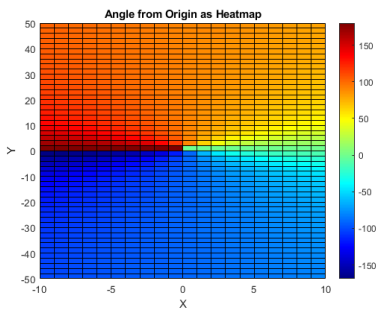
- Suppose we have a grid of points and want to find the angle to each of them from the origin.

```
for iX = 1:length(x)
    for iY = 1:length(y)
        angles(iY,iX) = atan2d(y(iY), x(iX));
    end
end
```



Data Matrices

- This can also be used for Data Matrices. Each row is a different variable, and each column is a different subject.
- We can use a nested loop to perform our Gambler's Ruin game many times and collect statistics!

```
for iSim = 1:num_sims
    money = initial_money;
    for iPlay = 1:num_plays
        if money <= 0           % Did we go broke? Game Over
            money = 0;
            break;
        elseif rand < win_prob      % WINNER!
            money = money + 1;
        else                        % LOSER :(
            money = money - 1;
        end
        money_track(iPlay, iSim) = money; % Update our matrix
    end
    final_money(iSim) = money;      % Save our final money before next sim
end

% What is the average amount of money we ended with?
mean(final_money)       % Computes mean value of the vector we input
```

- In this game we had an average final money count of $5.31

Recursive Relationships

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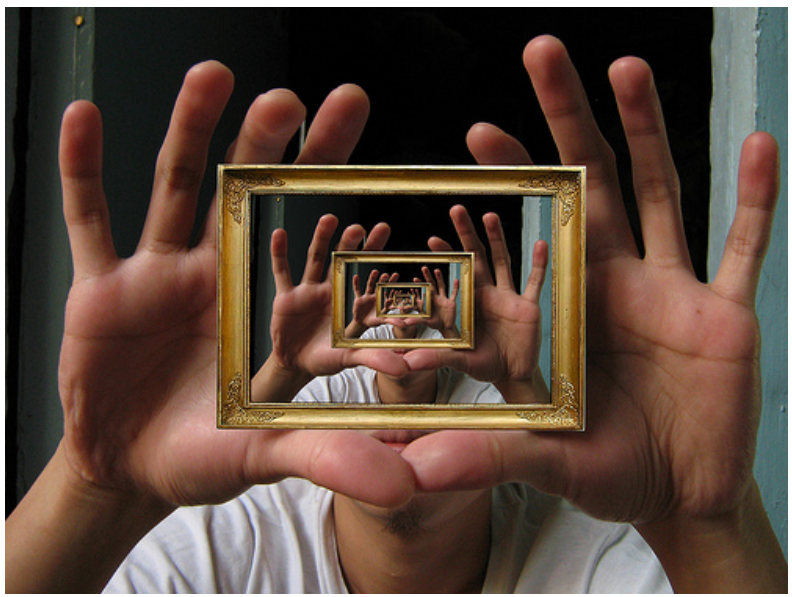

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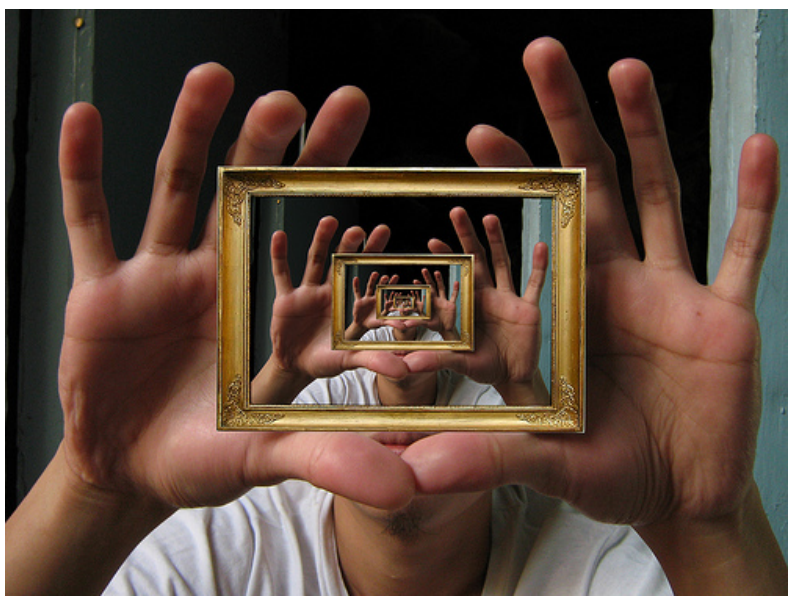

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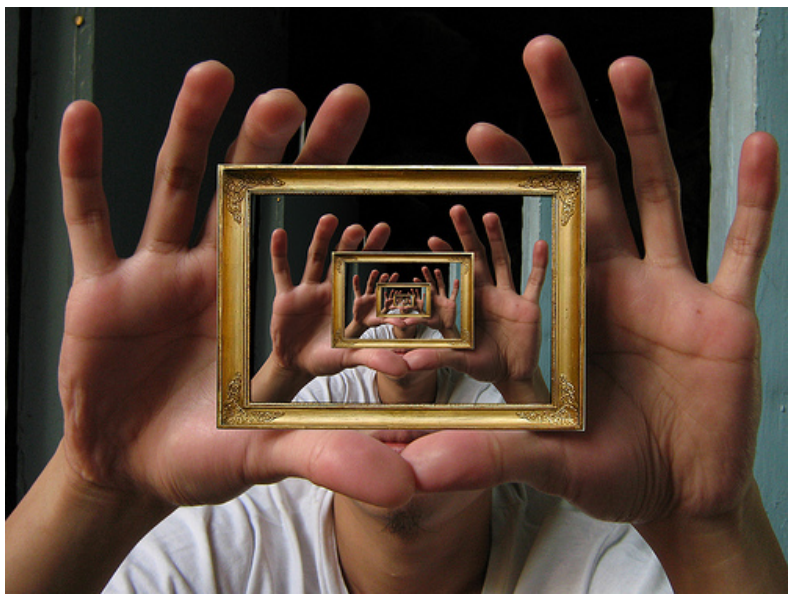
- In these cases, rather than writing a loop, we can create a **recursive function**
  - This is a function that calls on itself
  - Each time we call the function, we will get closer to some base case
  - The answer will "flow up" through all the previous function calls to give us our desired result in the end.



Bisection Method

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- This gives us a new, smaller possible range, so we do Bisection Method on that range
- We continue until $f(m)$ is sufficiently small.