MATH-151 Lab 6: Differentiation Methods

Due: Monday, October 9, 2023, 10:00am

Please perform the following tasks using Matlab, submitting all relevant code. You are welcomed to work with other students, however each student must submit their own unique code.

Task 1: A Speeding Object!

Similar to what we saw last week, it is very common to use these methods with real world measurements to estimate derivatives without having a known function. For this problem, please download Lab06_Data.mat from the course Canvas page and make sure it is located in the same folder as your script. You can load this data into Matlab using `load('Lab06_Data.mat')`, this should add vectors `time` and `pos` to your workspace.

a) Using the central difference method, calculate and plot your estimate of this object's speed. *(Note: Because the central difference method needs your x value to be be in the center you may use the appropriate finite difference methods for the first and last values)*

b) Say we are also interested in this object's acceleration over time. Calculate and plot an estimate of the second derivative from this data.

Task 2: Method Comparison

We learned that finite difference and central difference methods of numerical differentiation have differing orders of error. Hopefully, this makes some sense mathematically but lets take a second to look at what this means in an example. We will consider the function

$$f(x) = \sin\left(\frac{x}{3}\right)^2$$

with derivative

$$f'(x) = \frac{2}{3}\left(\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)\right)$$

a) Use finite difference method to compute an approximation of $f'(x)$ from $-\pi$ to π using a step size of 0.5. Plot this approximation on the same axis as the true $f'(x)$ function given above.

b) Now approximate $f'(x)$ using the same x grid using the central difference method. Plot this function on the same axis used in part (a). How do these approximations compare? Which looks more accurate.

c) In a new figure, plot the errors (true values - estimated values) for each of these methods. Is there a difference between these methods? Which is more accurate?

Remember to label your plots correctly!