AOE/CS/ME 6444 Verification and Validation in Scientific Computing
Instructor: Dr. Chris Roy

2D Heat Conduction Code (for use w/ the project)

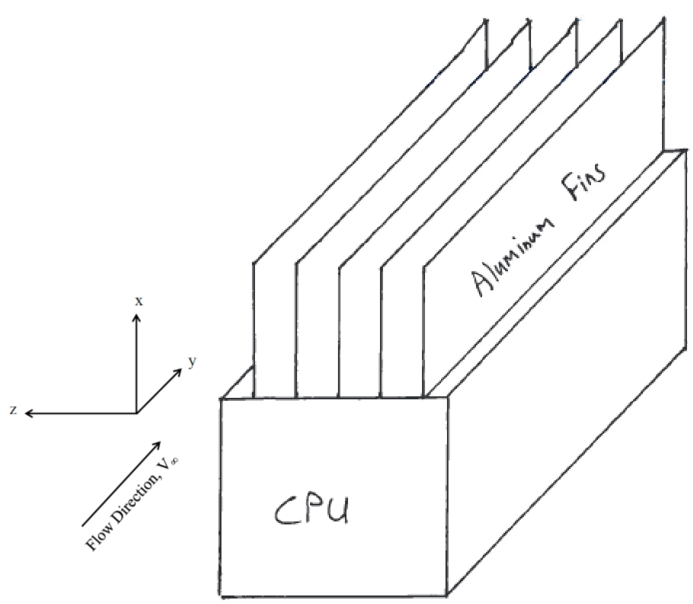
The code that is available to you is set up to solve the 2D version of the heat sink problem you were given in Homework #1. You are free to use the code to solve this problem or to modify it to solve some other 2D steady heat conduction problem. By removing the convective cooling source term, you can easily turn this code into a 2D code which solves for the temperature in either an insulated plate (in z direction) or an infinite slab (in z direction).

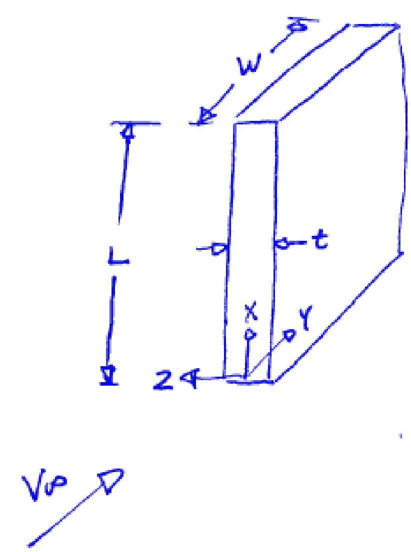
The current problem that the code is set up to solve is as follows. Consider that a high-performance CPU has a Thermal Design Power (*TDP*) of 125 W. An aluminum heat sink with 5 fins sits on top of the CPU and air blows over this heat sink at V_∞ = 1 m/s to dissipate the heat as shown in the left figure. The air flow is assumed to be laminar, and the convective cooling is found to be

$$h(y) = 0.332 k_{air} \Pr^{1/3} \left[\frac{\rho_{air} V_\infty}{\mu_{air}(y + t/2)} \right]^{1/2} \quad \frac{W}{m^2 K}$$

and the convective cooling is then a function of the spatial location as

$$\dot{q}_C(x, y) = h(y)\left[T(x, y) - T_\infty\right].$$





To simplify the analysis, only one aluminum fin is considered. The convective heat flux from the sides of the aluminum fin is found as specified above and results in a source term being added to the governing equation. The governing equation is thus

$$k_{Metal}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{2h(y)}{t}\left[T(x,y)-T_\infty\right]$$

where t is the thickness of the plate. Some example boundary conditions are fixed heat flux at the base ($x = 0$)

$$k_{Metal}\left.\frac{\partial T}{\partial x}\right|_{x=0}=-\frac{TDP}{W\cdot t\cdot N_{fins}}$$

and a convective cooling boundary condition at the end of the fin ($x = L$)

$$k_{Metal}\left.\frac{dT}{dx}\right|_{x=L}=-h(y)\left[T(x=L,y)-T_\infty\right].$$

Consider the conditions given in the table below (these are the default values in the MATLAB code I will supply). Note that the base temperature is returned by the function *heatcondsolve* as the variable *Tbase*. The default numerical parameters are: 33×33 nodes and a relative iterative convergence tolerance of 1×10^{-8}.

Table of Input Parameters

Thermal Design Power, *TDP*	125 W
Number of fins, *NumFins*	5
Total length of the fin, *L*	6 cm
Total width of the fin, *W*	8 cm
Fin thickness, *thick*	0.3 cm
Thermal conductivity of Aluminum, *k_Metal*	160 W/(m K)
Thermal conductivity of air, *k_air*	0.028 W/(m K)
Freestream air temperature, *Tinf*	300 K