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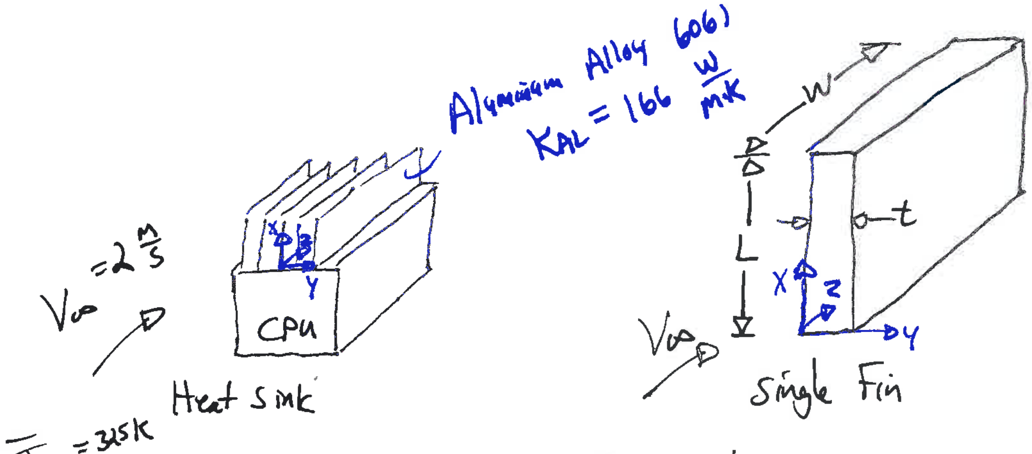
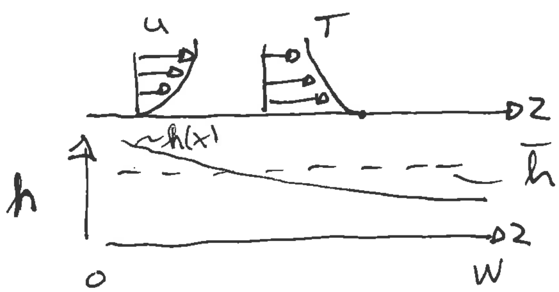


Thermal Design Power (TDP) = 100 W

Assumptions: 1) $t \ll L$ $\therefore$ neglect temp. variations in Y

2) integrated convective cooling coeff & T_∞ in Z



for laminar flow, we have

$$St \cdot Pr^{2/3} = \frac{C_f}{2} \quad \text{or} \quad \frac{Nu}{Re\,Pr} Pr^{2/3} = \frac{C_f}{2}$$

$$Nu = Re\,Pr^{1/3} \frac{C_f}{2}$$

Blasius: $C_f(x) = 0.664\, Re_x^{-1/2}$

$$\therefore \quad Nu(x) = Re(x) \cdot 0.332\, Pr^{1/3}\, Re(x)^{-1/2}$$

$$Nu(x) = 0.332\, Pr^{1/3} \left[Re(x)\right]^{1/2}$$

$$Re(x) = \frac{\rho V_\infty x}{\mu}, \quad Pr = \frac{\mu c_p}{k}, \quad Nu(x) = \frac{hx}{k}$$

$$\dot{q}_{conv} = -k \frac{\partial T}{\partial y}\Big|_w = h(T_w - T_\infty); \quad h(x) = \frac{k}{x}(0.332)\, Pr^{1/3} \left(\frac{\rho V_\infty x}{\mu}\right)^{1/2}$$

$$= 0.332 \cdot k \cdot Pr^{1/3} \left(\frac{\rho V_\infty}{\mu x}\right)^{1/2}$$

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$$\bar{h} = \frac{1}{L}\int_0^L h(x)\,dx = \frac{1}{L}(0.332)\cdot k\cdot P_r^{1/3}\left(\frac{\rho V_\infty}{\mu}\right)^{1/2}\int_0^L x^{-1/2}\,dx$$

$$\bar{h} = \frac{0.332}{L}\, k\, P_r^{1/3}\left(\frac{\rho V_\infty}{\mu}\right)^{1/2} 2x^{1/2}\Big|_0^L$$

$$= 0.664\,\frac{k P_r^{1/3}}{L}\left(\frac{\rho V_\infty L}{\mu}\right)^{1/2}$$

$V_\infty = 2$ m/s

$\therefore$, for $L = 6$ cm $= 0.06$ m (W)

Air: @~300K–350K

$\rho_{air} = \cancel{1.16}\ 1.05\ \frac{kg}{m^3}$

$C_p = 1.008\ \frac{kJ}{kg\cdot K}$

$\mu = 1.95E{-5}\ \frac{N\cdot s}{m^2}$

$k = 2.8E{-2}\ \frac{W}{m\cdot K}$

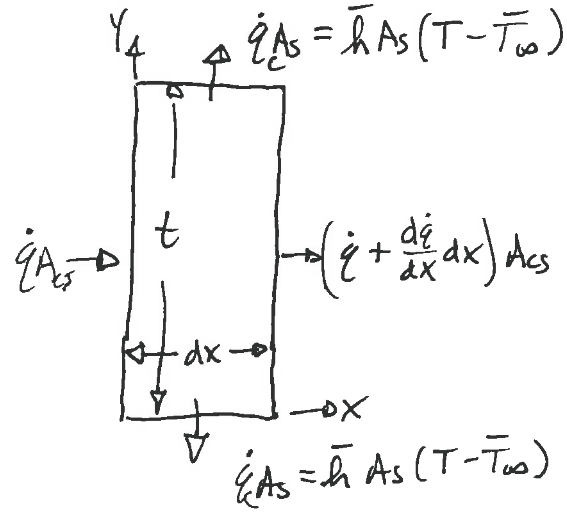
$P_r = 0.704$

$$\bar{h} = 0.664\,\frac{(2.8E{-2}\ \frac{W}{m\cdot K})(0.704)^{1/3}}{0.06\text{ m}}\left(\frac{1.05\ \frac{kg}{m^3}\cdot 2\frac{m}{s}\cdot 0.06\text{m}}{1.95E{-5}\ \frac{N\cdot s}{m^2}}\right)^{1/2}$$

$$\frac{\frac{kg}{m\cdot s}}{\frac{kg\cdot m}{m\cdot s}}$$ ✓

$$\bar{h} = 0.2756\,\frac{W}{m^2K}\left[6461.5\right]^{1/2} = \boxed{22.2\ \frac{W}{m^2 K} = \bar{h}}$$

* ~~Contact~~ Differential CV analysis [W is length into page (z-direction)]



$A_s = W\,dx$

$A_{cs} = Wt$

Energy Balance: $\left[\dot{q} - \left(\dot{q} + \frac{d\dot{q}}{dx}dx\right)\right] A_{cs} - 2\,\dot{q}_c A_s = 0$

$$-\frac{d\dot{q}}{dx} = 2\,\dot{q}_c\,\frac{A_s}{A_{cs}\,dx} = 2\,\dot{q}_c\,\frac{W dx}{W\cdot t\cdot dx} = \frac{2\,\dot{q}_c}{t}$$

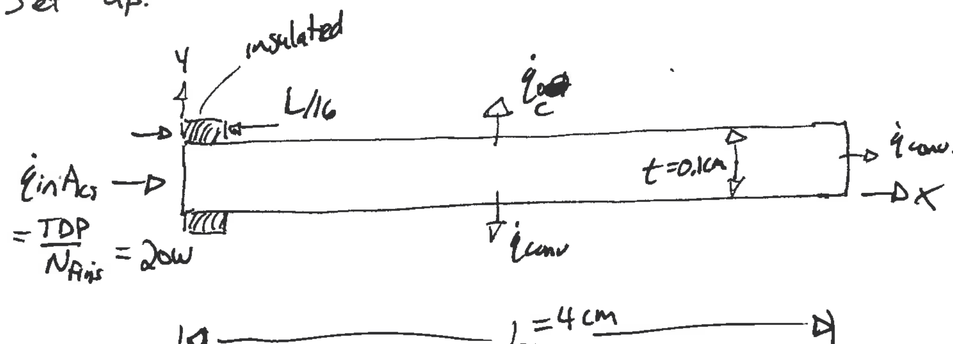
Since $\dot{q} = -k_{AL}\frac{dT}{dx}$,

$$-\frac{d}{dx}\left(-k_{AL}\frac{dT}{dx}\right) = \frac{2\dot{q}_c}{t}$$

$$k_{AL}\frac{d^2T}{dx^2} = \frac{2\dot{q}_c}{t} \qquad \dot{q}_c = \bar{h}(T - T_\infty)$$

Gov. Eqn $\quad k_{AL}\frac{d^2T}{dx^2} = \frac{2\bar{h}}{t}(T - T_\infty)$

Problem Set Up:



$N_{fins} = 5$

$\dot{q} = -k\frac{dT}{dx}$

$$k_{AL}\frac{d^2T}{dx^2} = \frac{2\bar{h}}{t}(T - T_\infty) \qquad \bar{h} = \begin{cases} 0 & 0 \le x < \frac{L}{16} \\ 22.2\,\frac{W}{m^2K} & \frac{L}{16} \le x \le L \end{cases}$$

BCs: $x = 0$:

$$\dot{q}_{in}A_{cs} = \frac{TDP}{N_{fins}} = 20\,W$$

$$\dot{q}_{in}A_{cs} = -k_{AL}\frac{dT}{dx} = 20W$$

$$\boxed{\left.\frac{dT}{dx}\right|_{x=0} = -\frac{TDP}{k_{AL}N_{fins}} = \frac{-20W}{k_{AL}}}$$

$x = L$: $\quad \dot{q}_{out}A_{cs} = A_{cs}\bar{h}(T - T_\infty)$

$$-k_{AL}\left.\frac{dT}{dx}\right|_{x=L} = \bar{h}(T - T_\infty)$$

$$\boxed{\left.\frac{dT}{dx}\right|_{x=L} = \frac{-\bar{h}}{k_{AL}}(T_L - T_\infty)}$$

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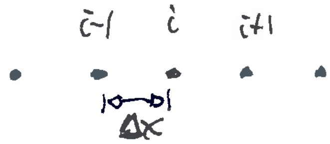


Discretization: finite difference $\qquad 1 \le i \le i_{max}$

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} = S(x_i)$$

$$S(x_i) = \frac{2\bar{h}}{k_{AL} \cdot t}\left(T_i - T_\infty\right)$$

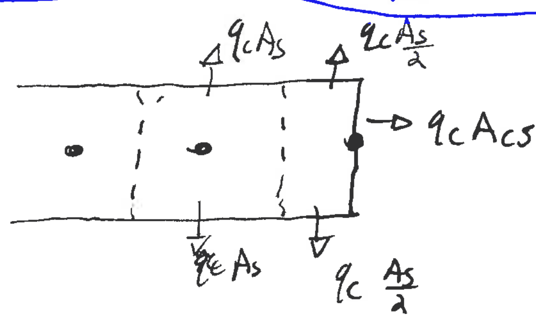
BCs: $i=1 \quad (x=0) \qquad \left.\frac{dT}{dx}\right|_{x=0} = \frac{-TDP}{k_{AL} \cdot N_{fins}}$

$$\boxed{i=1 \qquad \frac{-T_{i+2} + 4T_{i+1} - 3T_i}{2\Delta x} = \frac{-TDP}{k_{AL} \cdot N_{fins}}}$$

$i=i_{max} \quad (x=L) \qquad \left.\frac{dT}{dx}\right|_{x=L} = -\frac{\bar{h}}{k_{AL}}\left(T(x=L) - T_\infty\right)$

$$\boxed{i=i_{max} \qquad \frac{3T_i - 4T_{i-1} + T_{i-2}}{2\Delta x} = -\frac{\bar{h}}{k_{AL}} * \left(\frac{\Delta x}{t}+1\right)\left(T_i - T_\infty\right)}$$

Note: for end:



$A_s = w\Delta x$

$A_{cs} = wt$

$\therefore$ $x=L$ BC should be $-k_{AL}\left.\frac{dT}{dx}\right|_{x=L} \cdot A_{cs} = 2\left(\dot{q}_c \frac{A_s}{2}\right) + \dot{q}_c A_{cs}$

$$\left.\frac{dT}{dx}\right|_{x=L} = \frac{-\dot{q}_c A_s}{k_{AL} A_{cs}} - \frac{\dot{q}_c}{k} = \frac{-\dot{q}_c}{k}\left(\frac{\Delta x}{t}+1\right)$$

$$\dot{q}_c = \bar{h}\left(T - T_\infty\right)$$

$$\left.\frac{dT}{dx}\right|_{x=L} = -\frac{\bar{h}}{k_{AL}}\underbrace{\left(\frac{\Delta x}{t}+1\right)}_{*\text{new factor}}\left(T(x=L) - T_\infty\right)$$

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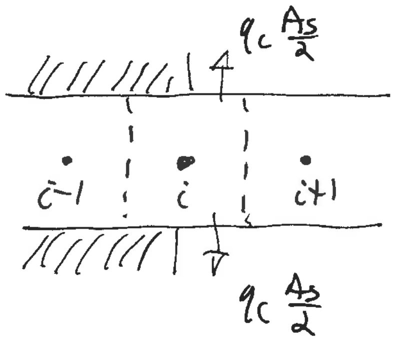
Note: for point @ end of insulation:



∴ need to divide source term by 2

$\frac{\cancel{J}\ \cancel{m}}{s \cdot \cancel{m} \cdot K} \frac{m^{\cancel{3}\,2}}{\cancel{kg}} \frac{\cancel{kg} \cdot \cancel{K}}{\cancel{J}} =$

$\alpha_{AL} = \frac{k}{\rho C_p} = 6.9E\text{-}5 \ \frac{m^2}{s}$ | $S(x_{i=\text{interface}}) = \frac{\bar{h}}{k_{AL} \cdot t}(T_i - \bar{T}_\infty)$

== Inputs ==

* Material Parameters *

- $K_{AL} = 166 \ \frac{W}{m \cdot K}$
- $\rho_{AL} = 2.7 \frac{g}{cm^3} = \frac{1 kg}{1000 g}\left(\frac{100 cm}{1 m}\right)^3 = 2700 \frac{kg}{m^3}$
- $C_{P_{AL}} = 895 \ \frac{J}{kg \cdot K}$

* Model Parameters *

- $\bar{h} = 22.2 \ \frac{W}{m^2 \cdot K}$
- $\bar{T}_\infty = 325$ K (average air temp)

* System Parameters *

- $V_\infty = 2$ m/s
- TDP = 100 W (Thermal Design Power)
- $N_{fins} = 5$
- W = 6 cm
- L = 4 cm
- t = 0.1 cm
- $L_{ins} = \frac{L}{16} = 0.25$ cm

* Discretization Parameters *

- $i_{max} =$ ~~16·N+1, N=1,2,3,~~ ≥ 17 (17, 33, 65, 129, etc)

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* Stability

1D unsteady heat conduction

$$\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = 0$$

$$\alpha = \frac{K}{\rho C_p}$$

$$\Delta t \le \frac{1}{2} \frac{\Delta x^2}{\alpha}$$

rewrite eqn as: $\frac{\partial T}{\partial t} - \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial x^2} = \text{src.}$

$$\rho C_p \frac{\partial T}{\partial t} - K \frac{\partial^2 T}{\partial x^2} = -S(T) = -\frac{2\bar{h}}{t}(T - T_\infty)$$

$$\frac{K}{\alpha} \frac{\partial T}{\partial t} - K \frac{\partial^2 T}{\partial x^2} = -S(T)$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - \frac{\alpha}{K} S(T)$$

$$S(T) = \frac{2\bar{h}}{t}(T - T_\infty)$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - \frac{2\alpha \bar{h}}{K \cdot t}(T - T_\infty) = RHS$$

$$\frac{T_i^{K+1} - T_i^K}{\Delta t} = RHS_i$$

$$T_i^{K+1} = T_i^K + \Delta t * RHS_i$$

BC Implementation

2/2/2022

$*\ i=1$

$$3T_i - 4T_{i+1} + T_{i+2} = 2\,\Delta x\,\frac{TDP}{K_{AL}\,N_{fins}}$$

$$T_i = \frac{1}{3}\left[4T_{i+1} - T_{i+2} + 2\,\Delta x\,\frac{TDP}{K_{AL}\,N_{fins}}\right]$$

$*\ i = i_{max}$

$$3T_i - 4T_{i-1} + T_{i-2} = -\underbrace{\frac{2\,\Delta x\,\bar{h}}{K_{AL}}\left(\frac{\Delta x}{t}-1\right)}_{\psi}(T_i - T_\infty)$$

$$(3+\psi)\,T_i = 4T_{i-1} - T_{i-2} + \psi T_\infty$$

$$T_i = \frac{1}{3+\psi}\left[4T_{i-1} - T_{i-2} + \psi T_\infty\right]$$