

Confirmatory Structure Analysis Demonstration

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```
library(lavaan)
library(lavaanPlot)
library(jmv)
library(psych)
```

Load in Data

```
wiscsem <- read.csv("wiscsem.csv")[,-c(1:2,13)]
cogability <- read.csv("cogability.csv")[,c(2:10)]
```

Descriptive Statistics

```
describe(wiscsem)
```

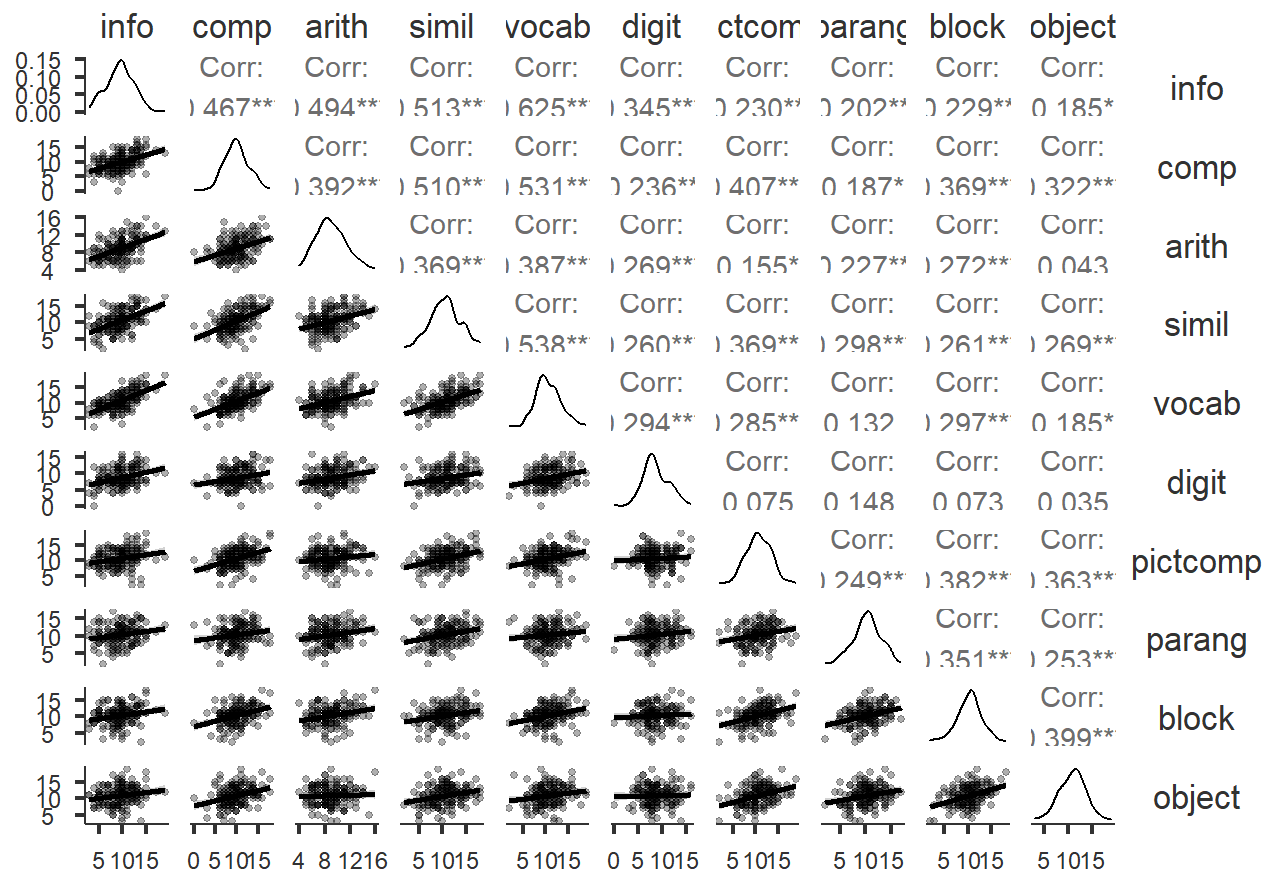
```
##          vars   n mean   sd median trimmed  mad min max range  skew kurtosis
## info         1 175  9.50 2.91    10    9.50 2.97   3  19   16  0.08   -0.08
## comp         2 175 10.00 2.97    10    9.95 2.97   0  18   18  0.09    0.33
## arith         3 175  9.00 2.31     9    8.89 2.97   4  16   12  0.39   -0.18
## simil         4 175 10.61 3.18    11   10.62 2.97   2  18   16  0.02   -0.23
## vocab         5 175 10.70 2.93    10   10.61 2.97   2  19   17  0.27    0.29
## digit         6 175  8.73 2.70     8    8.65 1.48   0  16   16  0.27    0.07
## pictcomp      7 175 10.68 2.93    11   10.70 2.97   2  19   17 -0.07    0.29
## parang        8 175 10.37 2.66    10   10.43 2.97   2  17   15 -0.20   -0.06
## block         9 175 10.31 2.71    10   10.36 2.97   2  18   16 -0.22    0.50
## object       10 175 10.90 2.84    11   10.94 2.97   3  19   16 -0.12    0.15
##              se
## info         0.22
## comp         0.22
## arith        0.17
## simil        0.24
## vocab         0.22
## digit        0.20
## pictcomp     0.22
## parang       0.20
## block        0.20
## object       0.21
```

```
describe(cogability)
```

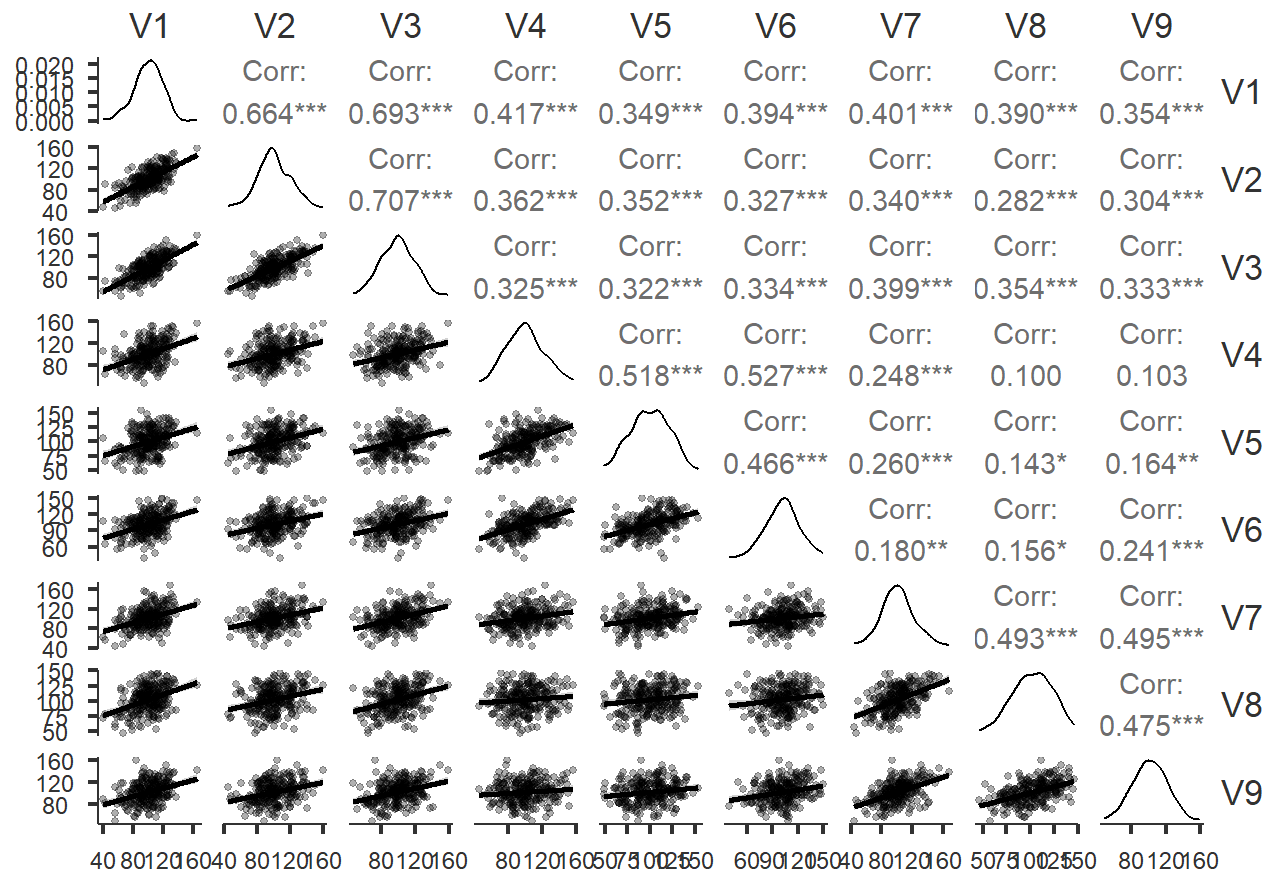
```
##      vars    n   mean    sd median trimmed   mad   min    max range  skew
## V1      1 250  99.91 18.60 100.91  100.63 17.99 39.86 165.46 125.60 -0.29
## V2      2 250 100.21 19.89  99.44  100.04 19.43 44.66 159.19 114.54  0.06
## V3      3 250  99.21 19.36  98.88   99.26 20.52 47.98 158.36 110.38  0.01
## V4      4 250 100.51 21.55  99.66   99.99 21.13 45.82 157.00 111.17  0.20
## V5      5 250  99.99 21.19  99.80  100.26 20.79 48.95 153.97 105.02 -0.09
## V6      6 250 101.84 19.95 101.95  101.92 19.28 38.42 150.38 111.96 -0.10
## V7      7 250 101.16 20.58 101.38  100.72 19.44 44.60 167.07 122.47  0.19
## V8      8 250 102.37 20.81 103.58  102.80 21.92 46.30 145.86  99.56 -0.19
## V9      9 250 101.34 19.63 101.58  101.41 20.93 48.87 158.29 109.42 -0.03
##      kurtosis   se
## V1      0.45 1.18
## V2      0.09 1.26
## V3     -0.23 1.22
## V4     -0.35 1.36
## V5     -0.57 1.34
## V6      0.02 1.26
## V7      0.26 1.30
## V8     -0.51 1.32
## V9     -0.26 1.24
```

CorrMatrix() output

```
corrMatrix(wiscsem,plots = TRUE,plotDens = TRUE,plotStats = TRUE)$plot
```



```
corrMatrix(cogability, plots = TRUE, plotDens = TRUE, plotStats = TRUE)$plot
```



Confirmatory Factor Analysis (CFA)

Model Specification (i.e., Measurement Model)

One-factor Model:

```
onefactor <- ' # Latent variable
g =~ info + comp + arith + simil + vocab + digit + pictcomp + parang + block + object

# Residuals for manifest variables
info~~info
comp~~comp
arith~~arith
simil~~simil
vocab~~vocab
digit~~digit
pictcomp~~pictcomp
parang~~parang
block~~block
object~~object

# Variances for latent variables
g~~1*g
'
```

Two-factor (Uncorrelated) Model:

```
twofactor <- ' # Latent variables
verbal =~ info + comp + arith + simil + vocab + digit
performance =~ pictcomp + parang + block + object

# Residuals for manifest variables
info~~info
comp~~comp
arith~~arith
simil~~simil
vocab~~vocab
digit~~digit
pictcomp~~pictcomp
parang~~parang
block~~block
object~~object

# Variances for latent variables
verbal~~1*verbal
performance~~1*performance
'
```

Two-factor (Correlated) Model:

```
ctwofactor <- ' # Latent variables
verbal =~ info + comp + arith + simil + vocab + digit
performance =~ pictcomp + parang + block + object

# Latent variable correlation
verbal ~~ performance

# Residuals for manifest variables
info~~info
comp~~comp
arith~~arith
simil~~simil
vocab~~vocab
digit~~digit
pictcomp~~pictcomp
parang~~parang
block~~block
object~~object

# Variances for latent variables
verbal~~1*verbal
performance~~1*performance
'
```

Run the model, summarize the parameter estimates, and plot:

```
# Run the model: One Factor Model
fit <- lavaan(onefactor,
              data = wiscsem,
              std.lv = TRUE)

# Get parameter estimates and fit statistics
summary(fit,
        fit.measures = TRUE,
        standardized = TRUE)
```



```

## lavaan 0.6-19 ended normally after 23 iterations
##
## Estimator ML
## Optimization method NLMINB
## Number of model parameters 20
##
## Number of observations 175
##
## Model Test User Model:
##
## Test statistic 101.635
## Degrees of freedom 35
## P-value (Chi-square) 0.000
##
## Model Test Baseline Model:
##
## Test statistic 503.222
## Degrees of freedom 45
## P-value 0.000
##
## User Model versus Baseline Model:
##
## Comparative Fit Index (CFI) 0.855
## Tucker-Lewis Index (TLI) 0.813
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0) -4082.864
## Loglikelihood unrestricted model (H1) -4032.046
##
## Akaike (AIC) 8205.727
## Bayesian (BIC) 8269.023
## Sample-size adjusted Bayesian (SABIC) 8205.689
##
## Root Mean Square Error of Approximation:
##
## RMSEA 0.104
## 90 Percent confidence interval - lower 0.081
## 90 Percent confidence interval - upper 0.128

```

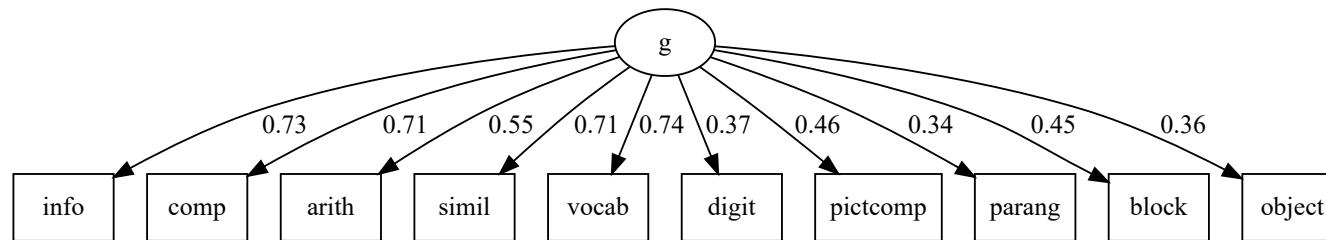
```

## P-value H_0: RMSEA <= 0.050      0.000
## P-value H_0: RMSEA >= 0.080      0.957
##
## Standardized Root Mean Square Residual:
##
## SRMR                                0.081
##
## Parameter Estimates:
##
## Standard errors                      Standard
## Information                          Expected
## Information saturated (h1) model      Structured
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## g =~
##   info      2.110   0.203  10.417   0.000   2.110   0.727
##   comp      2.102   0.208  10.116   0.000   2.102   0.711
##   arith      1.271   0.172   7.370   0.000   1.271   0.552
##   simil      2.265   0.223  10.168   0.000   2.265   0.714
##   vocab      2.172   0.202  10.734   0.000   2.172   0.743
##   digit      1.004   0.212   4.726   0.000   1.004   0.372
##   pictcomp    1.355   0.225   6.011   0.000   1.355   0.463
##   parang      0.898   0.210   4.272   0.000   0.898   0.339
##   block      1.229   0.209   5.890   0.000   1.229   0.455
##   object      1.017   0.224   4.542   0.000   1.017   0.359
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .info      3.981   0.529   7.523   0.000   3.981   0.472
## .comp      4.325   0.563   7.683   0.000   4.325   0.495
## .arith      3.677   0.425   8.643   0.000   3.677   0.695
## .simil      4.946   0.646   7.656   0.000   4.946   0.491
## .vocab      3.833   0.522   7.337   0.000   3.833   0.448
## .digit      6.263   0.689   9.095   0.000   6.263   0.861
## .pictcomp    6.725   0.754   8.913   0.000   6.725   0.786
## .parang      6.226   0.681   9.145   0.000   6.226   0.885
## .block      5.791   0.648   8.933   0.000   5.791   0.793

```

##	.object	7.007	0.769	9.116	0.000	7.007	0.871
##	g	1.000				1.000	1.000

```
lavaanPlot(fit,  
  coefs = TRUE,  
  stand = TRUE)
```



```
# Run the model: Two Factor Model
fit2 <- lavaan(twofactor,
               data = wiscsem,
               std.lv = TRUE)

# Get parameter estimates and fit statistics
summary(fit2,
        fit.measures = TRUE,
        standardized = TRUE)
```

```

## lavaan 0.6-19 ended normally after 24 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters    20
##
##      Number of observations        175
##
## Model Test User Model:
##
##      Test statistic                93.217
##      Degrees of freedom            35
##      P-value (Chi-square)          0.000
##
## Model Test Baseline Model:
##
##      Test statistic                503.222
##      Degrees of freedom            45
##      P-value                        0.000
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)    0.873
##      Tucker-Lewis Index (TLI)      0.837
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)  -4078.655
##      Loglikelihood unrestricted model (H1) -4032.046
##
##      Akaike (AIC)                  8197.309
##      Bayesian (BIC)                 8260.605
##      Sample-size adjusted Bayesian (SABIC) 8197.271
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                        0.097
##      90 Percent confidence interval - lower 0.074
##      90 Percent confidence interval - upper 0.122

```

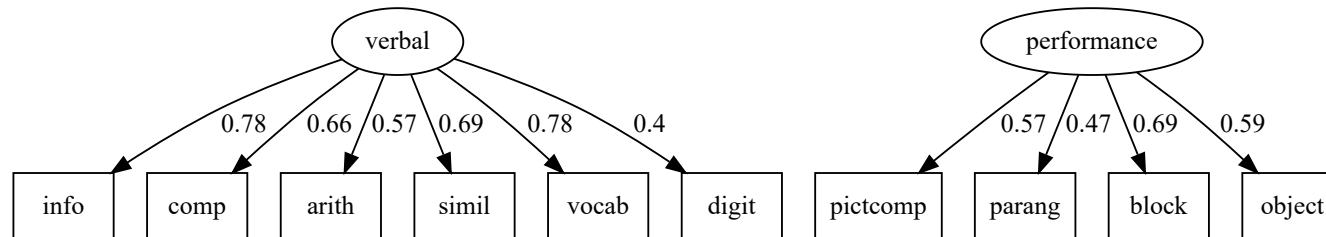
```

## P-value H_0: RMSEA <= 0.050          0.001
## P-value H_0: RMSEA >= 0.080          0.892
##
## Standardized Root Mean Square Residual:
##
## SRMR                                0.160
##
## Parameter Estimates:
##
## Standard errors                      Standard
## Information                          Expected
## Information saturated (h1) model      Structured
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## verbal =~
##   info      2.269   0.199  11.389   0.000   2.269   0.781
##   comp      1.958   0.214   9.160   0.000   1.958   0.662
##   arith      1.309   0.172   7.591   0.000   1.309   0.569
##   simil      2.179   0.227   9.589   0.000   2.179   0.686
##   vocab      2.277   0.201  11.335   0.000   2.277   0.779
##   digit      1.085   0.212   5.118   0.000   1.085   0.402
## performance =~
##   pictcomp   1.664   0.255   6.518   0.000   1.664   0.569
##   parang     1.244   0.233   5.347   0.000   1.244   0.469
##   block      1.868   0.241   7.753   0.000   1.868   0.691
##   object     1.669   0.248   6.732   0.000   1.669   0.589
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##   .info      3.284   0.501   6.550   0.000   3.284   0.389
##   .comp      4.907   0.615   7.979   0.000   4.907   0.561
##   .arith      3.579   0.420   8.521   0.000   3.579   0.676
##   .simil      5.331   0.685   7.782   0.000   5.331   0.529
##   .vocab      3.366   0.510   6.599   0.000   3.366   0.394
##   .digit      6.093   0.675   9.021   0.000   6.093   0.838
##   .pictcomp   5.790   0.807   7.174   0.000   5.790   0.676
##   .parang     5.486   0.676   8.110   0.000   5.486   0.780
##   .block      3.813   0.730   5.224   0.000   3.813   0.522

```

##	.object	5.255	0.760	6.915	0.000	5.255	0.653
##	verbal	1.000				1.000	1.000
##	performance	1.000				1.000	1.000

```
lavaanPlot(fit2,  
  coefs = TRUE,  
  stand = TRUE)
```



```
# Run the model: Two Factor Model
fit2c <- lavaan(ctwofactor,
                data = wiscsem,
                std.lv = TRUE)

# Get parameter estimates and fit statistics
summary(fit2c,
        fit.measures = TRUE,
        standardized = TRUE)
```



```

## lavaan 0.6-19 ended normally after 23 iterations
##
##   Estimator                      ML
##   Optimization method          NLMINB
##   Number of model parameters    21
##
##   Number of observations        175
##
## Model Test User Model:
##
##   Test statistic                55.305
##   Degrees of freedom            34
##   P-value (Chi-square)          0.012
##
## Model Test Baseline Model:
##
##   Test statistic                503.222
##   Degrees of freedom            45
##   P-value                       0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)    0.954
##   Tucker-Lewis Index (TLI)      0.938
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)   -4059.699
##   Loglikelihood unrestricted model (H1) -4032.046
##
##   Akaike (AIC)                   8161.397
##   Bayesian (BIC)                  8227.858
##   Sample-size adjusted Bayesian (SABIC) 8161.357
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                          0.060
##   90 Percent confidence interval - lower 0.028
##   90 Percent confidence interval - upper 0.088

```

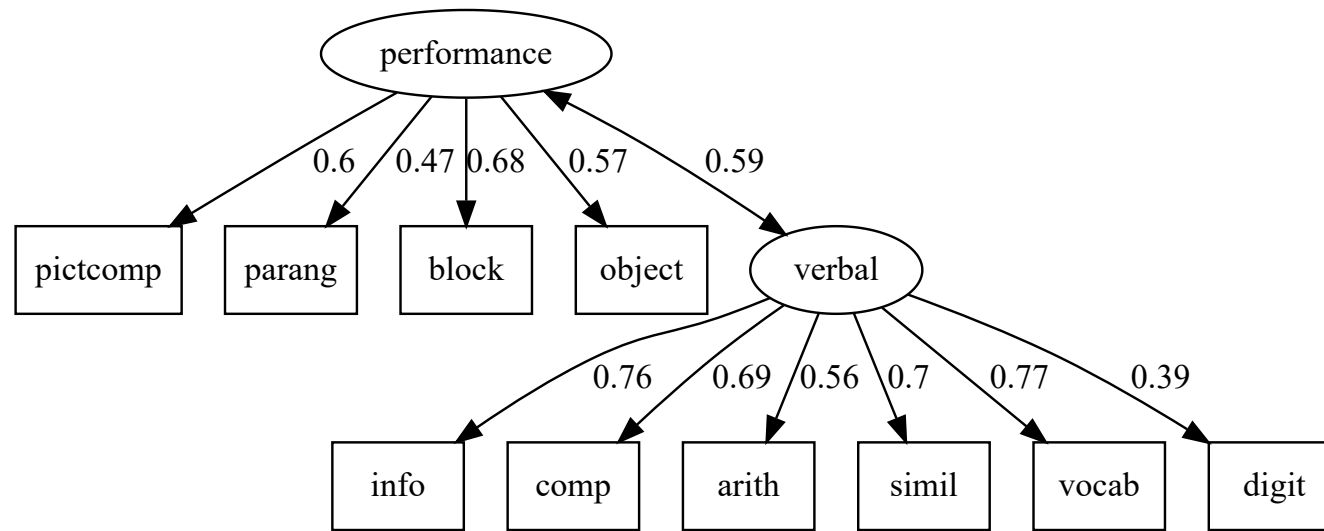
```

## P-value H_0: RMSEA <= 0.050      0.268
## P-value H_0: RMSEA >= 0.080      0.124
##
## Standardized Root Mean Square Residual:
##
## SRMR      0.055
##
## Parameter Estimates:
##
## Standard errors      Standard
## Information          Expected
## Information saturated (h1) model      Structured
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## verbal =~
##   info      2.207   0.200  11.031   0.000   2.207   0.760
##   comp      2.042   0.210   9.710   0.000   2.042   0.691
##   arith      1.299   0.172   7.550   0.000   1.299   0.565
##   simil      2.234   0.224   9.951   0.000   2.234   0.704
##   vocab      2.249   0.201  11.218   0.000   2.249   0.769
##   digit      1.051   0.212   4.960   0.000   1.051   0.390
## performance =~
##   pictcomp    1.761   0.242   7.266   0.000   1.761   0.602
##   parang      1.251   0.225   5.570   0.000   1.251   0.472
##   block       1.833   0.222   8.242   0.000   1.833   0.678
##   object      1.604   0.236   6.794   0.000   1.604   0.566
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## verbal ~~
##   performance    0.589   0.075   7.806   0.000   0.589   0.589
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##   .info         3.564   0.507   7.032   0.000   3.564   0.423
##   .comp         4.572   0.585   7.815   0.000   4.572   0.523
##   .arith        3.604   0.420   8.573   0.000   3.604   0.681
##   .simil        5.087   0.661   7.696   0.000   5.087   0.505

```

##	.vocab	3.491	0.507	6.891	0.000	3.491	0.408
##	.digit	6.165	0.681	9.057	0.000	6.165	0.848
##	.pictcomp	5.460	0.757	7.217	0.000	5.460	0.638
##	.parang	5.467	0.659	8.298	0.000	5.467	0.777
##	.block	3.943	0.641	6.155	0.000	3.943	0.540
##	.object	5.468	0.720	7.595	0.000	5.468	0.680
##	verbal	1.000				1.000	1.000
##	performance	1.000				1.000	1.000

```
lavaanPlot(fit2c,
            coefs = TRUE,
            stand = TRUE,
            covs = TRUE)
```



Direct Model Fit Comparisons (Chi-square Difference Test)

Can compare fit of our 2-factor models because they are nested (same items) AND one has fewer degrees of freedom
`anova(fit2, fit2c)`

```
##
## Chi-Squared Difference Test
##
##           Df      AIC      BIC  Chisq Chisq diff  RMSEA Df diff Pr(>Chisq)
## fit2c 34 8161.4 8227.9 55.305
## fit2 35 8197.3 8260.6 93.217      37.912 0.45927      1 7.402e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(fit, fit2c)
```

```
##
## Chi-Squared Difference Test
##
##           Df      AIC      BIC  Chisq Chisq diff  RMSEA Df diff Pr(>Chisq)
## fit2c 34 8161.4 8227.9 55.305
## fit 35 8205.7 8269.0 101.635      46.33 0.50895      1 9.993e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Then we can call for the summary of the "best" model and the model fit indices
summary(fit2c,
        fit.measures = TRUE,
        standardized = TRUE)
```

```

## lavaan 0.6-19 ended normally after 23 iterations
##
## Estimator ML
## Optimization method NLMINB
## Number of model parameters 21
##
## Number of observations 175
##
## Model Test User Model:
##
## Test statistic 55.305
## Degrees of freedom 34
## P-value (Chi-square) 0.012
##
## Model Test Baseline Model:
##
## Test statistic 503.222
## Degrees of freedom 45
## P-value 0.000
##
## User Model versus Baseline Model:
##
## Comparative Fit Index (CFI) 0.954
## Tucker-Lewis Index (TLI) 0.938
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0) -4059.699
## Loglikelihood unrestricted model (H1) -4032.046
##
## Akaike (AIC) 8161.397
## Bayesian (BIC) 8227.858
## Sample-size adjusted Bayesian (SABIC) 8161.357
##
## Root Mean Square Error of Approximation:
##
## RMSEA 0.060
## 90 Percent confidence interval - lower 0.028
## 90 Percent confidence interval - upper 0.088

```

```

## P-value H_0: RMSEA <= 0.050      0.268
## P-value H_0: RMSEA >= 0.080      0.124
##
## Standardized Root Mean Square Residual:
##
## SRMR      0.055
##
## Parameter Estimates:
##
## Standard errors      Standard
## Information          Expected
## Information saturated (h1) model      Structured
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## verbal =~
##   info      2.207   0.200  11.031   0.000   2.207   0.760
##   comp      2.042   0.210   9.710   0.000   2.042   0.691
##   arith      1.299   0.172   7.550   0.000   1.299   0.565
##   simil      2.234   0.224   9.951   0.000   2.234   0.704
##   vocab      2.249   0.201  11.218   0.000   2.249   0.769
##   digit      1.051   0.212   4.960   0.000   1.051   0.390
## performance =~
##   pictcomp    1.761   0.242   7.266   0.000   1.761   0.602
##   parang      1.251   0.225   5.570   0.000   1.251   0.472
##   block       1.833   0.222   8.242   0.000   1.833   0.678
##   object      1.604   0.236   6.794   0.000   1.604   0.566
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## verbal ~~
##   performance    0.589   0.075   7.806   0.000   0.589   0.589
##
## Variances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
##   .info      3.564   0.507   7.032   0.000   3.564   0.423
##   .comp      4.572   0.585   7.815   0.000   4.572   0.523
##   .arith      3.604   0.420   8.573   0.000   3.604   0.681
##   .simil      5.087   0.661   7.696   0.000   5.087   0.505

```

##	.vocab	3.491	0.507	6.891	0.000	3.491	0.408
##	.digit	6.165	0.681	9.057	0.000	6.165	0.848
##	.pictcomp	5.460	0.757	7.217	0.000	5.460	0.638
##	.parang	5.467	0.659	8.298	0.000	5.467	0.777
##	.block	3.943	0.641	6.155	0.000	3.943	0.540
##	.object	5.468	0.720	7.595	0.000	5.468	0.680
##	verbal	1.000				1.000	1.000
##	performance	1.000				1.000	1.000

```
fitMeasures(fit2c)
```



```

##          npar          fmin          chisq
##          21.000          0.158          55.305
##          df          pvalue    baseline.chisq
##          34.000          0.012          503.222
##    baseline.df    baseline.pvalue          cfi
##          45.000          0.000          0.954
##          tli          nnfi          rfi
##          0.938          0.938          0.855
##          nfi          pnfi          ifi
##          0.890          0.673          0.955
##          rni          logl    unrestricted.logl
##          0.954        -4059.699        -4032.046
##          aic          bic          ntotal
##          8161.397        8227.858          175.000
##          bic2          rmsea    rmsea.ci.lower
##          8161.357          0.060          0.028
##    rmsea.ci.upper    rmsea.ci.level    rmsea.pvalue
##          0.088          0.900          0.268
##    rmsea.close.h0    rmsea.notclose.pvalue    rmsea.notclose.h0
##          0.050          0.124          0.080
##          rmr          rmr_nomean          srmr
##          0.439          0.439          0.055
##    srmr_bentler    srmr_bentler_nomean          crmr
##          0.055          0.055          0.061
##    crmr_nomean    srmr_mplus    srmr_mplus_nomean
##          0.061          0.055          0.055
##          cn_05          cn_01          gfi
##          154.790          178.391          0.940
##          agfi          pgfi          mfi
##          0.902          0.581          0.941
##          ecvi
##          0.556

```

Structural Equation Model

Model Specification (Step 1 SEM: Measurement Model)

Correlated Three Factor Model:

```
cthreefactor <- ' # Latent variables
Fluid =~ V1 + V2 + V3
Verbal =~ V4 + V5 + V6
Visuospatial =~ V7 + V8 + V9

# Latent variable correlation
Fluid ~~ Verbal
Fluid ~~ Visuospatial
Verbal ~~ Visuospatial

# Residuals for manifest variables
V1 ~~ V1
V2 ~~ V2
V3 ~~ V3
V4 ~~ V4
V5 ~~ V5
V6 ~~ V6
V7 ~~ V7
V8 ~~ V8
V9 ~~ V9

# Variances for latent variables
Fluid~~1*Fluid
Verbal~~1*Verbal
Visuospatial~~1*Visuospatial
'
```

Model Specification (Step 2 SEM: Full Structural Model)

Fluid Intelligence Predicts Verbal/Visuospatial Model:

```
gfpredicts <- ' # Latent variables
Fluid =~ V1 + V2 + V3
Verbal =~ V4 + V5 + V6
Visuospatial =~ V7 + V8 + V9

# Regressions

Verbal + Visuospatial ~ Fluid

# Residuals for manifest variables
V1 ~~ V1
V2 ~~ V2
V3 ~~ V3
V4 ~~ V4
V5 ~~ V5
V6 ~~ V6
V7 ~~ V7
V8 ~~ V8
V9 ~~ V9

# Variances for latent variables
Fluid~~1*Fluid
Verbal~~1*Verbal
Visuospatial~~1*Visuospatial
'
```

Fluid Intelligence Predicts Verbal/Visuospatial Correlated Model:

```
gfpredictsc <- ' # Latent variables
Fluid =~ V1 + V2 + V3
Verbal =~ V4 + V5 + V6
Visuospatial =~ V7 + V8 + V9

# Correlated Latent variables
Verbal ~~ Visuospatial

# Regressions

Verbal + Visuospatial ~ Fluid

# Residuals for manifest variables
V1 ~~ V1
V2 ~~ V2
V3 ~~ V3
V4 ~~ V4
V5 ~~ V5
V6 ~~ V6
V7 ~~ V7
V8 ~~ V8
V9 ~~ V9

# Variances for latent variables
Fluid~~1*Fluid
Verbal~~1*Verbal
Visuospatial~~1*Visuospatial
'
```

Run the CFA measurement model, summarize the parameter

estimates, and plot

```
# Run the model: Three Factor Model
cfafit <- lavaan(cthreefactor,
                 data = cogability,
                 std.lv = TRUE)

# Get parameter estimates and fit statistics
summary(cfafit,
        fit.measures = TRUE,
        standardized = TRUE)
```

```

## lavaan 0.6-19 ended normally after 23 iterations
##
##      Estimator                      ML
##      Optimization method          NLMINB
##      Number of model parameters    21
##
##      Number of observations        250
##
## Model Test User Model:
##
##      Test statistic                31.927
##      Degrees of freedom            24
##      P-value (Chi-square)          0.129
##
## Model Test Baseline Model:
##
##      Test statistic                876.448
##      Degrees of freedom            36
##      P-value                        0.000
##
## User Model versus Baseline Model:
##
##      Comparative Fit Index (CFI)    0.991
##      Tucker-Lewis Index (TLI)      0.986
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)  -9523.464
##      Loglikelihood unrestricted model (H1) -9507.500
##
##      Akaike (AIC)                  19088.927
##      Bayesian (BIC)                 19162.878
##      Sample-size adjusted Bayesian (SABIC) 19096.306
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                          0.036
##      90 Percent confidence interval - lower 0.000
##      90 Percent confidence interval - upper 0.067

```

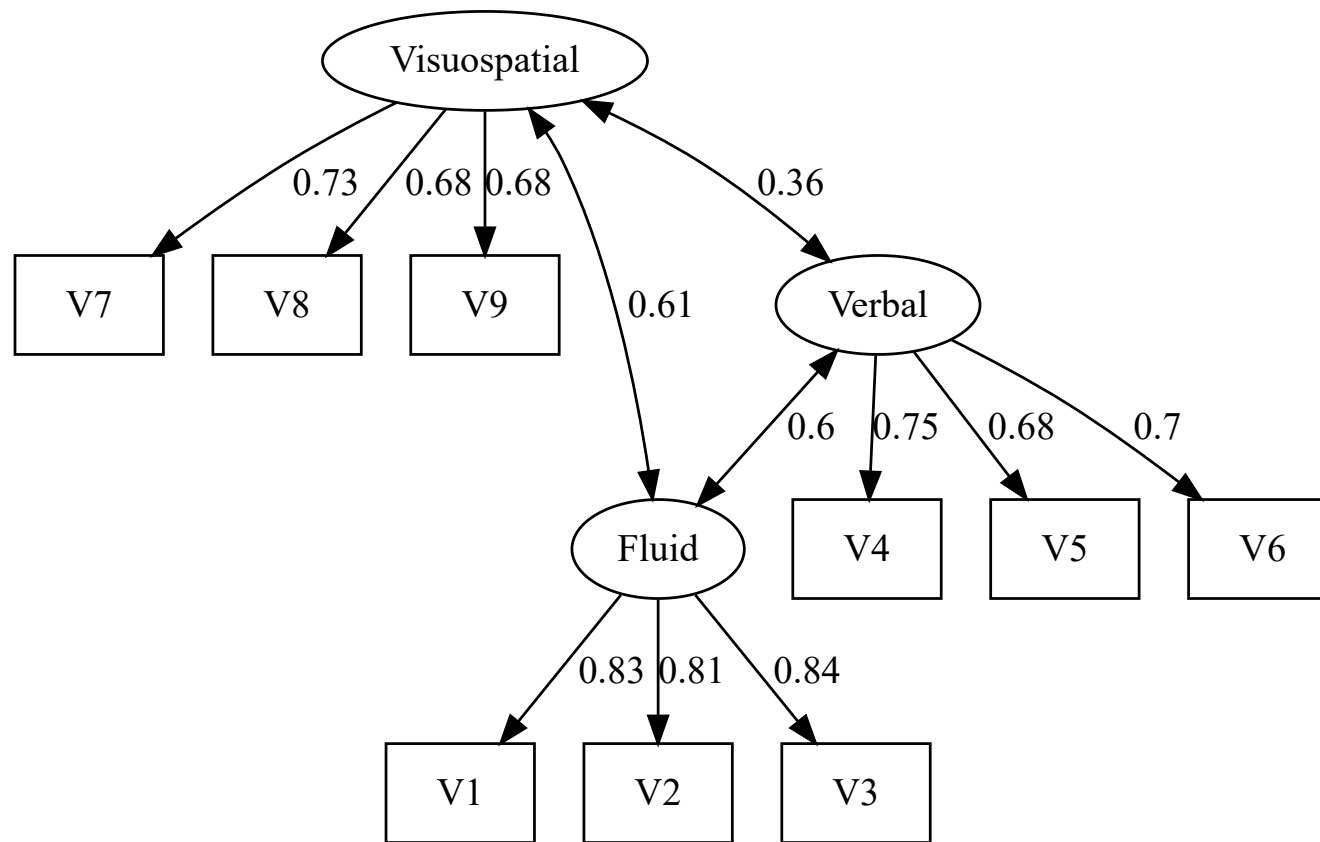
```

## P-value H_0: RMSEA <= 0.050 0.738
## P-value H_0: RMSEA >= 0.080 0.006
##
## Standardized Root Mean Square Residual:
##
## SRMR 0.032
##
## Parameter Estimates:
##
## Standard errors Standard
## Information Expected
## Information saturated (h1) model Structured
##
## Latent Variables:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## Fluid =~
## V1 15.442 1.007 15.333 0.000 15.442 0.832
## V2 16.124 1.088 14.824 0.000 16.124 0.812
## V3 16.311 1.041 15.664 0.000 16.311 0.844
## Verbal =~
## V4 16.213 1.358 11.943 0.000 16.213 0.754
## V5 14.416 1.351 10.669 0.000 14.416 0.682
## V6 13.854 1.268 10.924 0.000 13.854 0.696
## Visuospatial =~
## V7 15.075 1.317 11.450 0.000 15.075 0.734
## V8 14.152 1.342 10.546 0.000 14.152 0.681
## V9 13.294 1.266 10.500 0.000 13.294 0.679
##
## Covariances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## Fluid ~~
## Verbal 0.597 0.057 10.563 0.000 0.597 0.597
## Visuospatial 0.610 0.057 10.795 0.000 0.610 0.610
## Verbal ~~
## Visuospatial 0.358 0.077 4.656 0.000 0.358 0.358
##
## Variances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .V1 106.192 14.060 7.553 0.000 106.192 0.308

```

##	.V2	133.971	16.609	8.066	0.000	133.971	0.340
##	.V3	107.237	14.940	7.178	0.000	107.237	0.287
##	.V4	199.869	29.393	6.800	0.000	199.869	0.432
##	.V5	239.473	28.891	8.289	0.000	239.473	0.535
##	.V6	204.332	25.431	8.035	0.000	204.332	0.516
##	.V7	194.623	27.530	7.070	0.000	194.623	0.461
##	.V8	231.052	28.431	8.127	0.000	231.052	0.536
##	.V9	206.917	25.315	8.174	0.000	206.917	0.539
##	Fluid	1.000				1.000	1.000
##	Verbal	1.000				1.000	1.000
##	Visuospatial	1.000				1.000	1.000

```
lavaanPlot(cfafit,
            coefs = TRUE,
            stand = TRUE,
            covs = TRUE)
```

Run the SEM model(s), summarize the parameter estimates, and

plot

```
# Run the model: Gf Predicts SEM
semfit <- lavaan(gfpredicts,
                 data = cogability,
                 std.lv = TRUE)

# Get parameter estimates and fit statistics
summary(semfit,
        fit.measures = TRUE,
        standardized = TRUE)
```

```

## lavaan 0.6-19 ended normally after 21 iterations
##
## Estimator ML
## Optimization method NLMINB
## Number of model parameters 20
##
## Number of observations 250
##
## Model Test User Model:
##
## Test statistic 31.937
## Degrees of freedom 25
## P-value (Chi-square) 0.160
##
## Model Test Baseline Model:
##
## Test statistic 876.448
## Degrees of freedom 36
## P-value 0.000
##
## User Model versus Baseline Model:
##
## Comparative Fit Index (CFI) 0.992
## Tucker-Lewis Index (TLI) 0.988
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0) -9523.469
## Loglikelihood unrestricted model (H1) -9507.500
##
## Akaike (AIC) 19086.938
## Bayesian (BIC) 19157.367
## Sample-size adjusted Bayesian (SABIC) 19093.965
##
## Root Mean Square Error of Approximation:
##
## RMSEA 0.033
## 90 Percent confidence interval - lower 0.000
## 90 Percent confidence interval - upper 0.064

```

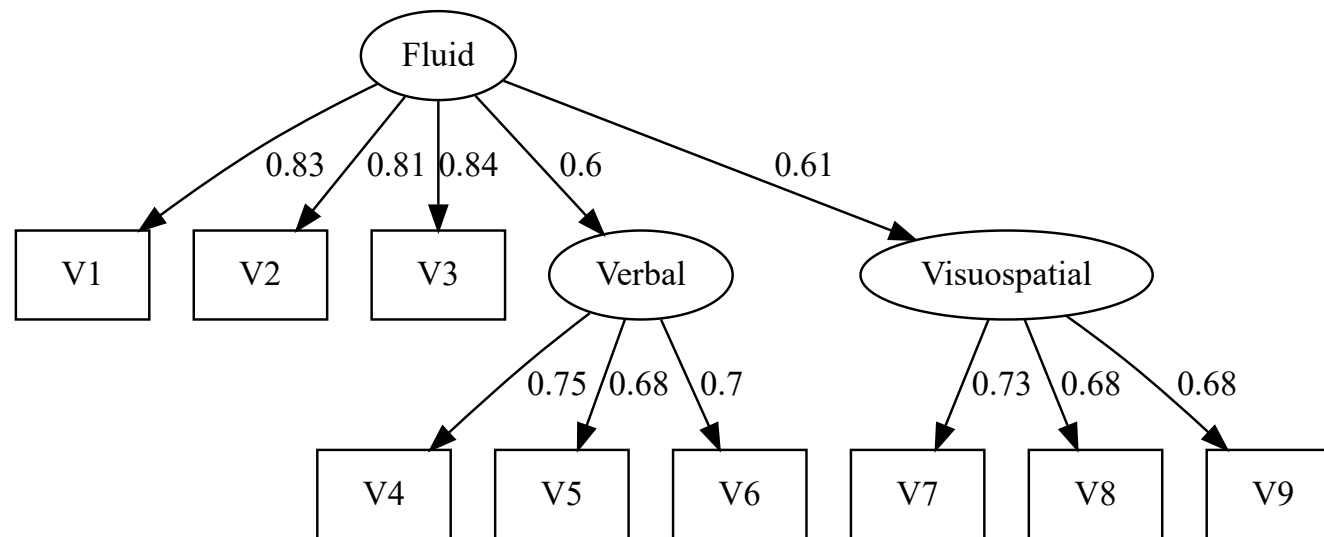
```

## P-value H_0: RMSEA <= 0.050 0.787
## P-value H_0: RMSEA >= 0.080 0.004
##
## Standardized Root Mean Square Residual:
##
## SRMR 0.032
##
## Parameter Estimates:
##
## Standard errors Standard
## Information Expected
## Information saturated (h1) model Structured
##
## Latent Variables:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## Fluid =~
## V1 15.440 1.007 15.329 0.000 15.440 0.832
## V2 16.127 1.088 14.828 0.000 16.127 0.813
## V3 16.314 1.041 15.669 0.000 16.314 0.844
## Verbal =~
## V4 13.006 1.220 10.660 0.000 16.206 0.753
## V5 11.572 1.176 9.841 0.000 14.420 0.682
## V6 11.120 1.109 10.028 0.000 13.857 0.696
## Visuospatial =~
## V7 11.960 1.176 10.169 0.000 15.085 0.734
## V8 11.213 1.168 9.602 0.000 14.143 0.681
## V9 10.539 1.101 9.573 0.000 13.293 0.679
##
## Regressions:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## Verbal ~
## Fluid 0.743 0.109 6.828 0.000 0.597 0.597
## Visuospatial ~
## Fluid 0.769 0.113 6.810 0.000 0.609 0.609
##
## Variances:
## Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .V1 106.272 14.063 7.557 0.000 106.272 0.308
## .V2 133.869 16.606 8.061 0.000 133.869 0.340

```

##	.V3	107.113	14.934	7.172	0.000	107.113	0.287
##	.V4	200.086	29.394	6.807	0.000	200.086	0.432
##	.V5	239.352	28.889	8.285	0.000	239.352	0.535
##	.V6	204.260	25.431	8.032	0.000	204.260	0.515
##	.V7	194.341	27.531	7.059	0.000	194.341	0.461
##	.V8	231.300	28.435	8.134	0.000	231.300	0.536
##	.V9	206.961	25.316	8.175	0.000	206.961	0.539
##	Fluid	1.000				1.000	1.000
##	.Verbal	1.000				0.644	0.644
##	.Visuospatial	1.000				0.629	0.629

```
lavaanPlot(semfit,
            coefs = TRUE,
            stand = TRUE,
            covs = TRUE)
```



```

# Run the model: Gf Predicts Correlated SEM
semfitc <- lavaan(gfpredictsc,
  data = cogability,
  std.lv = TRUE)

# Get parameter estimates and fit statistics
summary(semfitc,
  fit.measures = TRUE,
  standardized = TRUE)

```

```

## lavaan 0.6-19 ended normally after 21 iterations
##
## Estimator ML
## Optimization method NLMINB
## Number of model parameters 21
##
## Number of observations 250
##
## Model Test User Model:
##
## Test statistic 31.927
## Degrees of freedom 24
## P-value (Chi-square) 0.129
##
## Model Test Baseline Model:
##
## Test statistic 876.448
## Degrees of freedom 36
## P-value 0.000
##
## User Model versus Baseline Model:
##
## Comparative Fit Index (CFI) 0.991
## Tucker-Lewis Index (TLI) 0.986
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0) -9523.464
## Loglikelihood unrestricted model (H1) -9507.500
##
## Akaike (AIC) 19088.927
## Bayesian (BIC) 19162.878
## Sample-size adjusted Bayesian (SABIC) 19096.306
##
## Root Mean Square Error of Approximation:
##
## RMSEA 0.036
## 90 Percent confidence interval - lower 0.000
## 90 Percent confidence interval - upper 0.067

```

```

## P-value H_0: RMSEA <= 0.050      0.738
## P-value H_0: RMSEA >= 0.080      0.006
##
## Standardized Root Mean Square Residual:
##
## SRMR      0.032
##
## Parameter Estimates:
##
## Standard errors      Standard
## Information          Expected
## Information saturated (h1) model      Structured
##
## Latent Variables:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## Fluid =~
## V1      15.442  1.007  15.333  0.000  15.442  0.832
## V2      16.124  1.088  14.824  0.000  16.124  0.812
## V3      16.311  1.041  15.664  0.000  16.311  0.844
## Verbal =~
## V4      13.004  1.222  10.642  0.000  16.213  0.754
## V5      11.562  1.177   9.822  0.000  14.416  0.682
## V6      11.112  1.110  10.009  0.000  13.854  0.696
## Visuospatial =~
## V7      11.945  1.178  10.144  0.000  15.075  0.734
## V8      11.213  1.169   9.590  0.000  14.152  0.681
## V9      10.534  1.102   9.557  0.000  13.294  0.679
##
## Regressions:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## Verbal ~
## Fluid      0.745  0.110   6.795  0.000   0.597  0.597
## Visuospatial ~
## Fluid      0.770  0.114   6.777  0.000   0.610  0.610
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|) Std.lv Std.all
## .Verbal ~~
## .Visuospatial -0.011  0.102  -0.104  0.918  -0.011  -0.011

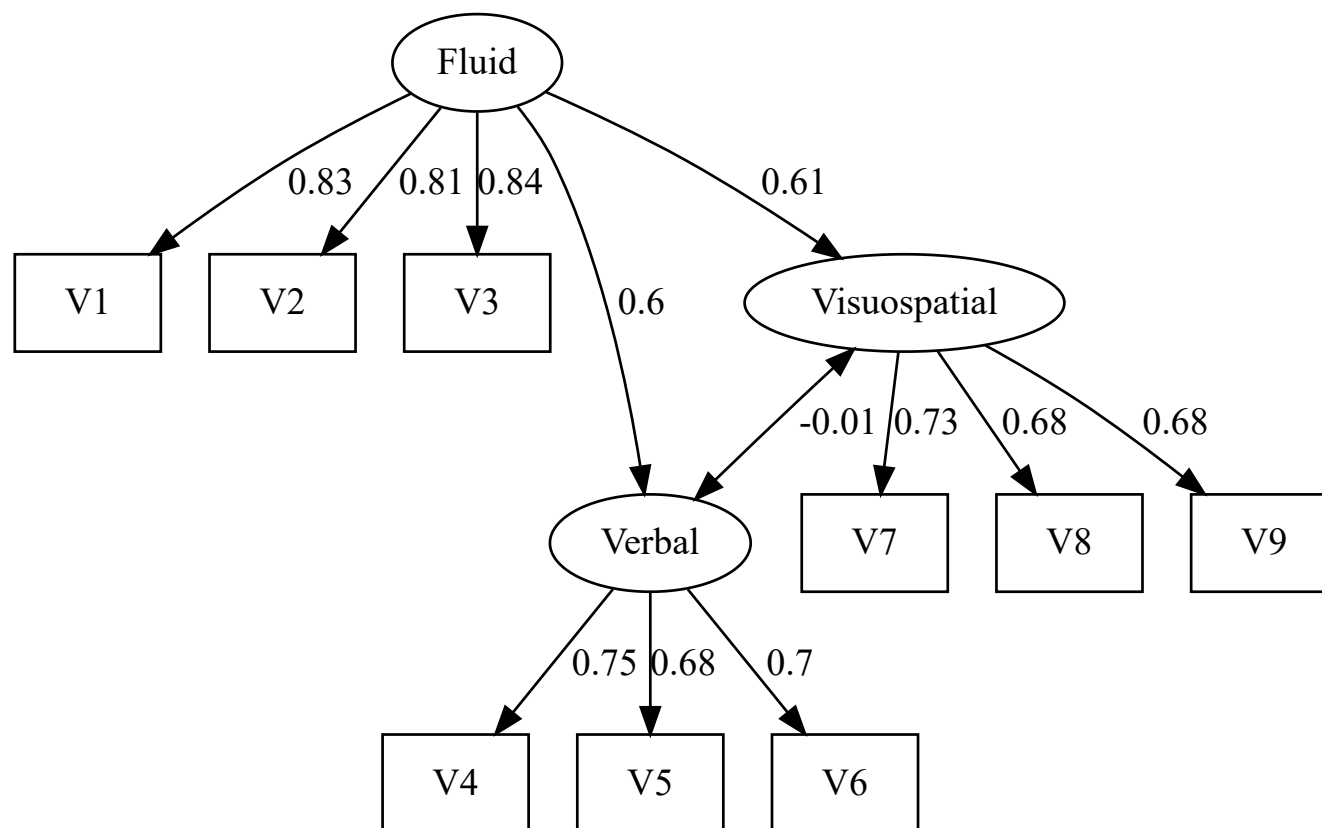
```



```
##
## Variances:
##
```

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
## .V1	106.192	14.060	7.553	0.000	106.192	0.308
## .V2	133.971	16.609	8.066	0.000	133.971	0.340
## .V3	107.237	14.940	7.178	0.000	107.237	0.287
## .V4	199.868	29.393	6.800	0.000	199.868	0.432
## .V5	239.473	28.891	8.289	0.000	239.473	0.535
## .V6	204.331	25.431	8.035	0.000	204.331	0.516
## .V7	194.623	27.530	7.070	0.000	194.623	0.461
## .V8	231.052	28.431	8.127	0.000	231.052	0.536
## .V9	206.917	25.315	8.174	0.000	206.917	0.539
## Fluid	1.000				1.000	1.000
## .Verbal	1.000				0.643	0.643
## .Visuospatial	1.000				0.628	0.628

```
lavaanPlot(semfitc,
            coefs = TRUE,
            stand = TRUE,
            covs = TRUE)
```



Direct Model Fit Comparisons (Chi-square Difference Test)

Can compare fit of our 2-factor models because they are nested (same items) AND one has fewer degrees of freedom
 anova(semfit,semfitc)

```
##
## Chi-Squared Difference Test
##
##      Df   AIC   BIC  Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
## semfitc 24 19089 19163 31.927
## semfit  25 19087 19157 31.937  0.010397    0      1    0.9188
```