Chutes and Ladders

An Exciting Up and Down Presentation for Statistics People!

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STAT 5400 Simulation Project

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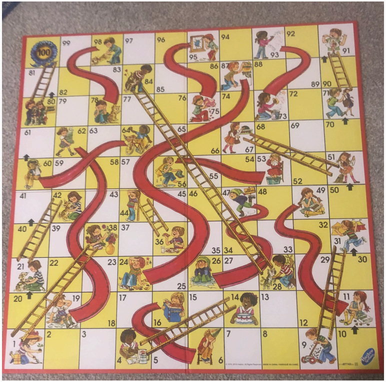
Intro

Game Play

- Game board consists of 100 squares
- Spinner determines how many spaces to advance (1 - 6)
- Ladders advance you, chute (slides) send you back
- First player to reach 100th square wins

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Overview

Why did we choose Chutes and Ladders to simulate?

- Simple game, ages 3+
- Independent player paths
- Purely a game of chance
- Spins are independent

Research Questions

Overview

1. What is the average number of turns for the winning player in a game of 2, 3, and 4 players?
2. Is there an advantage to going first?
3. Does taking the ladder to the 100th square significantly reduce the total number of turns for a player?
4. Can the number of trips up the ”big ladder” and trips down the ”big chute” be used to model the number of total turns for a player?

Winning path

In order for a player to win, they must land exactly on 100th square without going over.

Predictive Model

Simulation

Analysis

Average Winner Turns

1. What is the average number of turns for the winning player in a game of 2, 3, and 4 players?

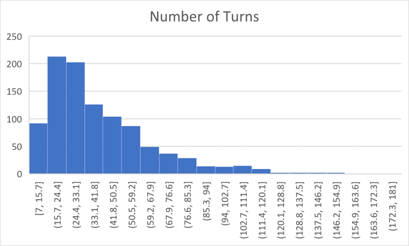
All Players	n	Avg Turns	SD Turns	Min Turns	Max Turns
2 Players	2000	39.41	24.318	7	231
3 Players	3000	39.75	26.541	7	311
4 Players	4000	38.69	24.538	7	234
All	**9000**	**39.20**	**25.231**		

Winnng Player	n	Avg Turns	SD Turns	Min Turns	Max Turns
2 Players	1000	26.641	13.157	7	98
3 Players	1000	21.926	9.887	7	74
4 Players	1000	19.153	7.721	7	51

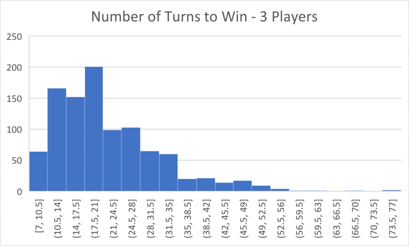
Avg Winner Turns

How are the number of turns distributed?

All Players



Winning Turns with 3 Players



Avg Winner Turns

Are the number of winning turns significantly different for 2, 3 and 4 player games?

2 Players	3 Players	4 Players
26.641	21.926	19.153

Table 1: ANOVA Table

	df	Sum Sq	Mean Sq	F-value	Pr(>F)
Players	2	28664	14332	130.1	< 2e-16
Residuals	2997	330142	110		

Table 2: Tukey's HSD test(95% level)

	Difference	Lower CL	Upper CL
2 vs. 3 plyr	-4.715	-5.816	-3.614
3 vs. 4 plyr	-2.773	-3.874	-1.672
2 vs. 4 plyr	-7.488	-8.589	-6.387

First Player Advantage?

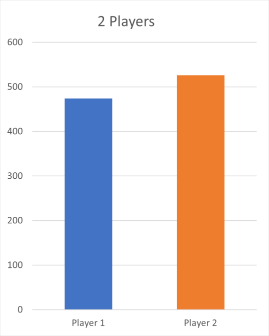
Is there an advantage to going first?

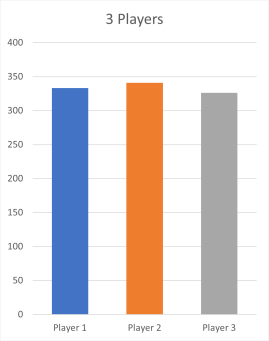
Player 1	Player 2	Player 3	Player 4
⋮	⋮	⋮	⋮
27	50	15	52
24	34	15	61
19	27	42	19
14	72	66	39
15	16	11	17
⋮	⋮	⋮	⋮

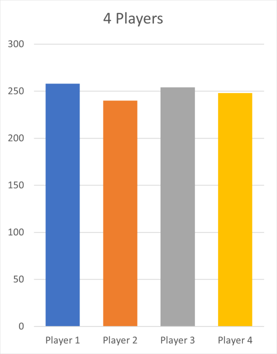
First Player Advantage

Proportion of Games Impacted

2 Players	3 Players	4 Players
0.012	0.017	0.030







First Player Advantage

Proportion of Games Impacted

2 Players	3 Players	4 Players
0.012	0.017	0.030

Results of prop.test in R

$\chi^2 = 8.58$ $df = 2$ p-value $= 0.0137$

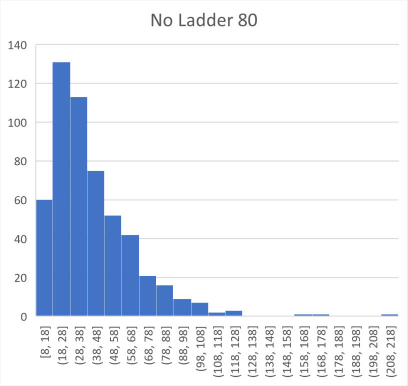
Sample Estimates

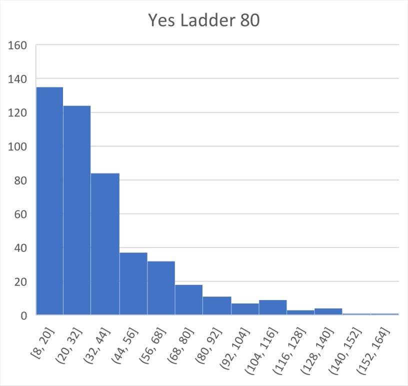
	2 Players	3 Players	4 Players
p-hat	0.01186	0.01672	0.02913

Ladder 80? Yes or No

Does taking the ladder to the 100th square significantly reduce the total number of turns for a player?

80 Ladder?	n	Avg Turns	Std. Dev
Yes	466	37.76	26.236
No	534	41.18	24.247





Ladder 80? Yes or No

Even though the distribution is not normal, because of the very large sample size, the Central Limit Theorem applies, so we will compare using a two-sample t-test

These are unpaired groups, and standard deviations for both samples are similar, and thus we use the Student t-test using a pooled variance

$$H_0 = \mu_{yes} = \mu_{no}$$

$$H_a = \mu_{yes} > \mu_{no}$$

Ladder 80? Yes or No

$$H_0 = \mu_{yes} = \mu_{no}$$

$$H_a = \mu_{yes} > \mu_{no}$$

$$s^2 = \frac{\sum_{i=1}^{n_{yes}} (x_i - \bar{x}_{yes})^2 + \sum_{j=1}^{n_{no}} (x_i - \bar{x}_{no})^2}{n_{no} + n_{yes} - 2} = 5.01735 \qquad \text{pooled variance}$$

$$t = \frac{\bar{x}_{yes} - \bar{x}_{no}}{\sqrt{s^2 \left(\frac{1}{n_{yes}} + \frac{1}{n_{no}} \right)}} = 10.7463$$

$$Pr(t_{998} \geq 10.7463) = 7.2557e - 26 \approx 0$$

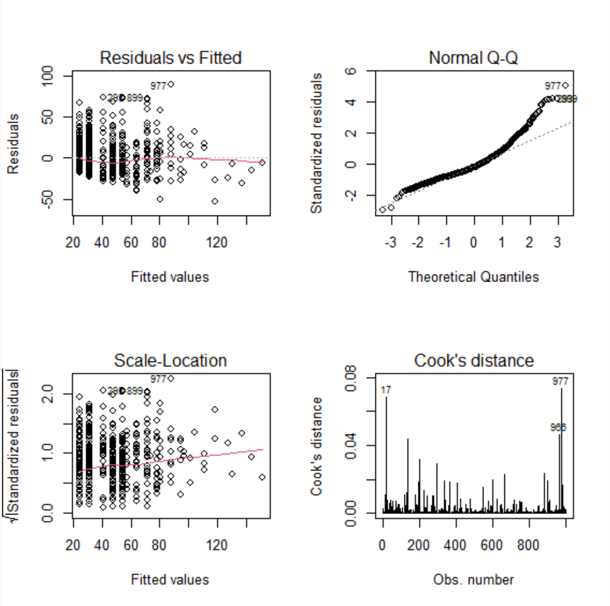
Therefore, reject the null hypothesis

Linear Model

Can the number of trips up the ”big ladder” and trips down the ”big chute” be used to model the number of total turns for a player?

turns $\sim$ bigL + bigC

	Estimates	Std. Error	t-value	Pr($>|t|$)
Intercept	31.3511	0.6944	45.150	<2e-16
bigL	-6.958	0.9212	-7.562	9.01e-14
bigC	23.4169	0.8047	29.101	<2e-16


Residuals vs Fitted
Residuals
Fitted values
977
899
Normal Q-Q
Standardized residuals
Theoretical Quantiles
Scale-Location
√|Standardized residuals|
Cook's distance
Obs. number
17
977

Linear Model

Breusch-Pagan Test: p-value = 4.697e-08; no heteroscedasticity
Shapiro-Wilk test: p-value < 2.2e-16; residuals are NOT normal

Potential Solutions?

- **Add variables**
  Modeled turns $\sim$ ladders + chutes + BigL + BigC
  All coefficients were significant, but still ran into trouble with non-normality.
- **Box-Cox transformation**
  Using a $\lambda = 0.2$, but still had non-normal residuals
- **Poisson Model**
  The turns data is discrete with no upper limit, however
  Mean(turns) = 39.689 and Var(turns) = 632.8952
  Very overdispersed, so Poisson model is not a good fit.
- **Other GLM?**

Linear Model

What are the conditions when playing a game of Chutes and Ladders?

In the end, we are going to use the original model. Even though some assumptions are violated, if the point is to give a quick calculation during game play of potential number of turns, the model will make that possible to do, even in ones head.

$$\text{Turns} = 31.35 + \text{bigC} * 23.42 - \text{bigL} * 6.97$$

Conclusions

Conclusions

- **Are the averages for the number of turns for the winning player different dependig on 2, 3, or 4 players?**
  Yes, the average for winning turns is different depending on how many players there are, with evidence to indicate that the more players there are, the fewer turns it will take for a single player to win.
- **Is there an advantage to going first?**
  There were only a small amount of scenarios in which this occurs, but it does appear it is more likely to give an advantage the more players there are.
- **Does taking the ladder to the 100th square significantly reduce the average number of turns for a player?**
  Yes, taking the ladder reduces the average number of turns.

Conclusions

continued

- Can the number of trips up the ”big ladder” and trips down the ”big chute” be used to model the number of total turns for a player?
  Yes, though some assumptions were violated in favor of creating a simple model that can allow estimation during game play in ones head.