Cyrus Simons
Ch 3.2 Q 3.6

(b) show: $\sqrt{a} - x_{n+1} = \frac{1}{2x_n}(\sqrt{a} - x_n)^2$

Using:
$$\alpha - x_{n+1} = (\alpha - x_n)^2\left[\frac{-f''(c_n)}{2(f'(x_n))}\right]$$

Since $\sqrt{\alpha}$ is the desired value, we compare:

where:
$$\left.\begin{aligned} f(x) &= x^2 \\ f'(x) &= 2x \\ f''(x) &= 2 \end{aligned}\right\}$$

$$\sqrt{\alpha} - x_{n+1} = (\sqrt{\alpha} - x_n)^2\left[\frac{-\cancel{2}}{\cancel{2}(2x_n)}\right]$$

$$\sqrt{\alpha} - x_{n+1} = (\sqrt{\alpha} - x_n)^2\left(-\frac{1}{2x_n}\right) \checkmark$$

because the true value of our root is $\sqrt{\alpha}$, we can simply subtract the estimated value to obtain the error & relative error.

s.t

$$err(x_{n+1}) = (err(x_n))^2\left(\frac{-1}{2x_n}\right)$$

and

$$Rel(x_{n+1}) = \frac{\sqrt{\alpha} - x_{n+1}}{\sqrt{\alpha}}\left(\frac{\sqrt{\alpha}}{1}\right) = \left(\frac{\sqrt{\alpha} - x_n}{\sqrt{\alpha}}\right)^2\left(-\frac{1}{2x_n}\right)\left(\sqrt{\alpha}\right)$$

$$Rel(x_{n+1}) = \left(\frac{\sqrt{\alpha} - x_n}{\sqrt{\alpha}}\right)^2\left(-\frac{\sqrt{\alpha}}{2x_n}\right)$$