CS 558 10 September 2024

Note Title 2024-09-10

No class Thursday 12 Sept. No office hours.

```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f v []     = v
foldr f v (x:xs) = f x (foldr f v xs)
```

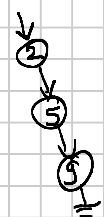
Example: f = (+)
v = 0
[x0, x1, x2]

sum = foldr (+) 0

```
  foldr (+) 0 [2, 5, 9]
= foldr (+) 0 (2 : (5 : (9 : [])))
```

f = (+)
v = 0
x = 2
xs = 5 : (9 : [])

```
= (+) 2 (foldr (+) 0 (5 : (9 : [])))
= (+) 2 ((+) 5 (foldr (+) 0 (9 : [])))
= (+) 2 ((+) 5 ((+) 9 (foldr (+) 0 [])))
= (+) 2 ((+) 5 ((+) 9 0))
= (+) 2 ((+) 5 9)
= (+) 2 14
= 16
```



foldr (:) [] = id

```
foldl :: (b -> a -> b) -> b -> [a] -> b
foldl f v []     = v
foldl f v (x:xs) = foldl f (f v x) xs
```

```
  foldl (+) 0 (2 : (5 : (9 : [])))
= foldl (+) ((+) 0 2) (5 : (9 : []))
= foldl (+) ((+) ((+) 0 2) 5) (9 : [])
= foldl (+) ((+) ((+) ((+) 0 2) 5) 9) []
= (+) ((+) ((+) 0 2) 5) 9
= (+) ((+) 2 5) 9
= (+) 7 9
= 16
```

In practice, we should use the strict (non-lazy) function foldl'
rather than foldl or foldr
when our lists are very long (but finite)

Properties of the form $\forall l :: [a].\ P(l)$

usually of the form $f\ l = g\ l$
for some functions $f, g :: [a] \to b$

Example. $\forall l :: [\text{Int}].\ \text{sum}\ l = \text{foldr}\ (+)\ 0\ l$

Structural induction

Principle of structural induction on finite lists:

To prove $\forall l :: [a] . P(l)$,
it suffices to prove

1. $P([\,])$ base case
2. $\forall x :: a . \forall xs :: [a] . P(xs) \Rightarrow P(x : xs)$ inductive case

Example.

Prove $\forall l :: [a] . \; \text{map id } l = l$ $(\text{map id} = \text{id})$

1. Base case: To prove $\text{map id } [\,] = [\,]$.

$$\begin{aligned} & \text{map id } [\,] \\ = & \quad \{\text{def. map}\} \\ & [\,] \end{aligned}$$ ✓

2. Inductive case

Assume arbitrary $x :: a$ and $xs :: [a]$,
to prove $P(xs) \Rightarrow P(x : xs)$ i.e. $\underbrace{\text{map id } xs = xs}_{IH} \Rightarrow \text{map id } (x : xs) = x : xs$

$$\begin{aligned} & \text{map id } (x : xs) \\ = & \quad \{\text{def. map}\} \\ & (\text{id } x) : (\text{map id } xs) \\ = & \quad \{\text{by } IH\} \\ & (\text{id } x) : xs \\ = & \quad \{\text{def. id}\} \\ & x : xs \end{aligned}$$ ✓

Example Prove that $\text{map } f \circ \text{map } g = \text{map } (f \circ g)$

$$\text{map } f \, (\text{map } g \; xs) = \text{map } (f \circ g) \; xs$$