

Syntax for binary operators

 $a \rightarrow b \rightarrow c$

Alphanumeric names

div mod

Prefix by default

div 7 2
→ 3

To use infix, we need backquotes

7 `div` 2

Non-alphanumeric names

: ==

Infix by default

"abc" # "def"

Can make infix specific by underlining in parentheses:

(++ "abc" "def")

Higher order functions over lists

"Functionals"

Helper function:

isCap :: Char → Bool

isCap c = 'A' ≤ c & c ≤ 'Z'

<= &&

keepOnlyCaps :: [Char] → [Char]

keepOnlyCaps [] = []

keepOnlyCaps (c:cs)

| isCap c = c : keepOnlyCaps cs

| otherwise = keepOnlyCaps cs

Example evaluation: keepOnlyCaps ['A', 'b', 'X']

= 'A' : keepOnlyCaps ['b', 'X']

= 'A' : keepOnlyCaps ['X']

= 'A' : 'X' : keepOnlyCaps []

= 'A' : 'X' : []

Higher order function filter

filter :: (a → Bool) → [a] → [a]

*Such an argument
"predicate"*

filter p [] = []

filter p (x:xs)

| p x = x : filter p xs

| otherwise = filter p xs

Now we can define keepOnlyCaps in two ways instead:

keepOnlyCaps :: [Char] → [Char]

keepOnlyCaps cs = filter isCap cs

or
keepOnlyCaps = filter isCap

keepAllButCaps = filter (not ∘ isCap)

List of all even positive integers:

filter ($\lambda n \rightarrow n \bmod 2 \equiv 0$) [1..]

λ a function which,
given some argument n (a number)
computes if it is even

$$[n * 2 \mid n \leftarrow [1, 2, 3]]$$

\downarrow
 $*2$
 $[2, 4, 6]$

$$\text{map} :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$$

$$\text{map } f [] = []$$

$$\text{map } f (x:xs) = f x : \text{map } f xs$$

$$\text{map } (\lambda n \rightarrow n * 2) [1, 2, 3]$$

$(*2)$

$$\text{map } (*2) [1, 2, 3]$$

$$= (*2) 1 : \text{map } (*2) [2, 3]$$

$$= 2 : \text{map } (*2) [2, 3]$$

$$= 2 : (*2) 2 : \text{map } (*2) [3]$$

$$= 2 : 4 : \text{map } (*2) [3]$$

$$= 2 : 4 : (*2) 3 : []$$

$$= 2 : 4 : 6 : []$$

$$\text{map isCap} ['X', 'b']$$

$$= [\text{True}, \text{False}]$$

$$\text{map isCap} :: [\text{Char}] \rightarrow [\text{Bool}]$$

Aggregation / Counting / Reducing / Folding

$$\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$$

combining operator /
aggregating from
starting
value
list to be
aggregated
result
of
aggregation

$$\text{foldr } f \ v \ [] = v$$

$$\text{foldr } f \ v \ (x:xs) = f \ x \ (\text{foldr } f \ v \ xs)$$

example implement list summation using foldr

$$\text{sum} :: [\text{Int}] \rightarrow \text{Int}$$

$$\text{sum} = \text{foldr } (+) \ 0$$

$$\text{sum } [1, 2, 3]$$

$$= \text{foldr } (+) \ 0 \ [1, 2, 3]$$

$$= (+) 1 \ (\text{foldr } (+) \ 0 \ [2, 3])$$

$$= (+) 1 \ ((+) 2 \ (\text{foldr } (+) \ 0 \ [3]))$$

$$= (+) 1 \ ((+) 2 \ ((+) 3 \ (\text{foldr } (+) \ 0 \ [])))$$

$$= (+) 1 \ ((+) 2 \ ((+) 3 \ 0))$$

$$= (+) 1 \ ((+) 2 \ 3)$$

$$= (+) 1 \ 5$$

$$= 6$$