Review for Midterm exam

$[(False, '0'), (True, '1')]$ :: $[(Bool, Char)]$

$(['a', 'b'], [False, True])$ :: $([Char], [Bool])$

twice :: $(a \to a) \to a \to a$

twice $f\ x = f\ (f\ x)$

add :: Num a $\Rightarrow$ a $\to$ a $\to$ a

add $x\ y = x * y$

evens :: Integral a $\Rightarrow [a] \to [a]$

evens :: $[Int] \to [Int]$

evens = filter even

↑ $(a \to Bool) \to [a] \to [a]$ ↰ Int → Bool

foldr $(+)$ 1 $[2, 3, 4]$

$[2, 3, 4] = 2 : (3 : (4 : []))$

$2 + (3 + (4 + 1))$

$\to 10$

foldr $(-)$ 1 $[2, 3, 4] = 2 - (3 - (4 - 1))$

foldl $(-)$ 1 $[2, 3, 4] = ((1 - 2) - 3) - 4 = -8$

-- sorted list insertion

insert :: Int $\to$ [Int] $\to$ [Int]

insert x [] = $[x]$

insert $x\ (y : ys) =$ if $x > y$ then y : insert $x\ ys$

else $x : y : ys$

and :: [Bool] $\to$ Bool

and [] = True

and $(b : bs) = b$ && and bs

reverse :: $[a] \to [a]$

reverse [] = []

reverse $(x : xs) =$ reverse xs ++ $[x]$

replicate :: Int $\to$ a $\to$ $[a]$

replicate 0 _ = []

replicate $n\ x = x$: replicate $(n-1)\ x$

sum $[n^2 \mid n \leftarrow [1..100]]$

$$\text{sum} \ [\, x \ast y \mid x \leftarrow [1..100], \ y \leftarrow [1..100] \,]$$

To prove

$$\text{map } f \ (\text{map } g \ xs) = \text{map } (f \circ g) \ xs$$

for all finite lists xs
all functions f, g

Base case

$\text{map } f \ (\text{map } g \ [\,])$

$= \quad \{ \text{def. of map} \}$

$\text{map } f \quad [\,]$

$= \quad \{ \text{def. of map} \}$

$\quad [\,]$

$\quad \{ \text{def. of map} \leftarrow \}$

$= \ \text{map } (f \circ g) \ [\,] \qquad \checkmark$

Inductive step $\qquad$ IH : $\text{map } f \ (\text{map } g \ xs) = \text{map } (f \circ g) \ xs$

$\text{map } f \ (\text{map } g \ (x : xs))$

$= \quad \{ \text{def map} \}$

$\text{map } f \ (g \ x : \text{map } g \ xs)$

$= \quad \{ \text{def map} \}$

$f \ (g \ x) : \text{map } f \ (\text{map } g \ xs)$

$= \quad \{ \text{IH} \}$

$f \ (g \ x) : \text{map } (f \circ g) \ xs$

$\quad \{ \text{def of } \circ \}$

$= \ (f \circ g) \ x : \text{map } (f \circ g) \ xs$

$\quad \{ \text{def of maps} \}$

$= \ \text{map } (f \circ g) \ (x : xs) \qquad \checkmark$