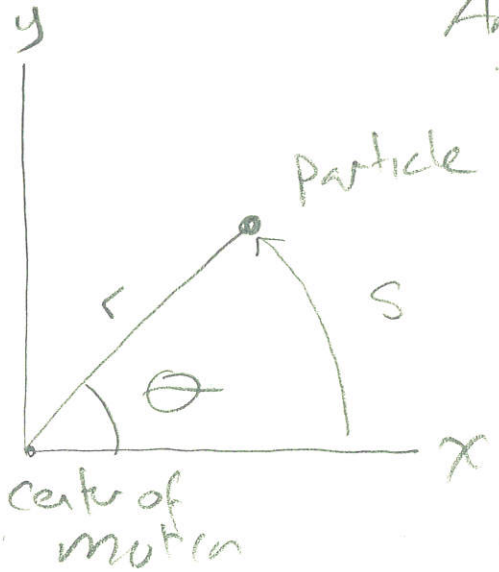
Angular Position



s = arc length
r = radius
θ = angle (radians)

Position described as

$$\theta = \frac{s}{r} \quad [\text{radians}]$$

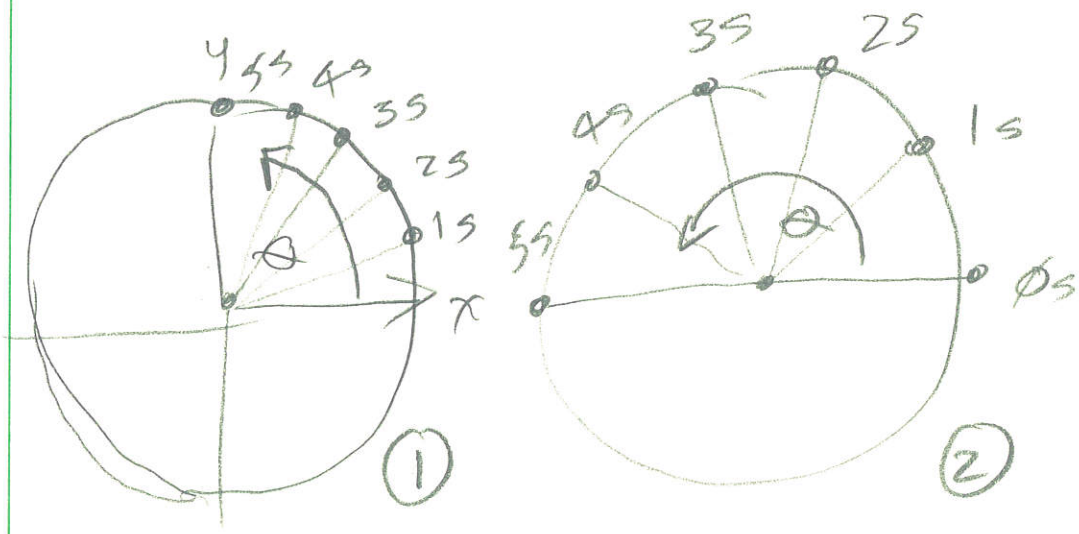
$$s = r\theta$$

- 1 rev = 360° = 2π radians

$$\theta_{\text{full circle}} = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

- $1 \text{ rad} = 1 \cancel{\text{rad}} \times \frac{360^\circ}{2\pi r \cancel{\text{rad}}} = 57.3^\circ$

$$\omega = \frac{\Delta\theta}{\Delta t} \quad [\text{rad/s}]$$ Angular Velocity



Find ω_1 & ω_2

$$\omega_1 = \frac{\Delta\theta_1}{\Delta t} = \frac{\pi/2 \text{ rad}}{5 \text{ sec}} = 0.314 \frac{\text{rad}}{\text{s}}$$

$$\omega_2 = \frac{\Delta\theta_2}{\Delta t} = \frac{\pi \text{ rad}}{5 \text{ sec}} = 0.628 \frac{\text{rad}}{\text{sec}}$$

Approximating

$1 \text{ rad} \approx 60^\circ$

$$\omega_1 \approx 0.3 \frac{\text{rad}}{\text{s}} \approx 0.3(60^\circ) \approx 20^\circ \text{ per sec}$$

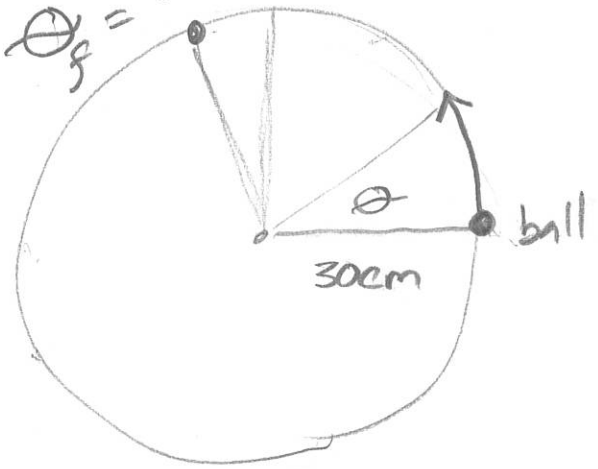
$$\omega_2 \approx \frac{0.6 \text{ rad}}{\text{s}} \approx 0.6(60^\circ) \approx 40^\circ/\text{sec}$$

$$\Delta\theta = \theta_f - \theta_i = \omega \Delta t \left[\frac{rad}{s} \cdot s\right]$$

$$\Delta x = x_f - x_i = v_x \Delta t \left[\frac{m}{s} \cdot s\right]$$

Example 2.2

0.34 rev or 120° or 2.1 rad



In 1.2 sec the ball turns 2 rev

a) find ω

b) $\theta_i = \emptyset$ what is position @ 2.0 sec

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\,rev}{1.2\,sec} = \frac{720°}{1.2\,sec} = \frac{4\pi\,rad}{1.2\,sec}$$

$$\omega = \frac{1.67\,rev}{sec} = \frac{600°}{sec} = \frac{12.54\,rad}{1.2\,sec} = \frac{10.47\,rad}{sec}$$

$$\Delta\theta = \omega \Delta t$$

$2(1.67\,rev)$ $\quad 2(600°)$ $\quad (10.47)_{rad} \cdot 2$

$$\Delta\theta = 3.34\,rev - 3.00\,rev \qquad 1200° - 1080° \qquad 20.94\,rad - 18.84 = 6\pi$$

$$\theta_f = 0.34\,rev \qquad 120° \qquad 2.10\,rad$$

Example 7.3

Crankshaft @ 3000 rpm
What is angular speed ω?

$$3000 \frac{\text{rev}}{\text{min}} \left| \frac{1\,\text{min}}{60\,\text{sec}} \right| = 50 \frac{\text{rev}}{\text{sec}}$$

Period
Frequency

$$\omega = \frac{2\pi \text{ rad}}{T} = 2\pi \text{ rad } f$$

T how long to go around a circle

f how much of a circle in one second

$$f = 50 \frac{\text{rev}}{\text{sec}}$$

$$\omega = 2\pi \text{ rad } f$$

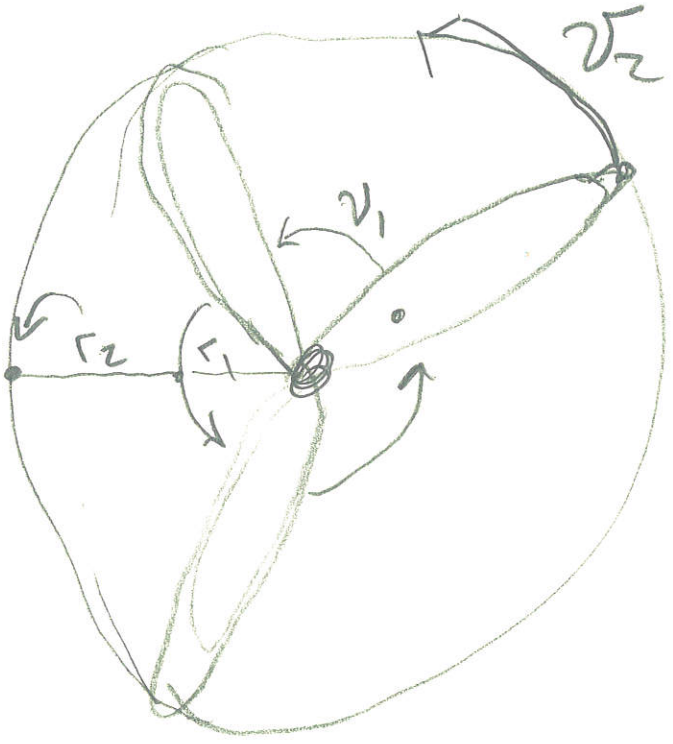
$$= 2\pi \text{ rad } 50 \frac{1}{\text{sec}}$$

$$= (2\pi)50 \frac{\text{rad}}{\text{sec}}$$

$$= 100\pi \frac{\text{rad}}{\text{sec}} = 314 \text{ rad/sec}$$

Relating speed & angular speed

$$v\,[m/s] = \omega\, r\,\left[\frac{rad}{s}\cdot m\right]$$



$r_1 = 25m$

$r_2 = 50m$

$$10\,rpm\left|\frac{min}{60\,sec}\right| = f$$

$$\frac{1}{6}\,sec = f$$

$$T = \frac{1}{f} = \frac{1}{\frac{1}{6}} = 6\,sec$$

$$\omega = \frac{2\pi\,rad}{T} = \frac{2\pi}{6.0\,sec} = 1.05\,\frac{rad}{s}$$

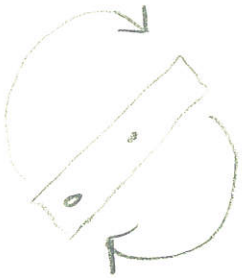
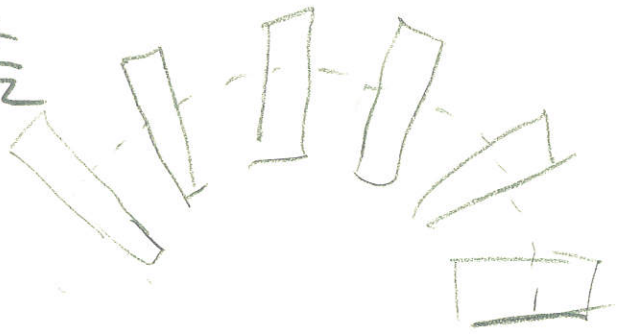
$$v_1 = \omega r_1 = 1.05\,\frac{rad}{sec}\cdot 25m$$

$$v_1 = 26\,\frac{m}{s}$$

$$v_2 = \omega r_2 = 1.05\,\frac{rad}{sec}\cdot 50m$$

$$v_2 = 52\,m/s$$

Rotation of a Rigid body

Translational motion +  = 

Rotational motion

combination motion

α Angular Acceleration

$$\alpha = \frac{\Delta w}{\Delta t} = \frac{\Delta \theta}{\Delta t^2}$$

Example 7.6
From Rest

$w_i = \emptyset$

$w_f = 5400 \text{ rpm} \left| \frac{\text{min}}{60 \text{sec}} \right| = 565 \text{ rad/sec}$ after 2 sec

find α angular acceleration

$$\alpha = \frac{\Delta w}{\Delta t} = \frac{w_f - w_i}{\Delta t} = \frac{565 - \emptyset \text{ rad}}{2 \text{ sec}}$$

$$\alpha = 282.5 \frac{\text{rad}}{\text{sec}^2}$$

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

$$(\Delta x) = (v_i \Delta t) + \frac{1}{2}a(\Delta t^2)$$

$\omega_i = \emptyset$ so

$$\Delta\theta = \frac{1}{2}\alpha \Delta t^2$$

$$= \frac{1}{2}\left(282.5 \frac{\text{rad}}{\text{sec}^2}\right)(2.00 \text{ sec})^2$$

$$= \frac{1}{2}\, 282.5 \cdot 4 \text{ rad}$$

$$\simeq \underline{565 \text{ rad}}$$

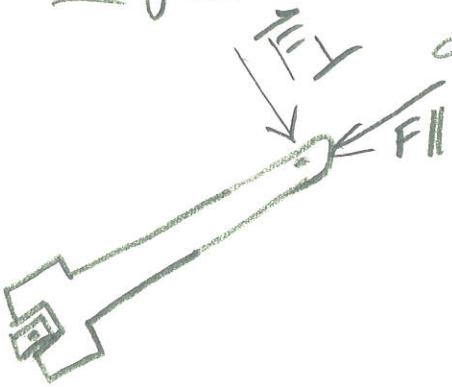
$1 \text{ rev} = 2\pi \text{ rad}$

$$\Delta\theta = 565 \text{ rad} \left| \frac{1 \text{ rev}}{2\pi \text{ rad}} \right|$$

$$= \frac{565}{6.28} \text{ rev}$$

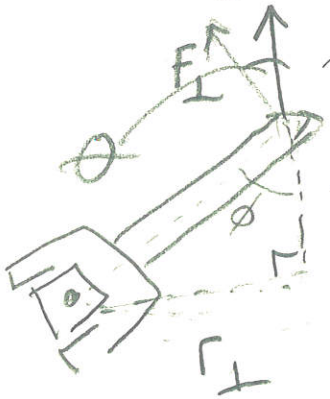
$$\Delta\theta = 90 \text{ rev}$$

Torque: Ability of a force to cause a rotation depends on



1) Magnitude of force
2) distance from the pivot
3) Angle @ which force is applied

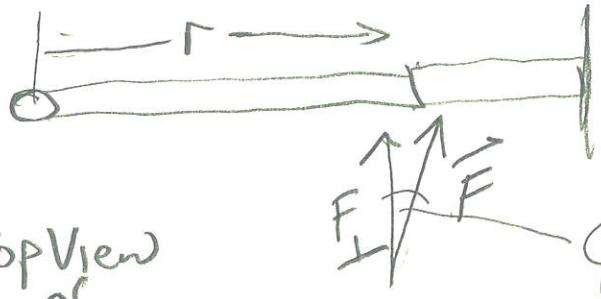
Together they make Torque



$\tau = r F_\perp$

$\phi + \theta = 90°$

Torque to open a stuck door



$r = 0.75\,m$

$\vec{F} = 240\,N$

Top View of door

What is torque on the door?

$F_\perp = \vec{F} \cdot \cos 20°\ N$

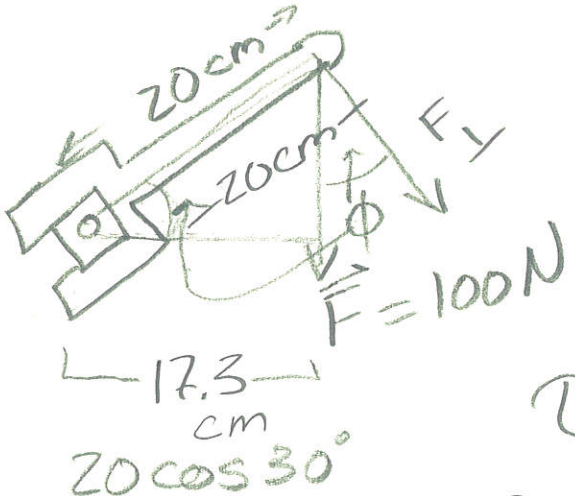
$= 240 \cdot \cos(20°)\ N$

$= 226\ N$

$$\tau = r F_\perp = (0.75\,m)(226\,N)$$

$$\tau = 170\,N\cdot m$$

Calculate the torque on a nut



$\phi = 30°$

$$\tau = r F_\perp = r_\perp F$$

$$\tau = 0.2\,m\left[100\cos 30°\right] = 0.173_{m}^{\sim}\,100N$$

$$\tau = -17\,N\cdot m = -17\,N\,m$$