

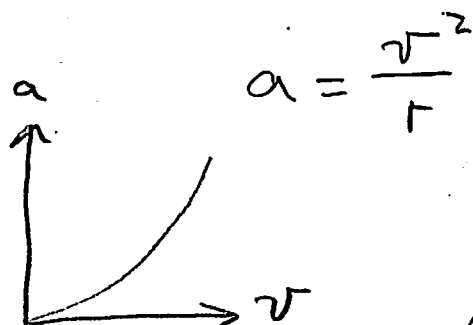
7 Feb 14

# Circular Motion

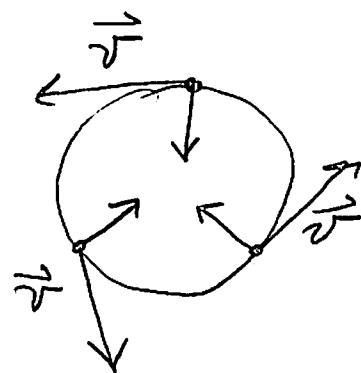
Ch 6

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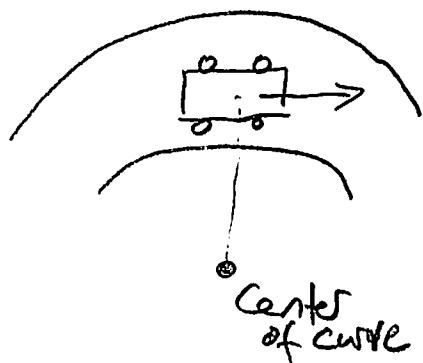
Speed  $\neq$  Velocity  
 $s \neq \vec{v}$



$$a = \frac{v^2}{r}$$



Centripetal acceleration  
 for uniform circular motion



$\vec{v} = \text{east}$

Where is  $\vec{a}$ ?

Towards the center

the acceleration is  
 due to the change  
 in direction, not the  
 change in speed

for a circle, motion is periodic,  $T$ . You go around the  
 same place again & again

Rather than expressing time for a revolution, we use  
frequency,  $f$

$$f = \frac{1}{T} \quad ; \quad \text{revolutions (count) per second}$$

If we know distance traveled, we know the radius,  $r$

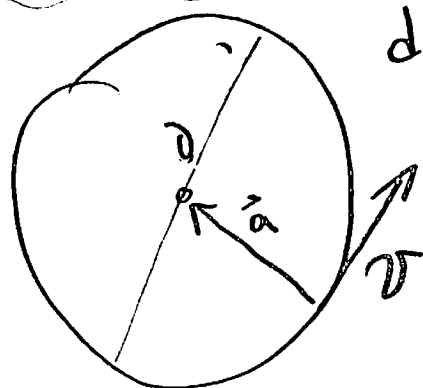
$$v = \frac{2\pi r}{T} \quad \text{or} \quad v = 2\pi f r$$

Tree/Dylan 120 PM

Combining equations:

$$a = \frac{v^2}{r} = \frac{(2\pi f)^2 r}{1}$$

$$a = \left( \frac{2\pi}{T} \right)^2 r$$



$d = 120 \text{ mm}$

$f = 540 \text{ rpm}$

What is the time  
of 1 revolution,  $T$   
velocity @ edge?  
acceleration?

Step 1 Convert to SI

$$d = 120 \frac{\text{mm}}{1000 \frac{\text{mm}}{\text{m}}} = 0.12 \text{ m}$$

$$f = 540 \frac{1}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 9 \frac{1}{\text{sec}}$$

$d = 0.12 \text{ m}$

$r = 0.06 \text{ m}$

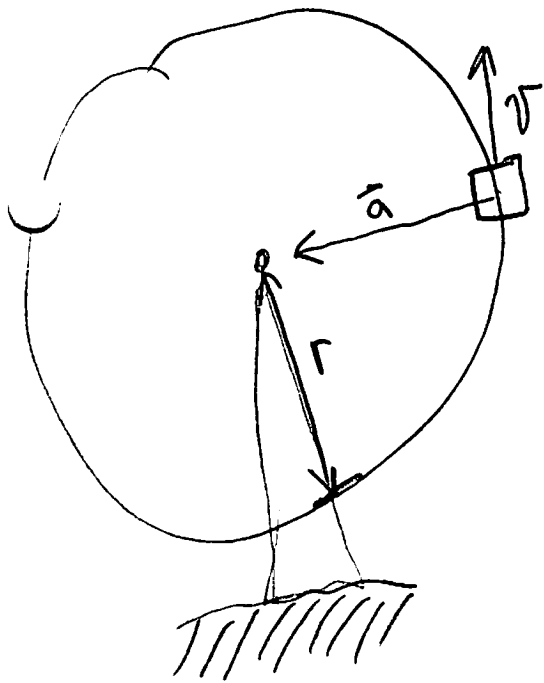
$f = 9 \frac{\text{rev}}{\text{sec}}$

$$T = \frac{1}{f} = \frac{1}{9 \frac{1}{\text{sec}}} = \frac{1}{9} \text{ sec} = 0.11 \text{ sec}$$

$$v = 2\pi f r = 2 \left( \frac{22}{7} \right) 9 \frac{1}{\text{sec}} \cdot 0.06 \text{ m} = 3.39 \frac{\text{m}}{\text{sec}}$$

$$\vec{a} = (2\pi f)^2 r = \left[ 2 \left( \frac{22}{7} \right) 9 \right]^2 \cdot 0.06 = 192 \frac{\text{m}}{\text{sec}^2}$$

So, Period is 0.11 sec, velocity is 3.4 m/sec, Accel is 190 m/sec<sup>2</sup>



$$r = 5\text{m}$$

$$a_{\text{max}} = 20 \text{ m/s}^2$$

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Find period,  $T$   
Velocity,  $v$

@ safe max speed

$$a = \frac{v^2}{r} \Rightarrow (ar)^{\frac{1}{2}} = v$$

$$\left(20 \frac{\text{m}}{\text{sec}^2} \cdot 5\text{m}\right)^{\frac{1}{2}} = v$$

$$\left(100 \frac{\text{m}^2}{\text{sec}^2}\right)^{\frac{1}{2}} = v$$

$$\underline{\underline{10 \text{ m/sec} = v}}$$

$$a = \left(\frac{2\pi}{T}\right)^2 r$$

$$\left(\frac{a}{r}\right)^{\frac{1}{2}} = \frac{2\pi}{T}$$

$$\sqrt{\frac{r}{a}} = \frac{T}{2\pi}$$

$$2\pi \sqrt{\frac{r}{a}} = T$$

$$\frac{2 \cdot 22}{7} \sqrt{\frac{5}{20}} = T$$

$$\underline{\underline{3.1 \text{ sec} = T}}$$

$$\left[\frac{\frac{\text{m}}{\text{sec}^2}}{\text{m}}\right]^{\frac{1}{2}} = \frac{1}{\text{sec}}$$

$$\left[\frac{1}{\text{sec}^2}\right]^{\frac{1}{2}} = \frac{1}{\text{sec}}$$

$$\left[\frac{1}{\text{sec}}\right] = \left[\frac{1}{\text{sec}}\right]$$

At maximum safe acceleration,  
the carriage travels @

$$10 \frac{\text{m}}{\text{sec}} \approx 25 \text{ mph}$$

and has periodicity = 3.1 sec

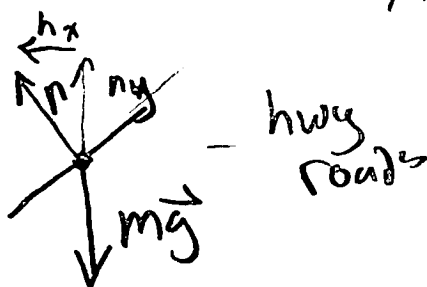
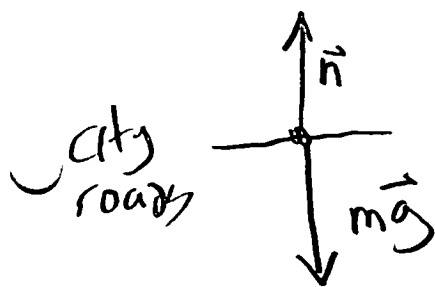
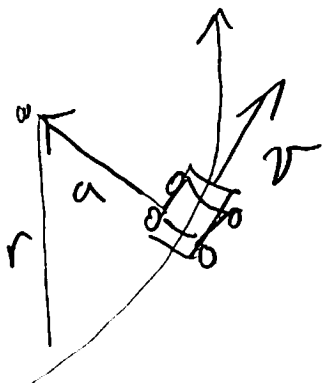
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Uniform motion Dynamics of Circular motion 4/4



$$F_{\text{net}} = m\vec{a} = \left( \frac{mv^2}{r}, \text{towards center} \right)$$

Newton's First: Object  
in motion stays  
in a straight line  
motion, unless...  
Acted upon by an  
outside force



$$\vec{f} = \mu \vec{n}$$

$\mu$  = coeff friction

$\mu_k$  = kinetic friction

$\mu_s$  = static friction

$\mu_r$  = rolling friction

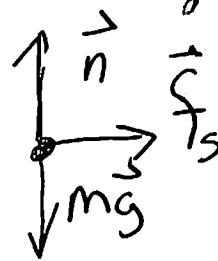
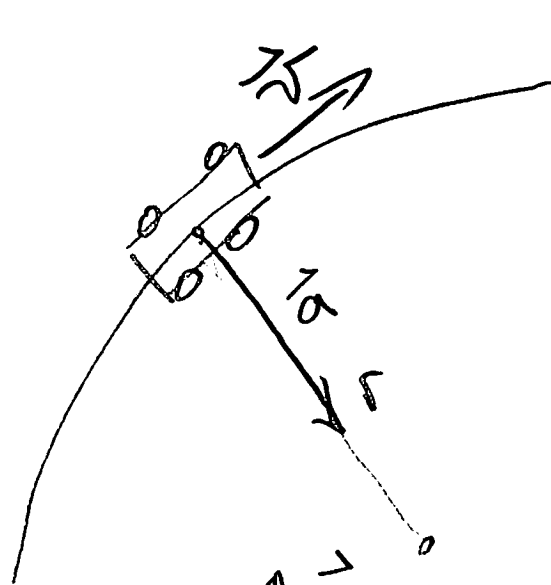
1500 kg car

$r = 20\text{m}$

Non banked road

[City road; intersection]

find the max velocity  
you can travel



$$\vec{f}_s = \mu_s \vec{n}$$

$\mu_s = 1.0$  for  
dry road;  
regular  
tires

$$\Sigma F_y: n - mg = 0$$

$$\Sigma F_x = f_s = \frac{mv^2}{r}$$

$$\Sigma F_y: n = mg$$

$$n = 1500(9.81)$$

$$\vec{n} = \underline{\underline{14715\text{ N}}}$$

$$\vec{f}_s = \mu \vec{n}$$
$$= (1.0)(14715\text{ N})$$

$$\vec{f}_s = \underline{\underline{14715\text{ N}}}$$

$$\Sigma F_x: \vec{f}_s = \frac{mv^2}{r} \Rightarrow$$

$$f_s \frac{r}{m} = v^2$$

$$\sqrt{14715 \frac{20}{1500}} = v$$

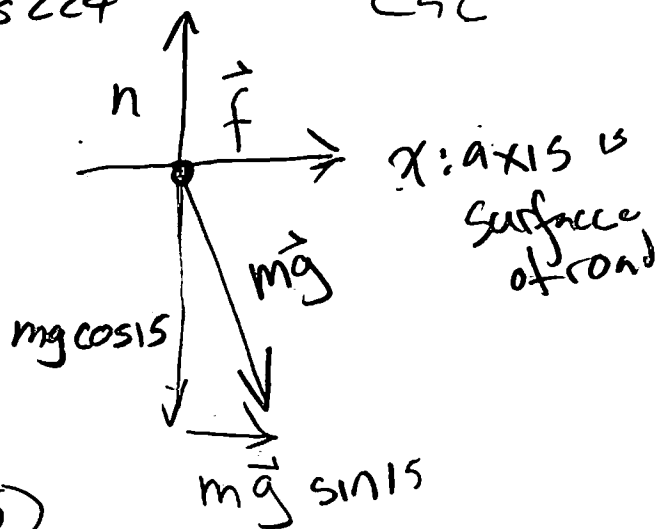
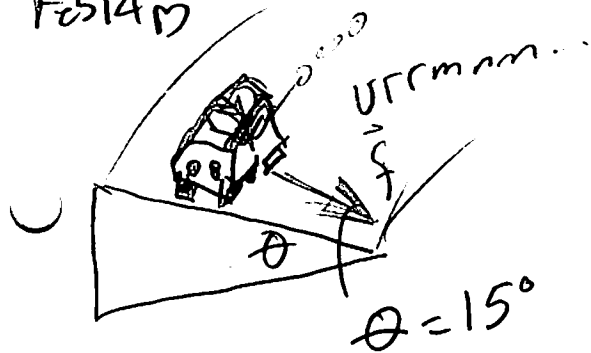
$$14\text{ m/s} = v$$
$$30\text{ mph} \approx v$$

Fes14m

Phys 224

C4C

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$$\Sigma F_x : \vec{f} + mg \sin(15)$$

$$\Sigma F_y : \emptyset = n - mg \cos 15$$

$$n = 1500 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cos(15)$$

$$\vec{n} = 14,213 \text{ N}$$

$$\vec{F}_x : \mu_s n + mg \sin(15)$$

$$14213 \text{ N} + 9,81 \cdot 1500 \cdot \sin(15)$$

$$\vec{F}_x = 18,021 \text{ N} = m \frac{v^2}{r}$$

$$18021 = \frac{1500}{20} v^2$$

$$240.3 = v^2$$

$$15.5 \text{ m/s} = v$$

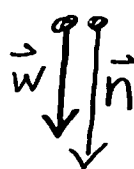
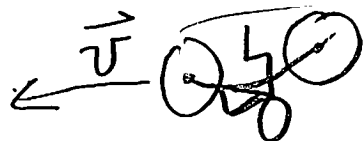
So, banking gets you a little more speed...

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Phys 2224  
Circular Motion CSC

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Ring of death



$$r = 2.2 \text{ m}$$

$$\vec{n} \geq \vec{w}$$

$$\sum F_x : mg + mg = \frac{mv_{\min}^2}{r}$$

$$2mg = \frac{mv_{\min}^2}{r}$$

$$2gr = v^2$$

$$(2gr)^{\frac{1}{2}} = v$$

$$\sqrt{2 \cdot 9.81 \cdot 2.2} = v$$

$$6.6 \text{ m/s} = v$$

The minimum speed to stay in the ring of death is

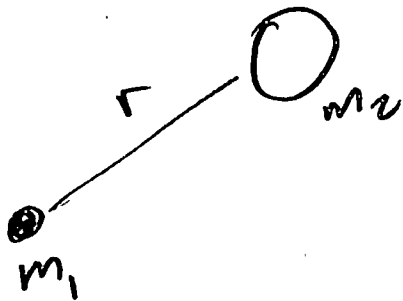
6.6 m/s or  $\approx 15$  mph

Fes14M

# Newton's Laws of Gravity <sup>CSC 8/9</sup>

Every object attracts another object  
Force is inversely proportional to

- (1) Square of dist apart
- (2) Square of the mass



$$F_{1on2} = F_{2on1} = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{Kg}^2}$$

Force on two students

Students = 65 kg

dist = 0.6 m

How much are they attracted to one another?



$$F_{1on2} = \frac{G \cdot m_1 m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \cdot 65 \cdot 65}{(0.6)^2}$$

$$= 7.48 \times 10^{-7} \text{ N}$$

So, the bigger the students are, the more they are attracted to one another



Fes14D

Phys 224

CSC

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Find your attraction  
to the earth ...

$$r = 6.37 \times 10^6 \text{ m}$$

$$M_{\text{earth}} = 5.98 \times 10^{24} \text{ Kg}$$

$$F_{\text{Earth on you}} = \frac{4 \cdot 65 \cdot 5.98 \times 10^{24}}{[6.37 \times 10^6]^2}$$

$$= \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{6.37^2 \times 10^{12}}$$

$$\frac{(6.67 \cdot 5.98) \times 10^{13}}{6.37^2 \times 10^{12}}$$

$$0.983 \times 10^1$$

$$9.83 \text{ m/sec}^2$$

w/rounding error,  
the same number...