Answers for Quiz 3

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1. $(p \land (p \to q) \land (q \to r)) \to r$ Note that this is also: $p, p \to q, q \to r \vdash r$.

$$\begin{array}{c|cccc} 1 & & p \\ 2 & & p \rightarrow q \\ 3 & & q \rightarrow r \\ 4 & & q & \rightarrow \text{E 1,2} \\ 5 & & r & \rightarrow \text{E 3,4} \\ \end{array}$$

2. (a) Given $A = \{2, 5, 7, 8\}$, $B = \{2, 5, 4\}$, and $C = \{5, 10, 1\}$:

$$B - A = \{4\}$$

$$C - A = \{1, 10\}$$

$$(B - A) \cup (C - A) = \{1, 4, 10\}$$

$$B \cup C = \{1, 2, 4, 5, 10\}$$

$$(B \cup C) - A = \{1, 4, 10\}$$

∴
$$(B - A) \cup (C - A) = (B \cup C) - A$$
.
(b) Show $(B - A) \cup (C - A) = (B \cup C) - A$:

$$(B-A)\cup(C-A)=(B\cap\overline{A})\cup(C\cap\overline{A}) \qquad \qquad \text{definition of set difference}$$

$$=(B\cup C)\cap\overline{A} \qquad \qquad \text{distribution}$$

$$=(B\cup C)-A \qquad \qquad \text{definition of set difference}$$