1. 3D variables

BLE Benney-Luke equations with non-dimensional variables $(t, x, y) \in \mathbb{R}^3$:

BLE1
$$\partial_t \Phi - \frac{\mu}{2}\partial_t \nabla^2 \Phi + \frac{\epsilon}{2}\left|\nabla\Phi\right|^2 + \eta = 0 \quad \text{in} \quad \Omega_h, \tag{1.1a}$$

BLE2
$$\partial_t \eta - \frac{\mu}{2}\partial_t \nabla^2 \eta + \nabla \cdot \big((1+\epsilon\eta)\,\nabla\Phi\big) - \frac{2}{3}\mu\nabla^4\Phi = 0 \quad \text{in} \quad \Omega_h, \tag{1.1b}$$

bd1
$$\mathbf{n} \cdot \nabla\Phi = 0 \quad \text{on} \quad \partial\Omega_h, \tag{1.1c}$$

bd2
$$\mathbf{n} \cdot \nabla(\nabla^2\Phi) = 0 \quad \text{on} \quad \partial\Omega_h. \tag{1.1d}$$

The way of scaling the non-dimensional variables (t, x, y) into dimensional variables $(\widehat{t}, \widehat{x}, \widehat{y})$ is as follows:

$$x = \frac{\sqrt{\epsilon}}{H_0}\widehat{x}, \quad y = \frac{\sqrt{\epsilon}}{H_0}\widehat{y}, \quad z = \frac{1}{H_0}\widehat{z}, \quad t = \frac{\sqrt{gH_0\epsilon}}{H_0}\widehat{t}, \quad \eta = \frac{1}{\epsilon H_0}\widehat{\eta} \tag{1.2}$$

where $H_0(m)$ is depth of water and g is the gravitational constant $9.8m/s^2$.