Summary of project on simulating surface plasmon gap waveguides

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During my Masters degree in Photonics (Lasers and Photonic Materials), I worked on two projects for my Masters' project course:

- I followed the proof of 3D weak form of the Conjugate Gradient FFT method (equations 1 to 51 of [1: *The Three-Dimensional Weak Form of the Conjugate Gradient FFT Method for Solving Scattering Problems, P. Zwamborn and P. M. van den Berg, IEEE Mic. Theo. Tech. 40 (1992) 1757*]) for finding the scattered EM wave from a general geometry, and simulated the 3D scattering from a metallic surface plasmon gap waveguides [2: *Simulation of practical nanometric optical circuits based on surface plasmon polariton gap waveguides, K. Tanaka, M. Tanaka, and T. Sugiyama, Opt. Express, 13 (2005) 256*] depicted in the following box (Fig. 4) in FORTRAN.

More details:

I have driven the following 3D scattering problem in free space (zero charge/current sources), presented in [1], via Maxwell equations and Green function formalism:

$$\vec{E}^{i}(\vec{x}) = \frac{\vec{D}(\vec{x})}{\varepsilon(\vec{x})} - \frac{1}{\varepsilon_0}\int_{\vec{x}'\in D^S} \frac{\exp(ik_0|\vec{x}-\vec{x}'|)}{4\pi|\vec{x}-\vec{x}'|}\frac{\varepsilon(\vec{x})-\varepsilon_0}{\varepsilon(\vec{x})}[k_0^2 D(\vec{x}')]d\vec{x}' - \frac{1}{\varepsilon_0}\int_{\vec{x}'\in D^S} \frac{\exp(ik_0|\vec{x}-\vec{x}'|)}{4\pi|\vec{x}-\vec{x}'|}\frac{\varepsilon(\vec{x})-\varepsilon_0}{\varepsilon(\vec{x})}\vec{\nabla}[\vec{\nabla} D(\vec{x}')]d\vec{x}'$$

Eq. (5)

where

$\boldsymbol{E}^i = (E_1^i,\ E_2^i,\ E_3^i)$ is the incident electric field at a general 3D coordinate $\vec{x}$,

$\boldsymbol{D} = (D_1,\ D_2,\ D_3)$ is the unknown electric flux density over the object domain D^S,

$\epsilon(\boldsymbol{x})$ is the absolute permittivity of over the 3D object, and

ϵ_0 is the permittivity of the vacuum, and $k_0 = \omega(\epsilon_0\mu_0)^{1/2}$ is the wavenumber of the incident electric field with time factor $\exp(-i\omega t)$ in vacuum.

Using following points, the above integral equation (Eq. (5)) leads to a linear matrix equation of the form $e^{i} = Ld$, which is solvable using GMRES (Generalized Minimal Residual) and FFT algorithms.

Points to convert scattering integral equation to a linear matrix form:

1. Sace discretization as following:

   For a general point $\boldsymbol{x} = (x_1, x_2, x_3)$ in space, with grid widths: Δx_1, Δx_2 and Δx_3, the discrete mesh points were considered as [1]: $x_1 = M\,\Delta x_1$, $x_2 = N\,\Delta x_2$ and $x_3 = P\,\Delta x_3$,

   $$\boldsymbol{x}_{M,N,P} = \{(M - \tfrac{1}{2})\,\Delta x_1,\ (N - \tfrac{1}{2})\,\Delta x_2,\ (P - \tfrac{1}{2})\,\Delta x_3\}$$

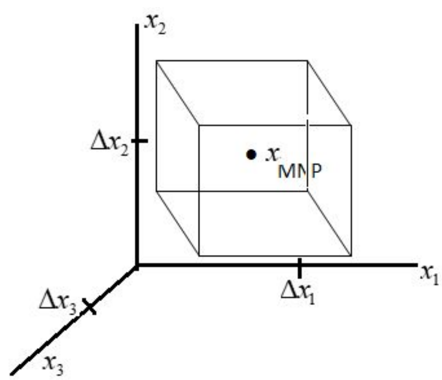
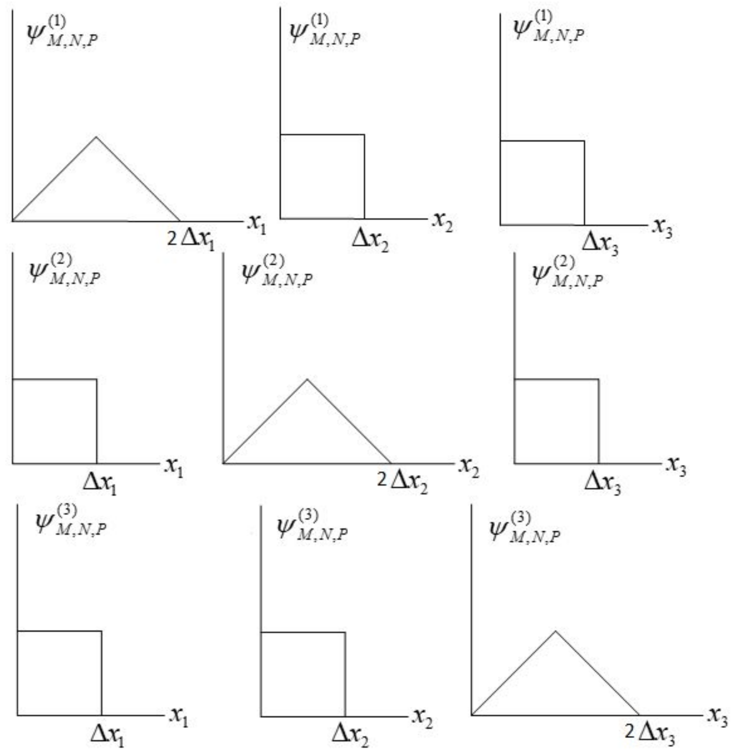


Fig. 3: the schematic of discretizing the space in and outside of the object

2. Roof-top functions, i.e the basis or expansion functions, were considered as following to simplify the grad-div operator in the second integral of Eq. (5).



3. The three parameters of Eq. (5), i.e. D, E^i, and the vector potential A:

$$\vec{A}(\vec{x}) = \frac{1}{\varepsilon_0}\int_{\vec{x}'\in D^S} \frac{\exp(ik_0|\vec{x}-\vec{x}'|)}{4\pi|\vec{x}-\vec{x}'|}\,\frac{\varepsilon(\vec{x})-\varepsilon_0}{\varepsilon(\vec{x})}\,D(\vec{x}')d\vec{x}'$$

are expanded on the basis functions introduced in point 3:

$$\boldsymbol{D}_q(\boldsymbol{x}) = \epsilon_0 \sum_{I,J,K} d^{(q)}_{I,J,K}\psi^{(q)}_{I,J,K}(\boldsymbol{x}) \qquad \text{for } \boldsymbol{x} \in \mathbb{D}^S,$$

$$\boldsymbol{A}_q(\boldsymbol{x}) = \sum_{I,J,K} \boldsymbol{A}^{(q)}_{I,J,K}\psi^{(q)}_{I,J,K}(\boldsymbol{x}) \qquad \text{for } \boldsymbol{x} \in \mathbb{D}^S,$$

$$\boldsymbol{E}^i_q(\boldsymbol{x}) = \sum_{I,J,K} \boldsymbol{E}^{i,(q)}_{I,J,K}\psi^{(q)}_{I,J,K}(\boldsymbol{x}) \qquad \text{for } \boldsymbol{x} \in \mathbb{D}^S.$$

4. Simplifying the discrete form of vector potential A using the convolution theorem of Discrete Fourier Transform (DFT)

We then tried to simulate the scattering of a plane wave around a metallic structure, called surface plasmon gap waveguide, depicted in Fig. 4 (or lower part: Fig 1 of [2]).

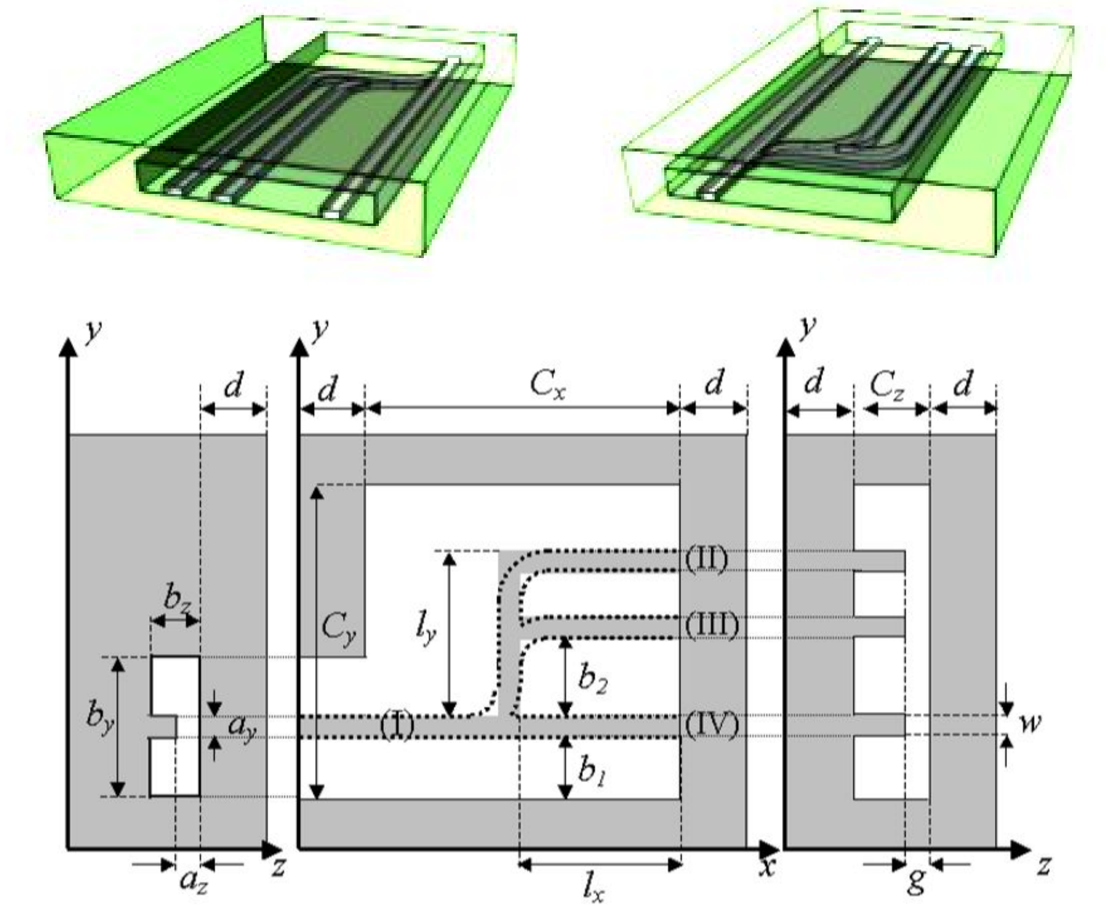


Fig. 4 The top two figures are made by me to show the 3D schematic of the circuit of the second row which is a Surface Plasmon Gap Waveguide studied in [2]. The exact dimensions marked one the network are considered in the simulation as those of in [2].

The absolute permittivity of the object in each space mesh was found according to Fig. 4, and the linear matrix equation for the scattering from such geometry was written in FORTRAN. Our final results of the simulation are open; however ref [2] shows such results as following:

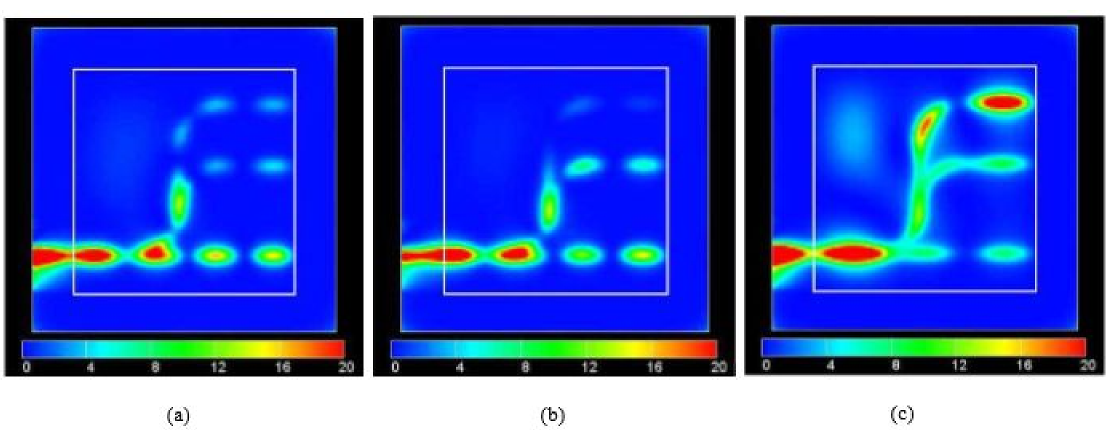


Fig. 5 (Fig. 5 of [2]). This graph shows that by changing the ridge height k_0h (h=C_z-g in Fig. 4) the input light from the left chooses different paths. Graph (a), (b), and (c) correspond to k_0h =1, 0.6, and 0.2, respectively.