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LETTER

Interpreting nonlinear semi-elasticities in reduced-form climate damage estimation

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Abstract Observed changes in weather can reveal marginal impacts of climate change on economic outcomes and the potential for adaptation. When modeling the nonlinear relationship between weather and changes in economic outcomes empirically, model choice can confound the interpretation of marginal and percentage effects and their respective confidence intervals. I present a simple solution for better characterizing semi-elasticities of nonlinear climate damages, and evaluate its relevance in interpreting empirical climate damages. For small marginal effects, the implications of this interpretation error is small; for larger effects, however, the misinterpretation error can be substantial.

1 Introduction

There is a quickly growing slate of empirical research that regresses changes in economic factors on realized weather outcomes (Carleton and Hsiang 2016; Dell et al. 2012, 2014; Burke et al. 2015). When using natural logarithms to transform dependent variables in regression equations, however, the marginal effects of discontinuous explanatory variables—such as dummy variables or temperature bins—cannot be interpreted directly as percentage changes. In this letter, I highlight the implications of improper interpretation of point estimates and their confidence intervals in climate damage estimation. I offer a simple solution and demonstrate its relevance with two illustrations.

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2 Modeling framework

In a representative, stylistic model, let Y_{it} be economic output in levels for country i at time t.[1] Often, we are interested in how weather may affect changes in the growth rate of economic output, Y_{it}, which can be facilitated by taking its log transform then first differencing the resulting quantities (e.g., $\ln(Y_{it})-\ln(Y_{it-1})$), which results in a semi-logarithmic reduced-form regression of the form:

$$\ln(Y^{\star}_{it}) \equiv \ln(Y_{it}/Y_{it-1}) = \beta f(T_{it}) + \gamma X_{it} + \varepsilon_{it} \quad \text{with } \varepsilon_{it} = u_i + \eta_{it}. \tag{1}$$

In this model, $f(T_{it})$ is some function of temperature T for country i at time t, X_{it} are other factors thought to influence economic growth, and ε_{it} is comprised of a unit-specific effect and an idiosyncratic error.

Within models of this form, we are primarily interested in estimating the population effect of temperature on economic growth.[2] That is, how does the growth rate of the economy for a given country change due to marginal changes in temperature? For continuous functions f, such as commonly used linear or quadratic specifications, a direct estimate of $\hat{\beta}$ provides the percentage effect on growth of marginal changes in temperature.

Many empirical papers, however, use piece-wise functions of temperature, number of days in temperature "bins", or other discontinuous forms of f to capture nonlinear responses to weather (e.g., Barreca et al. 2016; Burke et al. 2015; Burke and Emerick 2016; Deryugina and Hsiang 2014; Dell et al. 2012; Schlenker and Roberts 2006, 2009). Authors commonly specify bins at 1–3 °C intervals, 10 °F intervals, or fewer bins where observations beyond a given threshold are assigned a positive value.

For simplicity, assume $f(T_{it}) = D_{it}$, where $D_{it} = 1$ if average temperature at time t exceeds 90 °F and 0 otherwise. With discretized values of f, it is easy to make the mistake of interpreting $\hat{\beta}$ as a percentage effect of D_{it} on changes in outcomes. This nonlinear specification introduces problems for interpretation because $\partial \ln(Y^{\star}_{it})/\partial D_{it}$ is undefined—a problem in semi-logarithmic regression models with explanatory dummy variables that was first confronted by Halvorsen and Palmquist (1980). To see this, we can rewrite Eq. 1:

$$Y^{\star}_{it} = (1+g)^{D_{it}} \times \exp\left(\gamma X_{it} + \varepsilon_{it}\right), \tag{2}$$

where g is defined as the percentage effect of D_{it} on the dependent variable.[3] From Eq. 2, it is apparent that $g \neq \beta$. Rather, $\hat{\beta}$ provides an estimate of $\ln(1+g)$. That is, the marginal effect cannot be interpreted directly as a semi-elasticity. By assuming the original error is log-normally distributed, and that $\hat{\beta}$ is a consistent estimate of β, Kennedy (1981) proposed the following estimator for g:

$$\hat{g} = \exp\left(\hat{\beta} - \frac{1}{2}\hat{V}(\hat{\beta})\right) - 1, \tag{3}$$

[1] This framework extends to models with any logarithmic dependent variable.

[2] Alternatively, this equation could represent the effect of growing degree days on log agricultural yields, or the number of days above 90 °F on the log mortality rate.

[3] Formally, $g = [Y(D_{it}=1) - Y(D_{it}=0)]/Y(D_{it}=0)$.

where $\hat{V}(\hat{\beta})$ is an estimate of the variance of $\hat{\beta}$. van Garderen and Shah (2002) go on to propose the following simple approximate unbiased estimator for the variance of $\hat{g}$:

$$\tilde{V}(\hat{g}) = \exp(2\hat{g})\left[\exp(-\hat{V}(\hat{\beta})) - \exp(-2\hat{V}(\hat{\beta})\right]. \tag{4}$$

The insight thus far is not new, although it is often overlooked in empirical applications. Further, the issue extends beyond interpreting simple dummy variables in semi-logarithmic equations. Namely, I contend that semi-logarithmic equations with "continuous" binned weather variables, such as growing degree days or heating degree days, also fall prey to this interpretation issue. To see this in a simplified example, consider again the dummy variable D_{it} that equals one if temperatures exceed 90 °F on a given day and zero otherwise. Now, define a binned variable $N_{it} = \sum D_{it}$ over the course of some fixed time period, so that a value of 10 indicates that there was 10 instances of greater than 90 °F days for region i during time t. N_{it} is often claimed to better capture exposure to the full temperature distribution than simple dummy variables.

Authors then claim to estimate a regression on some outcome R_{it} of the form,

$$\ln(R_{it}) = \zeta_1 N_{it} + \gamma X_{it} + u_i + \eta_{it}, \tag{5}$$

and interpret ζ_1 as the marginal effect of substituting 1 day in the omitted temperature bin (less than 90 °F) for 1 day in the 90° temperature bin as a semi-elasticity (see, e.g., Barreca et al. 2016). This interpretation is incomplete; the correct interpretation of ζ_1 is the marginal effect of one additional day in the 90° bin *conditional on being in the 90° bin*. By construction of the "continuous" variable N_{it}, there is an implicit interactive term in Eq. 5 that confounds interpretation. That is, authors actually estimate

$$\ln(R_{it}) = \zeta_2 N_{it} D_{it} + \gamma X_{it} + u_i + \eta_{it}. \tag{6}$$

The marginal effect of an additional day in the 90° temperature bin is thus $\partial \ln(R_{it})/\partial N_{it} = \zeta_2 D_{it}$ and must account for the discontinuous effect of any day being realized in the 90° bin. $\partial \ln(R_{it})/\partial D_{it}$, which is undefined mathematically, picks up the discrete effect of moving from 0 to 1 day in that bin, while $\partial \ln(R_{it})/\partial N_{it}$ picks up the effect of moving from 1 to 2 days, 2 to 3 days, and so forth. So, estimated coefficients on temperature variables using the binned approach similar to the one described here, which are functions of dummy variables, cannot be interpreted as percent changes.

I now assess the magnitude of potential interpretation mistakes. By evaluating the estimator, $\hat{g}$, and its variance in Table 1, we see that for $\hat{\beta}$ close to zero, the error is relatively small. The percentage effect is larger in absolute magnitude for all estimates of $\hat{\beta}$, meaning the negative marginal impacts overestimate the negative percentage effect while positive marginal impacts underestimate the percentage effects. Differences between the percentage effect and the marginal effect also increase with the precision with which $\hat{\beta}$ is estimated. The implied error in the estimates of the variance for $\hat{g}$ are relatively large even for small marginal effects (i.e., within the interval [−0.10, 0.10]). For $\hat{\beta} = -0.05$ evaluated at a critical level of significance for $p < 0.05$, the percentage effect possesses a standard error 5.2% smaller than that of the marginal effect.

To illustrate the magnitude of these interpretation errors, in Fig. 1, I present marginal coefficient estimates relative to their corresponding percentage effects, using standard errors for inference at $p < 0.1$. The solid black line depicts a naive estimate of the percentage effect, whereas the dashed blue line represents the relevant percent change adjusted by

Table 1 Simulated effects of point estimates for $\hat{g}$, its variance, and interpretation errors

	Using standard errors for inference at:							
	$p < 0.20$	(% error)	$p < 0.10$	(% error)	$p < 0.05$	(% error)	$p < 0.01$	(% error)
Panel A: Simulated estimates of the percentage effect ($\hat{g}$) relative to the marginal effect ($\hat{\beta}$)								
$\hat{\beta} = -0.25$	−0.236	(6.0)	−0.230	(8.6)	−0.228	(9.9)	−0.226	(10.8)
$\hat{\beta} = -0.10$	−0.098	(2.1)	−0.097	(3.3)	−0.096	(3.8)	−0.096	(4.2)
$\hat{\beta} = -0.05$	−0.049	(1.0)	−0.049	(1.6)	−0.049	(1.9)	−0.049	(2.1)
$\hat{\beta} = -0.01$	−0.010	(0.2)	−0.010	(0.3)	−0.010	(0.4)	−0.010	(0.4)
$\hat{\beta} = 0$	–	–	–	–	–	–	–	–
$\hat{\beta} = 0.01$	0.010	(0.2)	0.010	(0.3)	0.010	(0.4)	0.010	(0.4)
$\hat{\beta} = 0.05$	0.050	(0.9)	0.051	(1.5)	0.051	(1.8)	0.051	(2.0)
$\hat{\beta} = 0.10$	0.102	(1.8)	0.103	(3.0)	0.104	(3.6)	0.104	(4.0)
$\hat{\beta} = 0.25$	0.260	(3.8)	0.269	(7.2)	0.274	(8.6)	0.277	(9.6)
Panel B: Simulated estimates of the standard error of the percentage effect ($[\hat{V}(\hat{g})]^2$)								
$\hat{\beta} = -0.25$	0.148	(32.1)	0.116	(30.6)	0.098	(30.0)	0.083	(29.5)
$\hat{\beta} = -0.1$	0.070	(11.0)	0.055	(10.8)	0.046	(10.7)	0.039	(10.7)
$\hat{\beta} = -0.05$	0.037	(5.2)	0.029	(5.2)	0.024	(5.2)	0.020	(5.2)
$\hat{\beta} = -0.01$	0.008	(1.0)	0.006	(1.0)	0.005	(1.0)	0.004	(1.0)
$\hat{\beta} = 0$	–	–	–	–	–	–	–	–
$\hat{\beta} = 0.01$	0.008	(1.0)	0.006	(1.0)	0.005	(1.0)	0.004	(1)
$\hat{\beta} = 0.05$	0.041	(4.8)	0.032	(4.8)	0.027	(4.8)	0.023	(4.8)
$\hat{\beta} = 0.1$	0.086	(9.1)	0.067	(9.3)	0.056	(9.3)	0.047	(9.4)
$\hat{\beta} = 0.25$	0.243	(19.9)	0.192	(20.8)	0.162	(21.2)	0.137	(21.4)

Eq. 3.[4] Beyond 0.5 and −0.5, the difference between marginal and percentage effects is substantial, but for small $\hat{\beta}$, the effects are similar.

Before proceeding, it is worth noting an important point about these potential interpretation errors. They are just that: errors in interpretation. The goal of this empirical literature is to estimate causal effects of weather fluctuations to infer something about likely impacts of climate change. Each of the papers using this approach cited in this manuscript provides plausible and consistent estimates of marginal effects within their respective econometric designs. Several papers in this literature, however, misinterpret marginal effects as percent changes. As seen in Table 1 however, for small marginal coefficients, the implied error is small relative to other sources of potential bias in econometric estimation.

3 Empirical illustrations

To illustrate this point more concretely, consider the model developed by Burke et al. (2015) (BHM), explaining nonlinear changes in global GDP growth per capita at the country-year

[4]Note that $\hat{g}$ and $\hat{\beta}$ need not be equivalent at $\hat{\beta} = 0$ because of the variance estimate that scales $\hat{g}$. For the purpose of this figure, I set $\hat{V}(\hat{\beta}) = 0$ for $\hat{\beta} = 0$ (see van Garderen and Shah (2002) for further discussion).

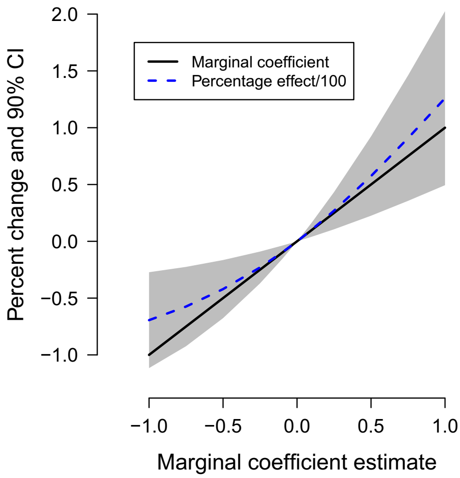

Fig. 1 Interpretation errors for marginal effects relative to percentage effects calculated with standard errors for inference at $p < 0.10$. 90% confidence intervals presented for the percentage effect

level by changes in temperature.[5] Results presented in Fig. 2a correspond to BHM's base estimates in model 1 of their Extended Data Table 1, inclusive of country fixed effects, year fixed effects, and quadratic country time trends, with errors clustered at the country level. Temperature is measured in degrees C and precipitation in meters.[6]

BHM model the function f in Eq. 1 as quadratic, which is replicated in Fig. 2a. By modeling the nonlinearities of weather more flexibly, similar in spirit to empirical models used by Barreca et al. (2016) and Schlenker and Roberts (2009), I construct a set of dummy variables representing 1-degree temperature bins spanning the observed temperature distribution instead of BHM's quadratic temperature specification. Including this set of dichotomous variables in lieu of a smooth function requires fewer ex ante assumptions about the relationship between temperature and our outcome variable.

Specifically, the model is specified as

$$\Delta Y_{it} = \sum_{s=1}^{S} \beta_s \mathbb{1}(Temp_{it} = s) + \lambda_1 P_{it} + \lambda_2 P_{it}^2 + \mu_i + \upsilon_t + \theta_i t + \theta_{i2} t^2 + \varepsilon_{it} \qquad (7)$$

where the ΔY_{it} is the first difference of the natural log of annual real (inflation-adjusted) gross domestic product per capita; $\mathbb{1}(Temp_{it} = s)$ is an indicator function equal to 1 if the temperature for country i at time t falls within temperature bin s, defined in 1 °C intervals, relative to the omitted bin: 15–16 °C; μ_i and υ_t are country and time fixed effects; and t and t^2 serve as a country-specific quadratic time trend.

The coefficients of interest are the set of S marginal effects, $\hat{\beta}_s$, and are presented in Fig. 2a. Each coefficient can be interpreted as the marginal effect on economic growth from one additional year with mean temperatures being in temperature bin s, relative to the omitted category. Of course, these coefficients should not be interpreted as percentage changes. By applying the estimator for $\hat{g}$ and its variance in Eqs. 3 and 4, we can translate the marginal effects into a set of S percentage effects. Both $\hat{g}_s$ and its 90% confidence

[5]For details on the specification of BHM main regression results, see their supplementary information section B.1.

[6]See Burke et al. (2015) for details. Replication code and data was accessed from: https://purl.stanford.edu/wb587wt4560 (Last accessed: Feb. 7, 2017).

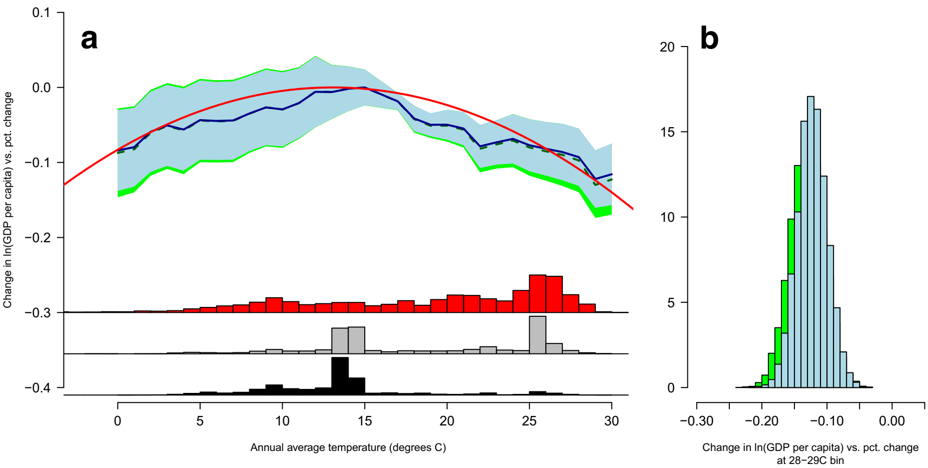

Fig. 2 Effect of annual average temperature on changes in economic growth. **a** Global nonlinear relationship between average temperature change and change in log gross domestic product (GDP) per capita (red line, relative to optimum) during 1960–2010, reproduced from Burke et al. (2015). Dashed green line presents coefficient estimates from a set of 1 °C temperature bins, relative to the omitted bin of 15–16 °C, and 90% confidence interval (green, clustered by country, $N = 6584$). Solid blue line is a set of percentage effects transformed by the estimator in Eq. 3, along with its 90% confidence interval (light blue). All models include country fixed effects, flexible trends, and precipitation controls. Histograms display global distributions of temperature exposure (red), population (gray), and income (black). **b** Frequency distribution of 10,000 draws from marginal and percentage effect distributions using estimated (green) and transformed (blue) means and variances, evaluated at the 28–29 °C bin

interval deviates slightly from those of $\hat{\beta}_s$ as shown in Fig. 2a, particularly in the tails of the temperature distribution. A Monte Carlo exercise of these parameters in Fig. 2b, evaluated at the 28–29 °C bin, reveals percentage effects with a smaller negative central tendency and a smaller degree of dispersion than those of the marginal effects. These effects, however, are quite similar.

As an additional illustrative example, this time implementing the “binned” approach as described in Eqs. 5 and 6 considers the primary model estimated in Barreca et al. (2015). They estimate the effect of temperature on the log mortality rate at the state level over the period of 1900–2004. By controlling flexibly for unobserved shocks at the state-by-month and year-month level, Barreca et al. are able to estimate a causal effect of extreme temperatures on human health outcomes conditional on the decile of long-run temperature distribution.

In Fig. 3, I recreate the primary results from Fig. 1 in Barreca et al. (2015), which are marginal coefficient estimates. I also present percent changes and their 95% confidence intervals after transforming the coefficients and standard errors as described earlier in this paper. As shown, for large coefficient estimates in the lower deciles, the percentage effect is several percentage points greater. For smaller marginal effects in the upper tail of the temperature distribution, however, the marginal and percentage effects are virtually identical. In the right panel, I show that the variance of the percentage effect is notably larger than that of the marginal effects, conditional on being in the first decile of temperature. This example has the added benefit of showing that this interpretation error extends to any econometric model with a logarithmic dependent variable and discontinuous explanatory variables.

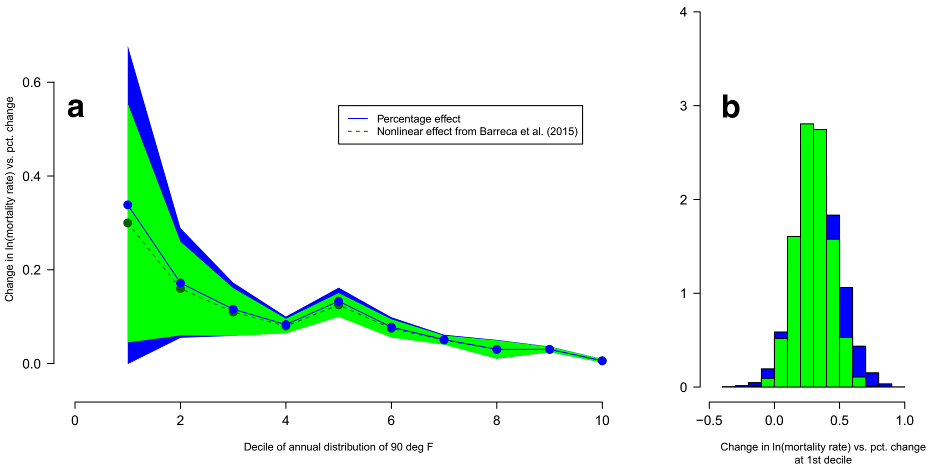

Fig. 3 Effect of days above 90 °F on changes in the mortality rate in the USA. **a** Marginal and percentage effects of days with temperature greater than 90 °F, by decile of its long-term distribution, 1900–2004. Marginal effects and their 95% confidence intervals are reproduced from Barreca et al. (2015), while percentage effects and their confidence intervals are calculated. **b** Frequency distribution of 10,000 draws from marginal and percentage effect distributions using estimated (green) and transformed (blue) means and variances, evaluated at the first decile

4 Implications

Overall, the implications of this interpretation error are small primarily because estimated marginal coefficients in this literature are small. Even small errors in interpretation, however, can lead to large welfare changes when considering such outcomes as growth rates of nations or mortality risk-reductions.

To show the effects of this error in a more extreme example, consider the working paper by Chan and Wichman (2017). In that paper, the authors model the dose-response function between outdoor recreation and weather by estimating the natural log of daily cycling demand as a function of temperature and precipitation variables, which are represented by dummy variables and emblematic of the empirical framework highlighted in Eq. 1. The authors estimate the marginal effect of an extremely wet day (with more than 1 in. of rainfall) to be an approximately 0.75 log-point reduction in the number of trips relative to a dry day. The corresponding percentage effect is only around a 55% reduction. A naive interpretation would overstate the impact of precipitation by more than 36%.

As a practical matter, there are several low-cost solutions. For estimated coefficients smaller than |0.10|, the implication of interpretation errors are likely trivial. For values greater than this, alternative model specifications can be implemented to account for nonlinear weather relationships to show agreement, as in e.g. Schlenker and Roberts (2009). For these cases, marginal coefficients should transformed into percentage effects for easy interpretation. Beyond marginal coefficients, transformed confidence intervals should also be computed for percentage effects. Alternatively, other transformations of dependent variables can be considered prior to estimation to avoid this issue entirely.

Although the differences in the examples here are relatively small, the illustrations show how improper interpretation of marginal effects as percentage effects can influence the accuracy of interpreting of climate impacts on changes in economic outcomes. Although careful econometricians are aware of this issue, the quickly growing literature in climate damage estimation needs to be sensitive to the implications of how model and specification choice can affect the interpretation of estimated parameters.

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