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Stock Selection System through Suitability Index and Fuzzy-Based Quantitative Characteristics

Jia-Hao Syu, Jerry Chun-Wei Lin*, Chi-Jen Wu, and Jan-Ming Ho

***Abstract*—With the rapid development of quantitative trading, stock selection is an ongoing task that requires consideration of the characteristics of stocks and investment strategies. Fuzzy set theory is excellent for modeling and describing abstract characteristics to establish the link between stocks and strategies. In this paper, we present our idea for a stock selection system called TripleS. It is based on the suitability index (SI) derived from fuzzy-set theory. The TripleS relies on the position size to extract the stock characteristics SI that describe not only the characteristics of each stock, but also the extent to which they are suitable for certain strategies. Then, the SI undergoes a transformation process in the hands of the developed fuzzy-set modules for stock selection and investment. There are some other proposed methods for stock selection respectively called TripleS-T1, TripleS-T2, TripleS-T2U, TripleS-EF and TripleS-EFT. Experimental results show that the proposed TripleS has minor improvement in precision but significant improvement in investment performance. The TripleS-EFT has outstanding profitability, the highest annual return in all strategies and the highest Sharpe ratio in two of the four strategies, and achieves an annual return of 10.01% and a Sharpe ratio of 1.032, which significantly outperform the benchmark system. The TripleS-EFT also significantly outperforms the state-of-the-art fuzzy-based system in measuring annual return and even has double the annual return in several strategies.**

***Index Terms*—Stock selection, trading system, position sizing, suitability index, momentum, contrarian, fuzzy-set**

I. INTRODUCTION

FINANCIAL markets are an indispensable environment to fully exploit capital and resources, and they have a profound impact on everyone. With the rapid progress of information technology and the explosion of data, institutional investors and many studies focused on quantitative trading in recent decades. Oliver-Muncharaz and García [1] noted that since 2015 there has been a sharp increase in the number published works on trading strategies, covering discussions in finance, economics, and also computer science. The use of objective and quantitative information to support investment decisions is known as quantitative trading, which is also known as algorithmic trading [2]. Potential artificial intelligence techniques for quantitative analysis include data mining, machine learning, and fuzzy-set theory.

Data mining is a powerful method for deriving useful and profitable rules from large amounts of historical data. A self-managed portfolio system that finds profitable features by using adaptive association mining was proposed by Syu et al. [3]. Machine learning approaches are also widely used in the financial industry. These techniques are used to perform very complicated analysis and prediction. Selvin et al. [4] predicted stock price by recurrent and convolutional deep neural networks. Jiang et al. [5]. In the context of deep reinforcement learning, Wu et al. [6] presented their proposal for portfolio management systems. The research published so far shows that artificial intelligence has the potential to help with financial investment and decision making.

Stock selection, on the other hand, is a continuous task that requires an examination of the characteristics of both stocks and investment techniques. This is due to the fact that different investment strategies are offered. Momentum and contrarianism are two well-known characteristics of strategies that are known to be at odds with each other; yet, the inability of these characteristics to relate to stocks presents a challenge when it comes to stock selection. Establishing a link between stocks and investment strategies is a natural approach for fuzzy-set theory because it can model and express abstract features so well in environments where precision is lacking. Buckley [7] used the fuzzy interest rates, fuzzy present and future values in mathematical finance. Through the use of random trading, Wu et al. [8] established a fuzzy system to measure the features of stocks. They were successful in achieving high prediction accuracy and 1.5 times the profitability of the benchmark. The extreme volatility of the financial market, on the other hand, may cause random trading algorithms to provide unpredictable outcomes in a variety of economic contexts. Therefore, stock selection systems and trading systems should be continually updated and adapted in order to receive the most recent information. In addition, the works concentrated only on the qualities of equities, but demonstrated only a tenuous connection to other investing techniques and trading systems.

To solve these problems, in this paper we propose a stock selection system (TripleS) based on the suitability index (SI) and fuzzy-set theory, which can be further developed into an automated trading system. First, we then design position sizing approach that is used to extract the characteristics of individual stocks. These are deterministic algorithms with high explainability and no randomness. The extracted characteristics express not only the momentum and contrarian characteristics, but also the suitability for the investment

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strategy, which is denoted by SI. Then, SI is transformed by the developed fuzzy-set modules to serve as a decision basis for stock selection and further portfolio management. Several stock selection mechanisms are proposed, and the TripleS with different mechanisms are denoted as TripleS-T1, TripleS-T2, TripleS-T2U, TripleS-EF and TripleS-EFT. Additionally, a rolling window method is used to update the system settings on a frequent basis and in an adaptable manner. This contributes to an improvement in the practicability and dependability of real-world investment.

The experimental results show that the proposed TripleS has only a small improvement in precision but a significant improvement in investment performance. The TripleS-EFT has outstanding profitability, the highest annual return in all strategies, the highest Sharpe ratio in two of the four strategies, achieves an annual return of 10.01% and a Sharpe ratio of 1.032, which significantly outperforms the benchmark system. Compared to the state-of-the-art fuzzy-based RandomFuzzy [8] system, the TripleS-EFT also significantly outperforms the RandomFuzzy [8] system in measuring annual returns and has even doubled the annual returns in several strategies. However, the Sharpe ratio is lower than RandomFuzzy [8] because the pre-selected list of stocks in [8] has the highest market capitalization and lower investment risk. In summary, the proposed TripleS shows a slight improvement in selection precision and excellent improvements in investment performance and portfolio management, especially in profitability. This paper makes the following contributions:

1) We develop position sizing algorithms to extract the characteristics of stocks, SI, without randomness.
2) We transform the SI through the developed fuzzy-set modules for stock selection and portfolio management.
3) A rolling window mechanism is used to update the system parameters regularly and adaptively.
4) The proposed TripleS has a small improvement in precision but a significant improvement in investment performance.
5) The TripleS-EFT achieves an annual return of 10.01% and a Sharpe Ratio of 1.032, significantly outperforming the benchmark system.

The paper is organized as follows. Section II reviews the literature on fuzzy-set theory, characteristics of stocks, and the financial indicators. Section III introduces the architecture of the proposed TripleS, and Section IV presents the designed fuzzy-set modules, including the membership functions, parameter optimization, and stock selection mechanisms. Section V states the descriptive statistics of the result, and then evaluates the investment performance. Section VI discusses and summarizes the results of the developed TripleS and future directions.

II. Literature Review

A. Fuzzy-set Theory and Membership Functions

Fuzzy-set theory [9] is concerned with the metaphorical assignment of an idea to a degree of membership. It differs from probability theory in that the idea can be multidimensional and there are no well-defined criteria [10]. The fuzzy set, in contrast to the crisp set, is better equipped to represent uncertainty in an imprecise environment and gives information to enhance human decision-making. Crisp sets are often used in situations when the level of precision is high. In the next sections, the primary elements of fuzzy-set theory, such as the type-1 fuzzy-set, the type-2 fuzzy-set, and membership functions, are described and discussed in detail.

1) Type-1 Fuzzy-set: The majority of ordinary fuzzy theories belong to the type-1 fuzzy-set. A type-1 fuzzy-set [11], F_1, translates a collection of elements (independent variables), X, to a set of type-1 membership values, $I = [0, 1]$, and may be represented as follows:

$$F_1 = \{(x, M_{F_1}(x)) \mid x \in X,\ M_{F_1}(x) \in I\}, \tag{1}$$

where M_{F_1} is the membership function of F_1.

2) Type-2 Fuzzy-set: The fuzzy-set of type-1, on the other hand, may contain uncertainties. Therefore, Zadeh [12] proposed as a higher order fuzzy-set of type-2, which is able to state uncertainty and collect more information than a unique membership value. Based on the formulation [11], a type-2 fuzzy-set F_2 maps a set of independent variables X to a set of type-2 membership values $I^2 = [0, 1] \to [0, 1]$ (ranges of type 1 membership degree) and can be expressed as follows:

$$F_2 = \{(x, M_{F_2}(x)) \mid x \in X,\ M_{F_2}(x) \in I^2\}, \tag{2}$$

where M_{F_2} is the membership function of F_2.

3) Membership Functions: These membership functions convert the independent variables into degrees of truth ranging from 0 (false) to 1 (true). They are similar to the probability [13], but conceptually different. Commonly used are the R-, L-, trigonometric (T-), and sigmoid (S-) functions, which are among the many membership functions available in [14].

Both the R and L functions are examples of symmetric Z-shaped functions with two boundaries ($a, b, a < b$). They have a linear transformation that is (approximately) linear inside the boundary, and they transfer the values that are outside the boundary to either 0 or 1, which is defined as follows:

$$\mathbf{R}(x) = \begin{cases} 1 & x < a \\ \frac{b-x}{b-a} & a \le x \le b \\ 0 & x > b \end{cases}, \tag{3}$$

$$\mathbf{L}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}. \tag{4}$$

The triangle (T-) function [15] contains a vertex (m) and two boundaries ($a, b,\ a < b$) and maps the values outside the boundary to 0. For the values inside the boundary, the T-function converts a value to a degree reflecting the distance of the value from the vertex, which is defined as follows:

$$\mathbf{T}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{m-a} & a \le x < m \\ \frac{b-x}{b-m} & m \le x \le b \\ 0 & x > b \end{cases}. \tag{5}$$

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It is important to note that the derivatives of the R-, L-, and T-functions do not exhibit continuous behavior. To get around the problem of discontinuity, the sigmoid function, also known as the S-function, can be used (especially the logistic function). The S-function is defined as follows:

$$\mathbf{S}(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+a^{-(x+b)}}, \tag{6}$$

where a and b are parameters to control the slope and offset.

B. Characteristics of Stocks and Trading Strategies

Among all financial trading strategies, momentum and contrarianism are two widely used and well-known categories that describe trading behavior [16], and represent two contrary concepts that often lead to opposite trading decisions. Momentum methods are based on the premise that fluctuations in stock prices are caused by momentum and that a price trend similar to the current one will continue in the not too distant future [17]. For example, one new momentum method is the so-called opening range breakout. This strategy uses the concept of a predetermined threshold to track market fluctuations and make appropriate investment decisions [18]. Several studies have shown that the opening range breakout strategy can make a profit on a variety of futures markets [19]. In addition, the use of multi-objective optimization [20] and evolutionary algorithms [21] can further improve the performance of the strategy. The phenomenon of mean reversion in stock prices, where it is found that the price tends to move towards the mean, is extended by contrarian strategies. A pre-dominant contrarian strategy is the Bollinger bands [22], [23], which uses price volatility to detect the mean-reversion signal.

After a stock position is established, the task of position sizing (or position adjustment) [24], [25] also includes the concepts of momentum and contrarian. The momentum-type position sizing increases position when it believes that the profit and loss increase with momentum. The contrarian-type position sizing, on the other hand, decreases the position when it earns profits, and increases the position when it gets losses, because it believes that the profit and loss will start to reverse as contrarian.

Fixed Ratio (FR) is an intuitive position sizing mechanism [26] that shows the relationship between the number of contracts (position size), as well as the amount of profit, in which it should be monitored through a positive fixed ratio, δ. Fixed ratio mechanisms are defined as follows and shown in Fig. 1. The current price is used to determine the value of the base price (BP) when the momentum-type fixed ratio mechanism is put into operation. Once the stock price reaches $\geq BP \times (1 + n \times \delta)$, the n^{th} grid price, the position should be extended to n fund units, where n is a positive integer. In other words, when the price increases by everyδ of BP, the position should be increased by one new unit of funds. Once the stock price reaches $\geq BP \times (1 + n \times \delta)$, the position should be decreased by n units of funds, which is the n^{th} grid price. This is part of the fixed ratio contrarian mechanism. Also, when the price increases by each δ of BP, one unit of funds should be taken out of the position to maintain the original risk level.

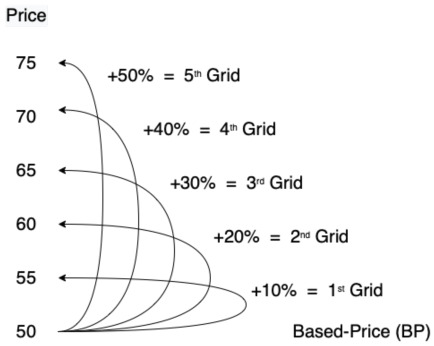

Fig. 1: Schematic diagram of the fixed ratio.

C. Financial Indicators

Several widely used indicators are used to evaluate investment performance from the perspective of profitability and risk measurement [27]. When calculating profitability, one of the most basic metrics is the annual return on investment. Let us assume that the total profit achieved over the entire period of the investment is $Profit$, the cost of the investment is $Cost$, and the total number of days invested is $Days$. The annual return is defined as:

$$\frac{Profit}{Cost \times \frac{Days}{252}}, \tag{7}$$

where $\frac{Days}{252}$ shows the number of investment years, and 252 indicates the average number of trading days of a year, which can thus be presented as: $\approx 365 \times \frac{5}{7}$.

The Sharpe Ratio [28] is an effective indicator of how much profit can be made per unit of risk, and it is useful when you are looking at the tradeoff between profitability and risk. $AnnRet$ represents the annual return, r_f represents the risk-free rate, and Std represents the standard deviation of daily returns. Here you can find an explanation of the Sharpe Ratio:

$$\frac{AnnRet - r_f}{Std \times \sqrt{252}}, \tag{8}$$

where $\sqrt{252}$ is considered as a multiplier that is used to adjust daily volatility to annualized volatility, and r_f is the risk-free return rate, which can be regularly replaced through the treasury yield or even ignored. This strategy is also considered in our designed model. In addition, investors appreciate a higher Sharpe ratio for their investment because they can achieve higher profits or lower investment risks.

III. Proposed TripleS: Stock Selection System

This section presents the architecture of the proposed TripleS. To increase the practicality of the investment, the TripleS is designed with rolling windows. Specifically, the TripleS trains with the data of the previous month to obtain the SI, selects and trades stocks for subsequent months as shown in Fig. 2.

The architecture of the proposed TripleS is shown in Fig. 3, which selects stocks and trades them with an investment strategy. For the strategy, the training data is first implemented

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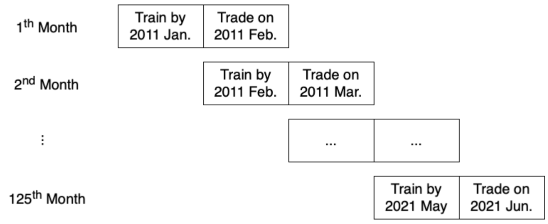

Fig. 2: The mechanism of rolling window.

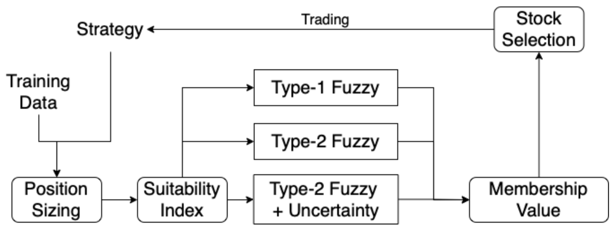

Fig. 3: Architecture of the proposed TripleS.

into the position sizing algorithms to obtain the SI for each stock. The SI of each stock is then transformed using fuzzy set theory (Section IV) for stock selection. The three membership values are used to quantify the suitability and select the stocks (Section IV-E), and TripleS utilizes this policy for the selected stocks.

A. Position Sizing

We develop algorithms for momentum and contrarian position sizing based on the fixed ratio [26]. To specify the grid size and the maximum number of position units, the algorithms require two parameters called δ and $MaxPos$. These parameters can be found in Section II-B, as well as Fig. 1. In addition, the algorithms need the daily opening price, highest price, and closing price of a stock, which are respectively denoted as OPEN, HIGH, and CLOSE. These three values are vectors with dimension D, where D is the number of days in the month. Finally, the algorithms return the momentum value of the stock SI (also known as *MSI*) and the contrarian value SI (also known as *CSI*). In the following experiments, note that the δ and $MaxPos$ settings are both set to 2% and 10, respectively.

The algorithm that defines the momentum position sizing is referred to as Algorithm 1. Starting from the first day, initialize the base price (BP) to the opening price of the T-th day (initialized to 1) and set the current grid index ($Index$, an integer) to 0, which records the grids the price has reached and also the stock units bought (line 4). Also set the last return rate ($LRet$) to 0 and the list of purchase costs ($Cost$) to empty (line 5).

The constant ratio encourages the acquisition of additional stock units at integer grids, but there is a break in continuity between the stock price on successive trading days. Therefore, we must pay attention to the grid of opening prices ($OGrid$, line 7). If the $OGrid > Index$, the algorithm buys new stock units at the opening price (even if it is not an integer grid, it is the closest and tradable price). In summary, once the $OGrid > Index$, the algorithm buys multiple stock units at the opening price until the $Index$ reaches the $OGrid$ (lines 9 to 11).

For the trading day, we then check the grid with the highest price ($HGrid$, line 8). After the $HGrid$ exceeds the $index$, the algorithm begins to buy several units of stock at each integer grid price. This process continues until the $Index$ reaches the $HGrid$ (lines 12 to 14). It is important to note that the prices from $OGrid$ to $HGrid$ are continuous and that the intervening integer grids can be traded. Also, the total position cannot be larger than $MaxPos$ and the cost of buying each position is recorded in $Cost$ (lines 11 and 14). As for the termination strategy, all positions are terminated and sold at the closing price as soon as the closing price $< BP \times (1 + (\text{Index} - 1) \times \delta)$, i.e., the price falls by one grid (lines 18 to 20). The fixed ratio strategy can be restarted from the next day, setting the day index T to the next day of termination (lines 19 and 24). If the position is not closed until the last day D, the remaining positions are sold at the price at which the market closed on the D-th trading day (lines 21 and 23).

DRet is where you will find an entry for the daily return of each day. This is the return you would get if you sold all positions at the closing price of the day, minus the previous day's return, $LRet$ (lines 15 to 17). Up to the last day, the momentum SI (*MSI*) is defined as the Sharpe ratio of the period (line 25), which is written as $\frac{\text{Mean(DRet)}\sqrt{252}}{\text{Std(DRet)}}$, where Mean($\cdot$) and Std($\cdot$) are the

Algorithm 1 Momentum Position Sizing

Input: δ, $MaxPos$, OPEN, HIGH, CLOSE, D
Output: *MSI*
1: T, DRet = 1, []; ▷ day index, daily return
2: **while** $T < D$ **do**
3: **if** True **then**
4: BP, $Index$ = OPEN[T], 0; ▷ base price, current grid
5: $Cost$, $LRet$ = [], 0; ▷ buying cost, last return
6: **for** d from T^{st} to D^{th} day **do**
7: $OGrid$ = $\lfloor$ ($\frac{\text{OPEN[d]}}{BP}$-1) / δ $\rfloor$;
8: $HGrid$ = $\lfloor$ ($\frac{\text{HIGH[d]}}{BP}$-1) / δ $\rfloor$;
9: **while** $OGrid > Index$ and len($Cost$) $< MaxPoS$ **do**
10: $Inedx$ += 1; ▷ add a position
11: $Cost$ += [OPEN[d]];
12: **while** $HGrid > Index$ and len($Cost$) $< MaxPoS$ **do**
13: $Inedx$ += 1; ▷ add a position
14: $Cost$ += [$BP \cdot$ (1+Index$\cdot\delta$)];
15: Ret = return rate of selling all stocks at CLOSE[d];
16: DRet += [Ret - $LRet$]; ▷ record the daily return
17: $LRet$ = Ret;
18: **if** CLOSE[d] $< BP \cdot (1 + (\text{Index}-1) \cdot \delta)$ **then**
19: $T = d$;
20: break; ▷ termination condition
21: **if** Not terminated yet **then**
22: Ret = return rate of selling all stocks at CLOSE[D];
23: DRet += [Ret - $LRet$];
24: T += 1;
25: $MSI = \frac{\text{Mean(DRet)}\sqrt{252}}{\text{Std(DRet)}}$;
26: Return *MSI*.

There is a definition for contrarian position sizing in the Algorithm 2. Set the current grid index ($index$, an integer) to 0, which records the grids that the price has reached and the units of stock sold (line 4), and initialize the base price (BP) to the opening price of the T-th day (initialized to 1).

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Since the contrarian-type fixed ratio is designed to reduce the position, the strategy first buys $MaxPos$ stock units at the opening price of the T-th day and records the cost in $Cost$ (line 5). In addition, the previous rate of return ($LRet$) should be corrected to 0 and the gain on sale ($Gain$) left blank (line 6). Once $HGrid$ is larger than $index$, the program begins selling multiple units of stock at each integer grid price until $Index$ reaches the value of $HGrid$ (lines 13 to 15). The profit made by selling each position is recorded in the $Gain$ variable (lines 12 and 15).

Under the termination strategy, all open positions are closed and sold at the closing price of $< BP \times (1 + (\text{Index} - 1) \times \delta)$, which means that the price has decreased by one grid (lines 19 to 21). The fixed ratio method can be reactivated from the day after tomorrow, setting the day index T to the next day of termination (lines 20 and 25). If the position is not terminated until the last day, D, the remaining positions are sold at the closing price of the D-th day (lines 22 and 24).

The daily return of each day is recorded in DRet (lines 16 to 18). Until the last day, the contrarian SI (*CSI*) is also defined as the Sharpe ratio of the period (line 26), denoted as $\frac{\text{Mean(DRet)}\sqrt{252}}{\text{Std(DRet)}}$. Note that a stock has two SI calculated by momentum position sizing (Algorithm 1) and contrarian position sizing (Algorithm 2), namely *MSI* and *CSI*.

Algorithm 2 Contrarian Position Sizing

Input: δ, $MaxPos$, OPEN, HIGH, CLOSE, D
Output: *CSI*
1: T, DRet = 1, []; ▷ day index, daily return
2: **while** $T < D$ **do**
3:   **if** True **then**
4:     BP, $Index$, = OPEN[T], 0, 0; ▷ base price, current grid
5:     $Cost$ = OPEN[T]; ▷ buying cost
6:     $Gain$, $LRet$ = [], 0; ▷ selling gain, last return
7:     **for** d from T^{st} to D^{th} day **do**
8:       $OGrid$ = $\lfloor$ ($\frac{\text{OPEN[d]}}{BP}$-1) / δ $\rfloor$;
9:       $HGrid$ = $\lfloor$ ($\frac{\text{HIGH[d]}}{BP}$-1) / δ $\rfloor$;
10:       **while** $OGrid > Index$ and len($Gain$) $< MaxPoS$ **do**
11:         $Inedx$ += 1; ▷ add a position
12:         $Gain$ += [OPEN[d]];
13:       **while** $HGrid > Index$ and len($Gain$) $< MaxPoS$ **do**
14:         $Inedx$ += 1; ▷ add a position
15:         $Gain$ += [$BP \cdot (1{+}\text{Index}{\cdot}\delta)$];
16:       Ret = return rate of selling all stocks at CLOSE[d];
17:       DRet += [Ret - $LRet$]; ▷ record the daily return
18:       $LRet$ = Ret;
19:       **if** CLOSE[d] $< BP \cdot (1 + (\text{Index}-1) \cdot \delta)$ **then**
20:         T = d;
21:         break; ▷ termination condition
22:     **if** Not terminated yet **then**
23:       Ret = return rate of selling all stocks at CLOSE[D];
24:       DRet += [Ret - $LRet$];
25:   T += 1;
26: $CSI = \frac{\text{Mean(DRet)}\sqrt{252}}{\text{Std(DRet)}}$;
27: Return *CSI*.

B. Example of Position Sizing

Fig. 4 provides a concise illustration of the momentum position sizing. In this figure, it is assumed that the δ is set as 10%. At the beginning of day T of the position sizing strategy, the grid index ($Index$) is initialized to 0 and the base price (BP) is set to the opening price of 50. $\lfloor(\frac{50}{50} - 1)/10\%\rfloor = 0$ and $\lfloor(\frac{58}{50} - 1)/10\%\rfloor = 1$ represent the $OGrid$ and $HGrid$ for the T-th day, respectively. Since $OHrid$ is equal to $Index$, there is no new position bought at the opening price. Since $HGrid > Index$, a new position is bought at the grid price 1^{st}, 55 dollars, and the $Index$ is updated to 1.

The $OGrid$ and $HGrid$ of the $T+1$-th day are $\lfloor(\frac{62}{50} - 1)/10\%\rfloor = 2$ and $\lfloor(\frac{68}{50} - 1)/10\%\rfloor = 3$. Since $OHrid > Index$, a new position is bought at the opening price, 62 dollars, and the $Index$ is the changed to 2. As $HGrid > Index$, a new position is bought at the 3^{rd} grid price, 65 dollars, and the $Index$ is updated to 3.

On $T+2$-th day, the $OGrid$, the $HGrid$, and the $Index$ are equal; therefore, no new position is added. Even if the lowest price of the $T+2$-th day is lower than the $(Index-1)^{\text{th}} = 2^{\text{nd}}$ grid price, the closing price is still higher than the 2^{nd} grid price, and the position is not terminated.

On $T+3$-th day, a new position is added, and the $Index$ is updated to 4. If the closing price of $T+4$-th day is lower than the grid price of $(Index - 1)^{\text{th}} = 3^{\text{rd}}$, all positions are closed and sold at the closing price of $T+4$-th day. The same method is applied from day $(T+5)$ to the last day, i.e. day D. Note that the size of the contrarian position is similar, except that it is sold instead of bought.

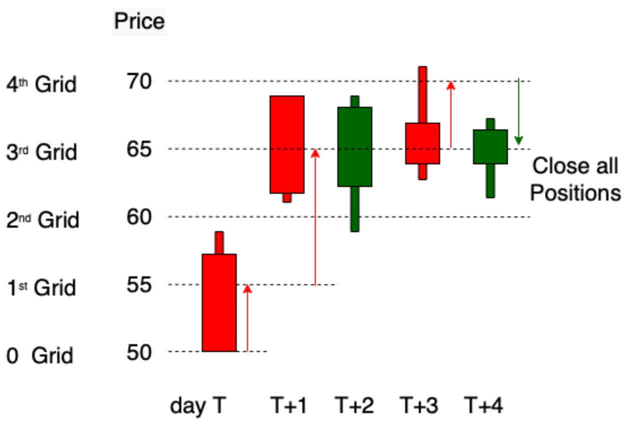

Fig. 4: Schematic diagram of the designed position sizing.

C. Suitability Index and Evaluation Strategies

As in Fig. 3, the proposed TripleS aims to quantify the suitability and select stocks for an investment strategy. In this paper, we use the well-known approach of applying four different strategies, each of which has both a momentum- and a contrarian-type concept (8 strategies in total). Gap strategy (**GAP**) [29] ensures that the signal of momentum GAP, GAP_Mom, (contrarian GAP, GAP_Con) is activated when the opening price is higher (lower) than the last closing price. Trading range break (**nHnL**) [30] indicates that the signal from momentum nHnL, nHnL_Mom, (contrarian nHnL, nHnL_Con) is triggered when the opening price is greater than the highest price (lower than the lowest price) of the last n days. In the following experiment, we set n to 1, 3, and 5, that is, the four evaluation strategies are **GAP**, **1H1L**, **3H3L**, and **5H5L**.

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The position sizing techniques described in the Section III-A are also included in the strategies described in Fig. 5. In particular, GAP_Mom and nHnL_Mom adopt the momentum position sizing of Algorithm 1, with line 3 replaced by the condition of GAP_Mom and nHnL_Mom, respectively. The momentum suitability index for GAP_Mom is given by *MSI-GAP*, while the momentum suitability index for nHnL_Mom is given by *MSI-nHnL*. Similarly, GAP_Con and nHnL_Con adopt the contrarian position sizing of Algorithm 2, by replacing the condition of GAP_Con and nHnL_Con (line 3, Algorithm 1). The contrarian suitability index for the GAP_Con is denoted as *CSI-GAP*, while the contrarian suitability index for the nHnL_Con is denoted as *CSI-nHnL*. For clarity, we list the abbreviations for strategies and suitability indexes in Table I.

TABLE I: Abbreviations of strategies and suitability indexes

Strategies	Description
GAP_Mom	Momentum GAP strategy
GAP_Con	Contrarian GAP strategy
GAP	Collective name of GAP_Mom and GAP_Con
nHnL_Mom	Momentum nHnL strategy, where n is 1 or 3 or 5
nHnL_Con	Contrarian nHnL strategy, where n is 1 or 3 or 5
nHnL	Collective name of nHnL_Mom and nHnL_Con
Suitability Indexes	Description
SI	Suitability index
MSI	Momentum SI
MSI-GAP	Momentum SI for GAP_Mom
MSI-nHnL	Momentum SI for nHnL_Mom
CSI	Contrarian SI
CSI-GAP	Contrarian SI for GAP_Mom
CSI-nHnL	Contrarian SI for nHnL_Mom

IV. Fuzzy-Set Quantifying Modules

In this section, the designed type-1, type-2, and type-2 fuzzy-set with uncertainty modules are presented, as shown in Sections IV-A to IV-C. Then, the rolling window method of parameter optimization is discussed in Section IV-D, in which the stock selection techniques are described in Section IV-E for more details.

A. Type-1 Fuzzy-Set Module

We then develop the momentum membership function for type-1: T1-MMF, to transform a *MSI* into a type-1 momentum membership value, *T1-MMV*, where *MSI* is in $\mathbb{R}$ and *T1-MMV* is in $[0, 1]$. Similarly, the contrarian type-1 membership function, or T1-CMF, is said to convert a *CSI* to a *T1-CMV*, where *CSI* is in the range $\mathbb{R}$ and *T1-CMV* is in the range $[0, 1]$. Note that the *T1-CMV* indicates a type-1 contrarian membership value. As you can see from the Eqs. 9 and 10, they are both intended to function as S-functions, also known as logistic functions.

$$\textit{T1-MMV} = \text{T1-MMF}(\textit{MSI}) = \frac{1}{1 + P1^{\ -\textit{MSI}+P2}} \tag{9}$$

$$\textit{T1-CMV} = \text{T1-CMF}(\textit{CSI}) = \frac{1}{1 + P3^{\ -\textit{SCI}+P4}} \tag{10}$$

The parameters $P1$ and $P3$ are used to define the slope of the function, while the parameters $P2$ and $P4$ are used to regulate the offsets of the functions. Following the procedure outlined in Section IV-D, the parameters ($P1$ to $P4$) are optimized utilizing the training data from each month.

B. Type-2 Fuzzy-Set Module

Due to the contradictory nature of the idea of momentum and the properties of contrarians, the linguistic formulation of momentum, also known as contrarian, may indicate the opposite of the term contrarian (momentum). Therefore, we use the value obtained by subtracting one from the momentum (contrarian) membership value to represent one side of the type-2 contrarian momentum (Contrarian) membership value. Below we explain how to develop the type-2 momentum and contrarian membership functions, also known as T2-MMF and T2-CMF.

The T2-MMF (T2-CMF) accepts both the *MSI* and *CSI* as inputs and maps a type-2 momentum membership value, *T2-MMV* (a type-2 contrarian membership value, *T2-CMV*), where *T2-MMV* (*T2-CMV*) is a range of $[0, 1] \rightarrow [0, 1]$. Moreover, the T2-MMF and the T2-CMF are both formulated as S-functions, as can be seen in the Eqs. 11 and 12.

$$\begin{aligned}\textit{T2-MMV} &= \text{T2-MMF}(\textit{MSI},\ \textit{CSI}) \\ &= \min(M,\ 1-C) \rightarrow \max(M,\ 1-C),\end{aligned} \tag{11}$$

where $M = \frac{1}{1+P1^{\ -\textit{MSI}+P2}}$ and $C = \frac{1}{1+P3^{\ -\textit{CSI}+P4}}$.

$$\begin{aligned}\textit{T2-CMV} &= \text{T2-CMF}(\textit{MSI},\ \textit{CSI}) \\ &= \min(C,\ 1-M) \rightarrow \max(C,\ 1-M),\end{aligned} \tag{12}$$

where $M = \frac{1}{1+P1^{\ -\textit{MSI}+P2}}$ and $C = \frac{1}{1+P3^{\ -\textit{CSI}+P4}}$.

The *T2-MMV* is a range from the minimum to the maximum of M and $1-C$, and the *T2-CMV* is a range from the minimum to the maximum of C and $1-M$. The M and C in Eqs. 11 and 12 are the membership values transformed by T1-MMF and T1-CMF with different parameters ($P1$ to $P4$), which are independently optimized by the training data of each month, as described in Section IV-D.

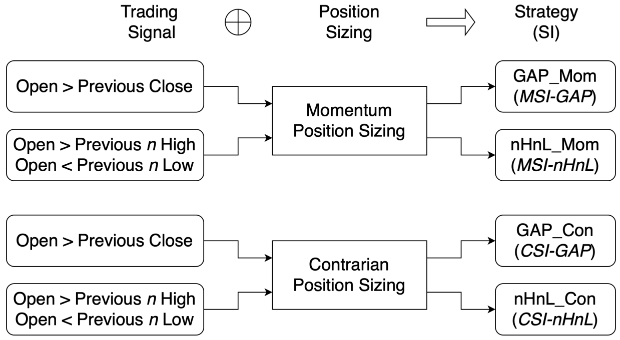

Fig. 5: Schematic diagram of the designed strategies.

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C. Type-2 Fuzzy-Set with Uncertainty Module

To further describe uncertainty, the third characteristic, uncertainty, was taken from [8]. The type-2 uncertain membership function (T2U-UMF) is designed to take both *MSI* and *CSI* as inputs, and maps to a type-2 uncertain membership value, *T2U-UMV*, representing a range of $[0, 1] \rightarrow [0, 1]$. The T2U-UMF is designed as a T-function, as shown in Eq. (13).

$$\begin{aligned} \textit{T2U-UMV} =& \text{T2U-UMF}(\textit{MSI},\ \textit{CSI}) = \\ & \min(UM,\ UC) \rightarrow \max(UM,\ UC), \end{aligned} \tag{13}$$

where the UM is:

$$UM = \begin{cases} 1 - \frac{abs(\textit{MSI}-P5)}{P6} & P5 - P6 \leq \textit{MSI} \leq P5 + P6 \\ 0 & \text{else} \end{cases} \tag{14}$$

and the UC is:

$$UC = \begin{cases} 1 - \frac{abs(\textit{CSI}-P7)}{P8} & P7 - P8 \leq \textit{CSI} \leq P7 + P8 \\ 0 & \text{else} \end{cases}. \tag{15}$$

The *T2-UMV* is a range from the minimum to the maximum of UM and UC. The $P5$ and $P7$ are parameters to determine the center of the T-function, and the $P6$ and $P8$ control the width of the T-function.

The momentum and contrarian membership functions of type-2 fuzzy-set with uncertainty module are denoted as T2U-MMF and T2U-CMF, which are defined in Eqs. 16 and 18, respectively. T2U-MMF (T2U-CMF) is designed to take both *MSI* and *CSI* as input, and maps to a type-2 membership value, *T2U-MMV* (*T2U-CMV*), which is a range of $[0, 1] \rightarrow [0, 1]$.

$$\begin{aligned} \textit{T2U-MMV} &= \text{T2U-MMF}(\textit{MSI},\ \textit{CSI}) \\ &= \begin{cases} MMV & \text{Mid}(MMV) > \text{Mid}(UMV) \\ 0 \rightarrow 0 & \text{else} \end{cases}, \end{aligned} \tag{16}$$

where the UMV and MMV are:

$$\begin{aligned} UMV &= \text{T2U-UMF}(\textit{MSI},\ \textit{CSI}) \\ MMV &= \text{T2-MMF}(\textit{MSI},\ \textit{CSI}). \end{aligned} \tag{17}$$

$$\begin{aligned} \textit{T2U-CMV} &= \text{T2U-CMF}(\textit{MSI},\ \textit{CSI}) \\ &= \begin{cases} CMV & \text{Mid}(CMV) > \text{Mid}(UMV) \\ 0 \rightarrow 0 & \text{else} \end{cases} \end{aligned} \tag{18}$$

where the UMV and CMV are:

$$\begin{aligned} UMV &= \text{T2U-UMF}(\textit{MSI},\ \textit{CSI}) \\ CMV &= \text{T2-CMF}(\textit{MSI},\ \textit{CSI}). \end{aligned} \tag{19}$$

In the designed type-2 fuzzy-set with uncertainty module, a stock is considered uncertain if the midpoint of the uncertain membership value , UMV, is greater than the midpoint of the momentum (contrarian) membership value, and the momentum (contrarian) membership value is set to $0 \rightarrow 0$. Parameters of T2U-UMF and T2U-MMF and T2U-CMF are optimized using the training data of each month, as described in Section IV-D.

D. Parameters Optimization

In the fuzzy-set modules presented, there are a number of parameters ranging from $P1$ to $P8$. Each of these parameters is separately optimized by the rolling window training data. Take **GAP** as an instance and the Fig. 6 visually represents the many stages of optimization. First, the system computes the momentum and contrarian suitability indices of all N stocks during the training month M. These indices are represented by the $[\textit{MSI-GAP}_1, \cdots, \textit{MSI-GAP}_N]$ and $\boldsymbol{CSI}_M$, respectively.

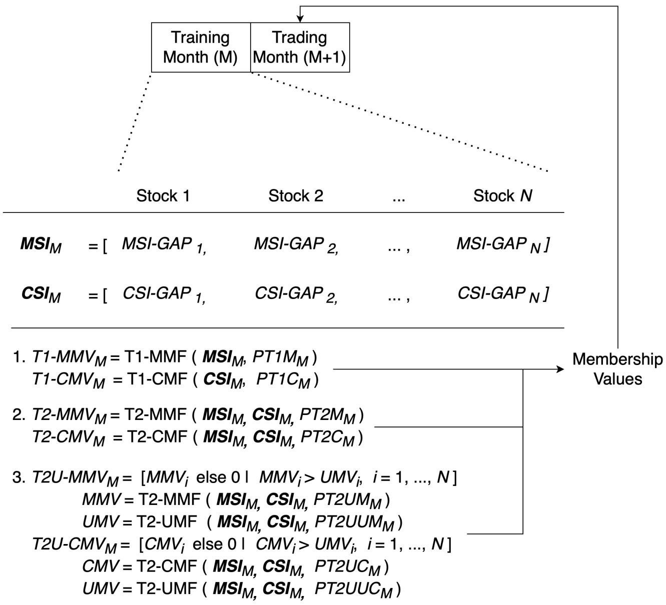

Fig. 6: Designed rolling window and parameter optimization.

The settings of the fuzzy-set modules can be optimized thanks to the TripleS. For the purposes of the type-1 fuzzy-set module, the parameters of the month M will be given as $\textit{PT1M}_M$ ($P1$ and $P2$ in T1-MMF, Eq. (9) and $\textit{PT1C}_M$ ($P3$ and $P4$ in T1-CMF, Eq. (10), defined in Eq. (20).

$$\begin{aligned} \textit{PT1M}_M &= \underset{P1,P2}{\arg\max}\, \text{OBJ}(\textit{T1-MMV}, \boldsymbol{MSI}_M), \\ \textit{PT1C}_M &= \underset{P3,P4}{\arg\max}\, \text{OBJ}(\textit{T1-CMV}, \boldsymbol{CSI}_M), \end{aligned} \tag{20}$$

where:

$$\begin{aligned} \textit{T1-MMV} &= \text{T1-MMF}(\boldsymbol{MSI}_{M-1}, [P1, P2]), \\ \textit{T1-CMV} &= \text{T1-CMF}(\boldsymbol{CSI}_{M-1}, [P3, P4]). \end{aligned} \tag{21}$$

The type-1 momentum membership $\boldsymbol{MSI}_{M-1}$ using T1-MMF with parameters $P1$ and $P2$. Similarly, the *T1-CMV* specifies the contrarian membership values of type-1, having been transformed from $\boldsymbol{CSI}_{M-1}$ by T1-CMF using parameters $P3$ and $P4$. The objective function is denoted by the OBJ, and its definition is as follows:

$$\text{OBJ}(MV, SI) = \frac{\text{Mean}(\overline{SI})}{\text{Std}(\overline{SI})}, \tag{22}$$

where:

$$\overline{SI} = [SI_i \mid MV_i > 0.5, \quad i = 1, \cdots, \text{length of } MV]. \tag{23}$$

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The idea of the Sharpe ratio, which takes into account both the average and the fluctuations in performance, served as the basis for OBJ. In summary, the T1-MMF and T1-CMF models look for the parameters that, when applied, convert the previous month's SI into membership values and have an optimally chosen performance for following months.

For the type-2 fuzzy-set module, the parameters of month M are represented as $\textit{PT2M}_M$ ($P1$ to $P4$ in T2-MMF, Eq. (11) and $\textit{PT2C}_M$ ($P1$ to $P4$ in T2-CMF, Eq. (12)), which are specified in Eq. (24).

$$\begin{aligned} \textit{PT2M}_M &= \underset{P1,P2,P3,P4}{\arg\max}\ \text{OBJ}(\text{Mid}(\textit{T2-MMV}), \boldsymbol{MSI}_M), \\ \textit{PT2C}_M &= \underset{P1,P2,P3,P4}{\arg\max}\ \text{OBJ}(\text{Mid}(\textit{T2-CMV}), \boldsymbol{CSI}_M), \end{aligned} \tag{24}$$

where:

$$\begin{aligned} \textit{T2-MMV} &= \text{T2-MMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P1, \cdots, P4]), \\ \textit{T2-CMV} &= \text{T2-CMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P1, \cdots, P4]). \end{aligned} \tag{25}$$

Eq. (24) is quite similar to Eq. (20). The major differences are in the membership functions (T2-MMF and T2-CMF) and the optimized parameters ($P1$ to $P4$). Since the value of type-2 membership value can be within a certain range, the objective value in Eq. (24) is calculated using the midpoints of the range, denoted by Mid($\cdot$).

For the type-2 fuzzy-set with uncertainty module, the parameters of month M are denoted as $\textit{PT2UM}_M$ ($P1$ to $P4$ in T2-MMF, Eq. (11), $\textit{PT2UC}_M$ ($P1$ and $P4$ in T2-CMF, Eq. (12)), $\textit{PT2UUM}_M$, and $\textit{PT2UUC}_M$ ($P5$ to $P8$ in T2-UMF, Eq. (13)). The $\textit{PT2UM}_M$ and $\textit{PT2UUM}_M$ are defined as:

$$\begin{aligned} &\textit{PT2UM}_M,\ \textit{PT2UUM}_M = \\ &\qquad \underset{P1,\cdots,P4,P5,\cdots,P8}{\arg\max}\ \text{OBJ}(\text{Mid}(\textit{T2U-MMV}), \boldsymbol{MSI}_M). \end{aligned} \tag{26}$$

The *T2U-MMV* is defined as:

$$\begin{aligned} \textit{T2U-MMV} = [&MMV_i \ \text{ else } 0 \rightarrow 0 \mid \\ &\text{Mid}(MMV_i) > \text{Mid}(UMV_i), \quad i = 1, \cdots, N], \end{aligned} \tag{27}$$

where:

$$\begin{aligned} MMV &= \text{T2-MMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P1, \cdots, P4]), \\ UMV &= \text{T2-UMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P5, \cdots, P8]). \end{aligned} \tag{28}$$

Similarly, the $\textit{PT2UC}_M$ and $\textit{PT2UUC}_M$ are difined as:

$$\begin{aligned} &\textit{PT2UC}_M,\ \textit{PT2UUC}_M = \\ &\qquad \underset{P1,\cdots,P4,P5,\cdots,P8}{\arg\max}\ \text{OBJ}(\text{Mid}(\textit{T2U-CMV}), \boldsymbol{CSI}_M). \end{aligned} \tag{29}$$

The *T2U-CMV* is defined as:

$$\begin{aligned} \textit{T2U-CMV} = [&CMV_i \ \text{ else } 0 \rightarrow 0 \mid \\ &\text{Mid}(CMV_i) > \text{Mid}(UMV_i), \quad i = 1, \cdots, N], \end{aligned} \tag{30}$$

where:

$$\begin{aligned} CMV &= \text{T2-CMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P1, \cdots, P4]), \\ UMV &= \text{T2-UMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P5, \cdots, P8]). \end{aligned} \tag{31}$$

Note that the parameters of the fuzzy-set modules are optimized in discrete spaces, as shown in Table II. Furthermore, for clarity, we list the abbreviations for fuzzy-set modules in Table III.

TABLE II: Search Space of Parameters

Parameter	Search Space		Meaning
$P1, P3$	$[1 + 0.2 \times i$	$i = 1, \cdots, 10]$	Slope of S-function
$P2, P4$	$[\ \ 0.5 \times i$	$i = -5, \cdots, 5]$	Offset of S-function
$P5, P7$	$[\ \ 0.1 \times i$	$i = -3, \cdots, 3]$	Center of T-function
$P6, P8$	$[\ \ 0.05 \times i$	$i = 1, \cdots, 5]$	Width of T-function

TABLE III: Abbreviations of fuzzy-set modules

Fuzzy-Set Modules	Description
T1	Type-1 fuzzy-set module
T2	Type-2 fuzzy-set module
T2U	Type-2 fuzzy-set with uncertainty module
Membership Values	**Description**
MV	Membership value
T1-MMV	Type-1 momentum *MV* in *T1*
T1-CMV	Type-1 contrarian *MV* in *T1*
T2-MMV	Type-2 momentum *MV* in *T2*
T2-CMV	Type-2 contrarian *MV* in *T2*
T2U-UMV	Type-2 uncertain *MV* in *T2U*
T2U-MMV	Type-2 momentum *MV* in *T2U*
T2U-CMV	Type-2 contrarian *MV* in *T2U*
Membership Functions	**Description**
MF	Membership function
T1-MMF	Type-1 momentum MF in T1, transforms a *MSI* to *T1-MMV*
T1-CMF	Type-1 contrarian MF in T1, transforms a *CSI* to *T1-CMV*
T2-MMF	Type-2 momentum MF in T2, transforms *MSI* and *CSI* to *T2-MMV*
T2-CMF	Type-2 contrarian MF in T2, transforms *MSI* and *CSI* to *T2-CMV*
T2U-UMF	Type-2 uncertain MF in T2U, transforms *MSI* and *CSI* to *T2U-UMV*
T2U-MMF	Type-2 momentum MF in T2U, transforms *MSI* and *CSI* to *T2U-MMV*
T2U-CMF	Type-2 contrarian MF in T2U, transforms *MSI* and *CSI* to *T2U-CMV*
Parameters	**Description**
PT1M	Parameters of T1-MMF in T1, $P1$ and $P2$
PT1C	Parameters of T1-CMF in T1, $P3$ and $P4$
PT2M	Parameters of T2-MMF in T2, $P1, \cdots, P4$
PT2C	Parameters of T2-CMF in T2, $P1, \cdots, P4$
PT2UM and *PT2UUM*	Parameters of T2U-MMF in T2U $P1, \cdots, P4$ and $P5, \cdots, P8$
PT2UC and *PT2UUC*	Parameters of T2U-CMF in T2U $P1, \cdots, P4$ and $P5, \cdots, P8$

E. Stock Selection Mechanisms

The optimized parameters are then utilized the transferred values by the membership functions of each stock, in which it shows the information that can be used for stock selection for the following trade months. Thus in this process, we then define 5 stock selection models, which are then denoted as T1, T2, T2U, EF, and EFT. In addition, TripleS with the stock selection models are then respectively denoted by TripleS-T1, TripleS-T2, TripleS-T2U, TripleS-EF, and TripleS-EFT. For example of **GAP**, the T1 (T2, T2U) model chooses the stocks with *T1-MMV* (*T2-MMV*, *T2U-MMV*), in which its value is larger than 0.5 for the GAP_Mom strategy, as well as chooses stocks with *T1-CMV* (*T2-CMV*, *T2U-CMV*) greater than 0.5 for the GAP_Con strategy.

The EF is an ensemble fuzzy mechanism, and selects stocks with average momentum membership value (*T1-MMV*, *T2-MMV*, and *T2U-MMV*) greater than 0.5 for the GAP_Mom strategy, and stocks with average contrarian membership value (*T1-CMV*, *T2-CMV*, and *T2U-CMV*) greater than 0.5 for

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the GAP_Con strategy. EFT is an advanced EF mechanism, and selects only the top-K stocks with the highest average momentum membership value for the GAP_Mom strategy, and selects the top-K stocks with the highest average contrarian membership value for the GAP_Con strategy. Note that the EFT mechanism also selects only stocks with average value higher than 0.5, and there would be less than K selected stocks.

V. Experimental Results

A. Data Usage

In this paper, we analyze stocks traded in Taiwan stock market, including exchange-traded funds, and apply our results to the TripleS model. From the beginning (1, January) of 2011 to at least June 30, 2021, there are a total of 1,230 stocks (2,574 trading days). Since the upper limit for price increases and decreases in a day in Taiwa stock market is $\pm 10\%$, we remove stocks whose highest (lowest) daily price is more than 1.1 times (less than 0.9 times) the previous closing price. In this way, we can avoid the impact of stock splits and mergers, as well as capital reductions and increases. In addition, we focus only on stocks that can be traded on all 2,574 trading days. Following the cleansing of the data, the list of stocks that have been integrated in TripleS has a total of 307 stocks.

Regarding the investment mechanism, for each strategy (**Gap**, **1H1L**, **3H3L** and **5H5L**), we integrate the results of momentum and contrarian strategies to create a neutral and thorough evaluation. In addition, the size of the membership value not only determines which stocks are available for selection, but also contributes to the management of the portfolio. We invest more weight in stocks with higher membership values. In particular, for momentum strategies, the system selects stocks based on the momentum membership value. The system determines the weight of stock $w_i = 2\times(MMV_i - 0.5)$ for the selected stock i that has a momentum membership value of MMV_i. This value represents twice the distance from MMV_i to 0.5. The fraction of a company's total assets invested in stock i is denoted by the notation i is $\frac{w_i}{\sum_i w_i}$. The same principle applies to the contrarian method. Note that we have ignored all transaction fees in this paper.

In addition, two benchmark models respectively called ALL [31] and RandomFuzzy [8] are used for comparison. Since ALL invests in all stocks with equal weight, it analyzes the performance of all stocks through both momentum and contrarian. The ALL is an effective investment technique that has gained widespread recognition in the financial study [32]. The RandomFuzzy [8] trading system is a state-of-the-art model of a fuzzy-based trading platform used to measure stock characteristics and build portfolios.

B. Descriptive Statistics

In this section, we evaluate the prediction performance using descriptive statistics. Since the system is designed for stock selection, correct selection leads to a profit, and the incorrect selection leads to a loss. The precision rate, the true positive rate, can also represent the rate of correct selection, which is also known as the investment win rate. Therefore, we use precision as a statistical measure to evaluate the performance of the proposed systems.

Table IV lists the precision of the benchmark system (ALL) and the five proposed systems (TripleS-T1, TripleS-T2, TripleS-T2U, TripleS-EF, and TripleS-EFT) during the testing period. The first two main-rows of the Table IV represent the precision of the momentum- and contrarian-type strategies, respectively. The third main-row is the combined precision of the momentum- and contrarian-type strategies. The bold values in Table IV are the best performance of the strategy (row). The experimental results show that TripleS-EF and TripleS-T1 perform best for the momentum-type strategies. In contrast, TripleS-EFT has the best precision in all contrarian-type strategies.

It can be seen that the proposed systems show only minor improvements in precision compared to the benchmark system, ALL. However, there could be a significant improvement in the investment performance, as the size of the membership value can further support the portfolio management. In addition, precision can only represent the win rate of the investment, but cannot provide information about the size of the profit and loss. Therefore, we will further explore the details of investment performance in the following sections.

C. Investment Performance

In this section, we evaluate the investment performance of the five proposed systems. Note that even though we have cleaned the data, ex-right and ex-dividends are still included in the stock price, which means that dividends are ignored in the following evaluations, and profitability is underestimated.

Table V lists the annual returns (before the slashes) and Sharpe ratios (after the slashes) of the strategies under different stock selection systems. The investment performance of each strategy in Table V is the combination of momentum- and contrarian-type strategies. Also, the bold values are the top-2 best performance for each strategy (row). The experimental results show that TripleS-EFT has excellent profitability with the highest annual return among all systems. In terms of Sharpe ratio, TripleS-EFT has the highest Sharpe ratio in the strategies **Gap** and **3H3L**, and has the second-highest value in the strategy **1H1L**. In summary, the TripleS-EFT is the best performing system with the highest annual return of 10.01% and the highest Sharpe ratio of 1.032, which significantly outperforms the benchmark system, ALL. In addition to TripleS-EFT, TripleS-T1 and TripleS-EF also performed well with an annual return of 6.19% and a Sharpe ratio of 0.988.

Fig. 7 shows the performance by the investment, as well as the equity curves of the systems. Among all strategies, the proposed systems perform best in the **Gap** and **3H3L** strategies. As shown in Fig. 7 (a) and (b), the annual returns and Sharpe ratios gradually increase when more complex modules are used (TripleS-T1 to TripleS-T2 to TripleS-T2U to TripleS-EF to TripleS-EFT). The same phenomenon can be observed for the separating equity curves and the prominent higher brown curve (the TripleS-EFT). These observations demonstrate the effectiveness of the proposed modules and systems. For the **5H5L** strategy, the proposed system outperforms the benchmark, but lacks significant improvements

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TABLE IV: The precision of the systems

	ALL [31]	TripleS-T1	TripleS-T2	TripleS-T2U	TripleS-EF	TripleS-EFT
GAP_Mom	29.1%	29.7%	30.5%	30.4%	**30.6%**	30.4%
1H1L_Mom	26.4%	27.4%	27.6%	27.5%	**27.7%**	26.9%
3H3L_Mom	25.0%	**26.2%**	25.1%	25.1%	25.0%	24.2%
5H5L_Mom	24.2%	**25.2%**	24.7%	24.6%	24.6%	22.6%
GAP_Con	51.0%	50.9%	50.7%	50.7%	50.7%	**52.1%**
1H1L_Con	52.4%	52.5%	51.5%	51.4%	51.6%	**53.3%**
3H3L_Con	52.8%	52.4%	52.1%	51.8%	52.1%	**54.2%**
5H5L_Con	53.1%	52.6%	53.1%	53.1%	53.2%	**53.6%**
GAP	40.9%	42.8%	44.1%	44.1%	**44.3%**	42.0%
1H1L	41.2%	43.1%	43.9%	43.9%	**44.0%**	41.9%
3H3L	41.7%	42.3%	**43.2%**	43.1%	43.1%	42.0%
5H5L	41.9%	42.0%	43.5%	**43.6%**	43.5%	41.4%

TABLE V: Investment performance (annual Return / Sharpe Ratio) of the systems on different strategies

	ALL [31]	TripleS-T1	TripleS-T2	TripleS-T2U	TripleS-EF	TripleS-EFT
GAP	4.26% / 0.497	5.31% / 0.592	6.15% / 0.698	6.08% / 0.691	**6.19%** / **0.703**	**10.01%** / **1.016**
1H1L	5.27% / 0.914	**6.03%** / **0.988**	5.65% / 0.916	5.58% / 0.907	5.83% / 0.950	**6.64%** / **0.952**
3H3L	3.73% / 0.809	**4.65%** / **0.924**	4.34% / 0.881	4.30% / 0.877	4.48% / 0.917	**5.72%** / **1.032**
5H5L	3.10% / 0.734	**3.74%** / 0.809	3.56% / 0.790	3.68% / **0.815**	3.65% / **0.812**	**3.85%** / 0.761

between modules, as shown by the closed curves in Fig. 7 (d). For the **1H1L** strategy, the proposed systems exhibit unstable performance, rarely having a worse Sharpe ratio than the benchmark, but always having better annual returns.

Table VI compares the proposed system, TripleS-EFT, with the state-of-the-art systems, ALL [31] and RandomFuzzy [8], during the period from October 1, 2019 to December 20, 2020 (300 trading days, aligned with [8]). The basis performance (buy-and-hold) of Taiwan Stock Exchange Weighted Index of the period has a high annual return of 29.32% and an extremely low Sharpe ratio of 0.17, indicating the extremely high trading risk. Note that we combine the two datasets [8] to create a similar list of stocks. The comparison shows that the proposed TripleS-EFT significantly outperforms RandomFuzzy in measuring annual returns, with annual returns actually doubling in **1H1L** and **3H3L**. However, the Sharpe ratios of TripleS-EFT are lower than those of RandomFuzzy, because the list of stocks in [8] are the stable stocks with the highest market capitalization and liquidity in the market (pre-selected stocks with lower investment risk). Moreover, the proposed TripleS-EFT consistently outperforms the ALL system on both measures of annual return and Sharpe ratio.

In the proposed TripleS-EFT, the average number of selected stocks in a day is 140.8 (with maximum and minimum numbers of 284 and 32), and the average number of trades is 34.6 (with maximum and minimum numbers of 236 and 0), indicating that only 24.6% of selectec stocks are traded.

D. Robustness Evaluation

To evaluate the robustness of the proposed TripleS, we investigate the performance of TripleS-EFT with different

TABLE VI: Comparison with state-of-the-art system

	ALL [31]	RandomFuzzy [8]	TripleS-EFT
GAP	16.30% / 1.431	20.63% / **4.424**	**29.16%** / 2.109
1H1L	8.14% / 1.017	6.70% / **2.148**	**15.92%** / 1.611
3H3L	3.71% / 0.600	3.10% / **1.310**	**7.86%** / 1.148
5H5L	2.16% / 0.374	2.78% / **1.291**	**5.15%** / 0.724

number of selected stocks K. With a smaller number of selected stocks (higher membership value), the investment performance should be better, proving the effectiveness and robustness of the membership value and the proposed TripleS. Table VII and Fig. 8 show the investment performance at different K, including 25, 50, 100, 150, 200, 250 and 300. The experimental results show a decreasing trend of annual returns (all the red curves in Fig. 8), which proves the effectiveness of the designed membership value to quantify the suitability and profitability of stocks for the strategy. However, Sharpe ratios sometimes increase with K, which violates the assumption.

The TripleS-EFT aims to select stocks with high *MSI* or *CSI*, which represent high Sharpe ratios for individual stocks. However, investing in stocks with high (individual) Sharpe ratios does not necessarily represent a portfolio with a high Sharpe ratio. In experiments, it can be seen that the annual returns are similar for large K. Based on the risk diversification [33], the more diversified the portfolio (more stocks, larger K), the lower the investment risk. Consequently, for a similar annual return and lower investment risk, the larger K, the higher Sharpe ratio. This phenomenon can be observed in the **1H1L** strategy with K greater than 150, the **3H3L** strategy with K greater than 100, and the **5H5L** strategy with K greater than 50, as shown in Fig. 8.

VI. Conclusions

In this paper, we present a stock selection method called TripleS, which is based on the suitability index (SI) and fuzzy-set theory. The developed system has the potential to be developed into an automated trading system that can be utilized in the real-world stock market. In the first step of our process, we develop position sizing algorithms to extract the features of stocks, known as SI. These features convey not only the momentum and contrarian aspects, but also its suitability for the approach. Then, SI is transformed by the specified fuzzy-set modules to facilitate stock selection and further portfolio management. There are some other proposed stock selection methods, which have been named TripleS-T1, TripleS-T2, TripleS-T2U, TripleS-EF and TripleS-EFT. In

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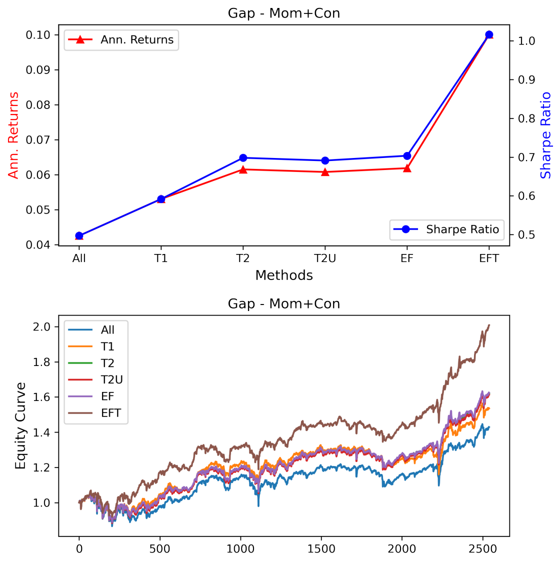
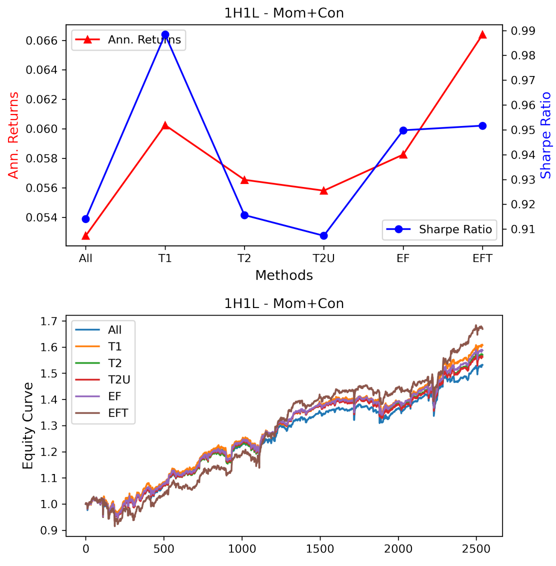
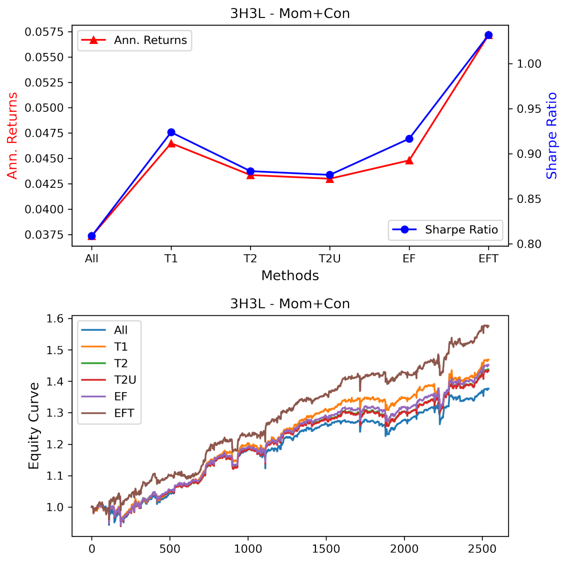


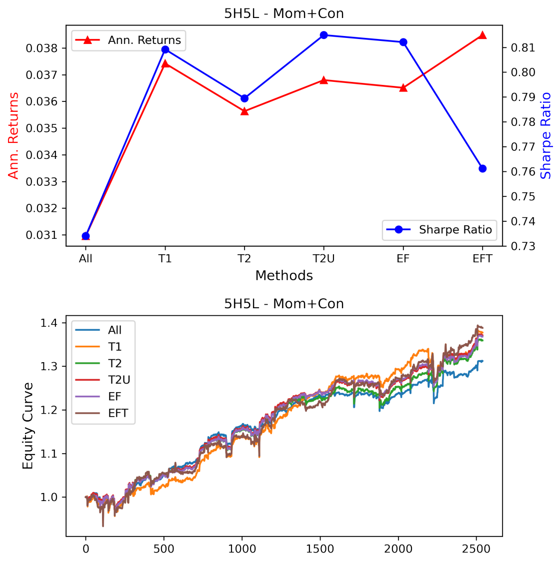
(a) Gap



(b) 1H1L



(c) 3H3L



(d) 5H5L

Fig. 7: Investment performance and equity curves of the systems on different strategies.

TABLE VII: Investment performance (annual return / Sharpe ratio) of TripleS-EFT under different K

K	25	50	100	150	200	250	300
GAP	**10.01%** / **1.016**	7.18% / 0.787	6.64% / 0.747	6.35% / 0.719	6.22% / 0.706	6.18% / 0.703	6.19% / 0.703
1H1L	**6.64%** / 0.952	6.51% / **0.990**	6.02% / 0.959	5.83% / 0.942	5.82% / 0.948	5.83% / 0.950	5.83% / 0.950
3H3L	**5.72%** / **1.032**	4.84% / 0.931	4.42% / 0.884	4.48% / 0.910	4.47% / 0.914	4.48% / 0.917	4.48% / 0.917
5H5L	**3.85%** / 0.761	3.61% / 0.765	3.66% / 0.805	3.66% / 0.812	3.68% / **0.818**	3.66% / 0.814	3.65% / 0.812

addition, we use a rolling window technique to periodically and dynamically adjust the settings of the system. The experimental results show that the proposed TripleS has only a small improvement in precision but a significant improvement in investment performance. The TripleS-EFT has outstanding profitability, the highest annual return in all strategies, and the highest Sharpe ratio in two of the four strategies, and achieves an annual return of 10.01% and a Sharpe ratio of 1.032, which significantly outperform the benchmark system. Additionally, the TripleS-EFT also significantly outperforms the state-of-the-art fuzzy-based system in measuring annual returns, and has even doubled the annual returns in several strategies. In summary, the proposed TripleS has a slight improvement in selection precision and excellent improvements in investment

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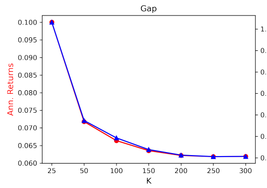
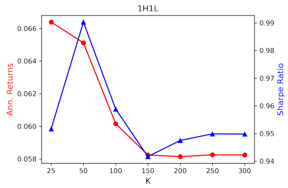
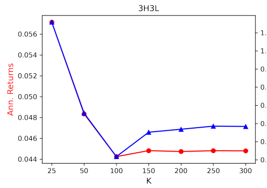


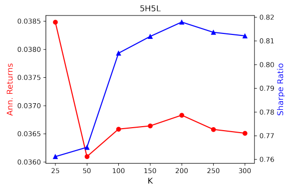
(a) Gap



(b) 1H1L



(c) 3H3L



(d) 5H5L

Fig. 8: Investment performance under different K

performance and portfolio management, especially in profitability.

In the future, we will investigate the temporal coherence of the selected stocks, as the historical selection may also contain valuable information. We will also use the proposed TripleS for portfolio management, i.e., we will extract comprehensive characteristics of all selected stocks to achieve further stability.

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REFERENCES

[1] J. Oliver-Muncharaz and F. García García, "Leading research trends on trading strategies," *Finance, Markets and Valuation*, vol. 6, no. 2, pp. 27–54, 2020.

[2] E. P. Chan, *Quantitative trading: how to build your own algorithmic trading business*. John Wiley & Sons, 2021.

[3] J.-H. Syu, Y.-R. Yeh, M.-E. Wu, and J.-M. Ho, "Self-management portfolio system with adaptive association mining: A practical application on taiwan stock market," *Mathematics*, vol. 9, no. 10, p. 1093, 2021.

[4] S. Selvin, R. Vinayakumar, E. Gopalakrishnan, V. K. Menon, and K. Soman, "Stock price prediction using lstm, rnn and cnn-sliding window model," in *2017 international conference on advances in computing, communications and informatics (icacci)*, 2017, pp. 1643–1647.

[5] Z. Jiang, D. Xu, and J. Liang, "A deep reinforcement learning framework for the financial portfolio management problem," *arXiv preprint arXiv:1706.10059*, 2017.

[6] M.-E. Wu, J.-H. Syu, J. C.-W. Lin, and J.-M. Ho, "Portfolio management system in equity market neutral using reinforcement learning," *Applied Intelligence*, pp. 1–13, 2021.

[7] J. J. Buckley, "The fuzzy mathematics of finance," *Fuzzy Sets and Systems*, vol. 21, no. 3, pp. 257–273, 1987.

[8] M. E. Wu, J. H. Syu, J. C. W. Lin, and J. M. Ho, "Effective fuzzy system for qualifying the characteristics of stocks by random trading," *IEEE Transactions on Fuzzy Systems*, 2021.

[9] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[10] H.-J. Zimmermann, *Fuzzy set theory—and its applications*. Springer Science & Business Media, 2011.

[11] H. Mo, F. Y. Wang, M. Zhou, R. Li, and Z. Xiao, "Footprint of uncertainty for type-2 fuzzy sets," *Information Sciences*, vol. 272, pp. 96–110, 2014.

[12] L. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—i," *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[13] J. Deng and Y. Deng, "Information volume of fuzzy membership function." *International Journal of Computers, Communications & Control*, vol. 16, no. 1, 2021.

[14] O. A. M. Ali, A. Y. Ali, and B. S. Sumait, "Comparison between the effects of different types of membership functions on fuzzy logic controller performance," *International Journal*, vol. 76, pp. 76–83, 2015.

[15] W. Pedrycz, "Why triangular membership functions?" *Fuzzy Sets and Systems*, vol. 64, no. 1, pp. 21–30, 1994.

[16] W. N. Goetzmann and M. Massa, "Daily momentum and contrarian behavior of index fund investors," *Journal of Financial and Quantitative Analysis*, vol. 37, no. 3, pp. 375–389, 2002.

[17] N. Jegadeesh and S. Titman, "Profitability of momentum strategies: An evaluation of alternative explanations," *The Journal of finance*, vol. 56, no. 2, pp. 699–720, 2001.

[18] J. H. Syu, M. E. Wu, S. H. Lee, and J. M. Ho, "Modified orb strategies with threshold adjusting on taiwan futures market," in *IEEE Conference on Computational Intelligence for Financial Engineering and Economics*, 2019, pp. 1–7.

[19] Y. C. Tsai, M. E. Wu, J. H. Syu, C. L. Lei, C. S. Wu, J. M. Ho, and C. J. Wang, "Assessing the profitability of timely ppening range breakout on index futures markets," *IEEE Access*, vol. 7, pp. 32 061–32 071, 2019.

[20] J. H. Syu and M. E. Wu, "Modifying orb trading strategies using particle swarm optimization and multi-objective optimization," *Journal of Ambient Intelligence and Humanized Computing*, pp. 1–13, 2021.

[21] M. E. Wu, J. H. Syu, J. C. W. Lin, and J. M. Ho, "Evolutionary orb-based model with protective closing strategies," *Knowledge-Based Systems*, vol. 216, p. 106769, 2021.

[22] T. Bollerslev, R. Y. Chou, and K. F. Kroner, "Arch modeling in finance: A review of the theory and empirical evidence," *Journal of Econometrics*, vol. 52, no. 1-2, pp. 5–59, 1992.

[23] J. M.-J. Leung and T. T.-L. Chong, "An empirical comparison of moving average envelopes and bollinger bands," *Applied Economics Letters*, vol. 10, no. 6, pp. 339–341, 2003.

[24] J. Ginyard, "Position-sizing effects on trader performance: An experimental analysis," *Uppsala University, Department of Psychology, master thesis, available at http://www. turtletrader. com/position_sizing. pdf*, 2001.

[25] S. Basak and D. Makarov, "Strategic asset allocation in money management," *The Journal of finance*, vol. 69, no. 1, pp. 179–217, 2014.

[26] R. Jones, *The trading game: playing by the numbers to make millions*. John Wiley & Sons, 1999, vol. 77.

[27] D. G. Uyemura, C. C. Kantor, and J. M. Pettit, "Eva® for banks: Value creation, risk management, and profitability measurement," *Journal of Applied Corporate Finance*, vol. 9, no. 2, pp. 94–109, 1996.

[28] W. F. Sharpe, "The sharpe ratio," *Journal of Portfolio Management*, vol. 21, no. 1, pp. 49–58, 1994.

[29] A. Polly. Gap trading strategies. [Online]. Available: https://school.stockcharts.com/doku.php?id=trading_strategies:gap_trading_strategies

[30] M. M. Lai and S. H. Lau, "The profitability of the simple moving averages and trading range breakout in the asian stock markets," *Journal of Asian Economics*, vol. 17, no. 1, pp. 144–170, 2006.

[31] Y. Plyakha, R. Uppal, and G. Vilkov, "Equal or value weighting? implications for asset-pricing tests," in *Financial Risk Management and Modeling*. Springer, 2021, pp. 295–347.

[32] V. DeMiguel, L. Garlappi, and R. Uppal, "Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy?" *The review of Financial studies*, vol. 22, no. 5, pp. 1915–1953, 2009.

[33] E. Qian, "Risk parity and diversification," *The Journal of Investing*, vol. 20, no. 1, pp. 119–127, 2011.