CIS 580, Machine Perception, Fall 2022
Homework 2
Due: Monday Oct 10 2022, 8pm ET

Instructions

- This is an individual homework and worth 100 points
- You must submit your solutions on Gradescope. We recommend that you use LaTeX, but we will accept scanned solutions as well.
- Start early! Please post your questions on Ed Discussion or come to office hours!

Submission

- You will submit a concise report to Gradescope that includes answers for Part 1 and discussion on the algorithms implemented as well as results from multiple frames for Part 2.
- You will also submit all python files you completed as .py files to Gradescope

1 Projective Invariants (30 pts)

1. (15pts) Consider Figure 1. With a ruler, you measure the lines AB = 4cm, BC = 2.5cm and DC = 1cm in length on the image. In world coordinates, we know $A_wB_w = B_wC_w = 3$m; find the distance C_wD_w. Focus only on the four yellow dots labelled as A, B, C, D.
2. (15pts) Consider Figure 2. You want to find the boy George's real-world height in cm, where the horizon and all relevant points and lines are labeled. Say the adult in the background has height 182.88cm, as labeled. Using a ruler, we measured line CD as 10cm and line DE as 5cm on the image. What is George's real-world height?

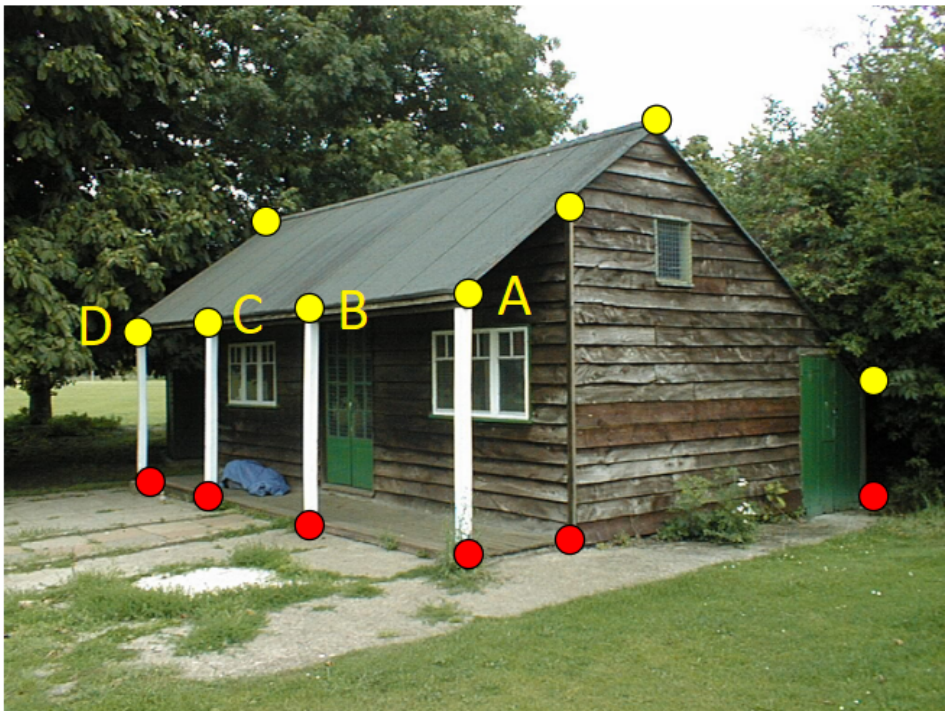


Figure 1: Problem 1.1

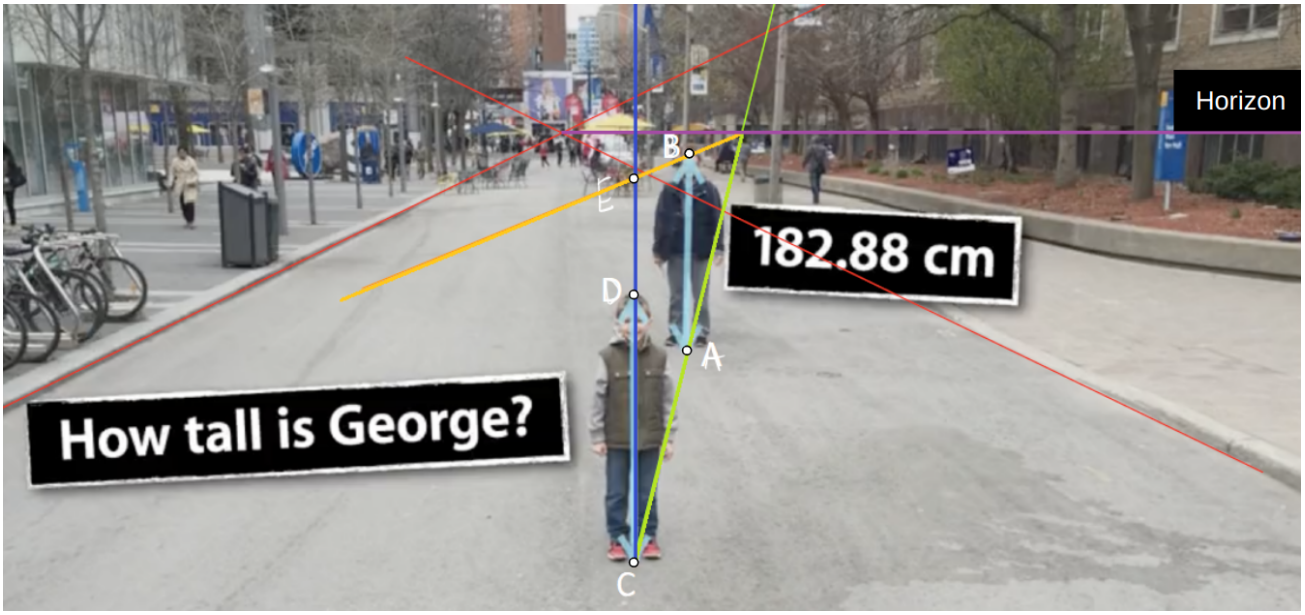


Figure 2: Problem 1.2

2 Augmented Reality with AprilTags (70pts)

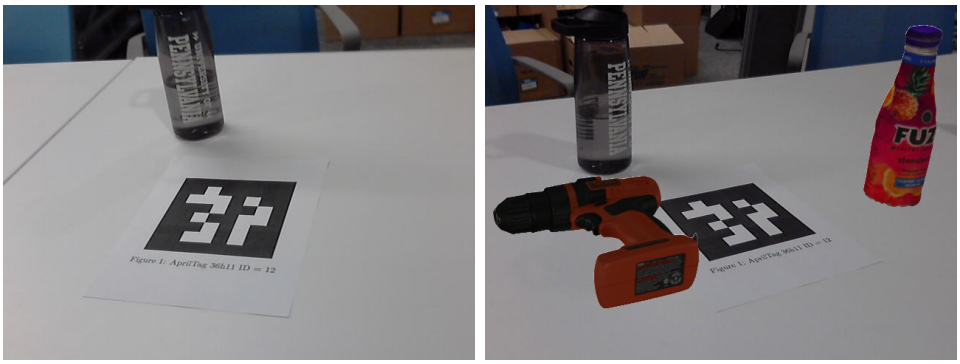
In this section, you will implement a simple augmented reality application. The deliverable is a video that contains several virtual object models as if they exist in the real world. Furthermore, you will be able to specify pixel positions to place an arbitrary object.

You are given a video with an AprilTags (https://april.eecs.umich.edu/software/apriltag) in each frame. These tags are usually used in robotics for determining the pose of the camera. To make things easier, we provide the 4 corners coordinates (in pixel) as well as the size of the tags.

Your main task in this problem is to recover the camera poses with two different approaches: 1) solving the Perspective-N-Point (PnP) problem with coplanar assumption and 2) solving the Persepective-three-point (P3P) and the Procrustes problem.

After retrieving the 3D relationship between the camera and world, we can place an arbitrary objects in the scene.

Note, although the Open-CV library is in the python environment requirements, when we ask you to implement the algorithm in the coding homework, you should implement the algorithm learned in the lecture by yourself instead of simply calling the opencv library implementation.



(a) sample image (b) sample result

Figure 3: Projecting objects on the table

2.1 Establish World Coordinate System (3 pts)

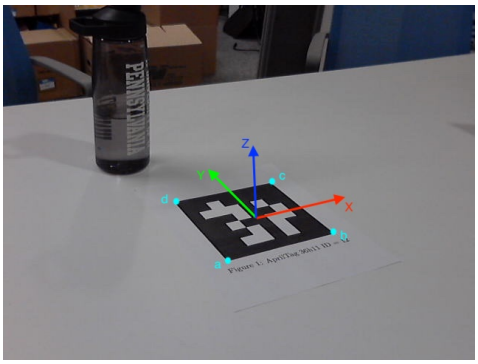


Figure 4: world coordinate setup

To know where the camera is in the world, we must first define a world coordinate system. We conveniently place the world coordinate at the center of the tag as shown in Figure 4.

From the lecture, we know that recovering camera pose from 2D-3D correspondence requires at least 3 points. We provide all 4 AprilTag corners in **corners.npy**. Theses corner coordinates will be used in **main.py**.

Let a, b, c and d be the four corners in the image. Your first task is to find the 3D world coordinates of a, b, c and d by completing **est_Pw.py** given the length of the side of the tag. We define the world coordinate system to be centered at the center of the AprilTag, with x, y direction pointing towards edge bc and cd. The z direction is oriented vertically up out of the tag. All tag points are assumed to be coplanar. Please refer to Figure 4 for the correct world coordinate configuration.

2.2 Solve PnP with Coplanar Assumption (15 pts)

In this section, you will estimate the camera pose from an AprilTag based on homography estimation. Recall that a homography is a projective transformation between planes, which you have implemented in hw1. The technique for solving camera pose follows closely the Lec07-slides.pdf (Our grader will check the solution following the algorithm on slide page-43, NOT page-44)

Since the world frame is conveniently placed on the tag, the z component of the corners should be zero. Following the slides, we have

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim K \begin{pmatrix} r_1, r_2, T \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

where K $\left(r_1, r_2, T\right)$ is a homography H. To recover H, feel free to re-use **est_homography.py** that you implemented in hw1.

Note that the homography matrix you calculate will have scale ambiguity, sometimes your solved H may, unfortunately, have a negative scale, which violates the fact that the world's origin is in front of the camera (think about this, why?). Hence, to get the camera pose you must normalize the homography matrix to get rid of the unreal situation (you should think about how to normalize the H in this homework).

After obtaining H, you should follow the slides to recover R and t. Note that in the slide, $R = R_w^c$ and $t = t_w^c$ whereas **est_homography.py** here should return R_c^w and t_c^w, which describe camera pose in the world. You should complete the function in **solve_pnp.py**

2.3 Solving Perspective-Three-Point problem (18 pts)

Here you will calculate the camera pose by first calculating the 3D coordinates of any 3 (out of 4) corners of the AprilTag in the camera frame. In Part 1, the 3D coordinates of the same points in the world frame have been calculated. You will use this correspondence to solve for camera pose R_c^w and t_c^w in the world frame by implementing solving the Procrustes problem. **Note**: you should first have a look at Prob 2.4 and finish the Procrustes function since you should call the Procrustes function in this question.

2.3.1 P3P Derivation

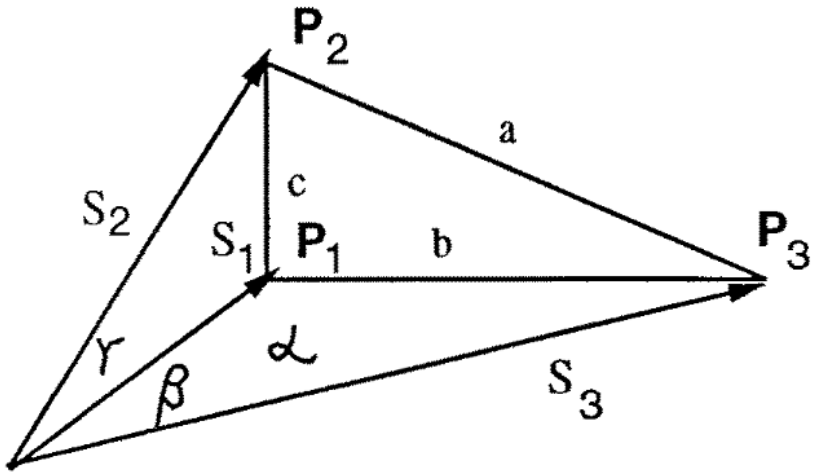


Figure 5: Illustrates the geometry of the three point space resection problem. The triangle side lengths a, b, c are known and the angles α, β, γ are known. The problem is to determine the lengths s_1, s_2, s_3 from which the 3D vertex point positions (in the camera frame) P_1, P_2, P_3 can be immediately determined.

You only need to solve up to a 4th degree polynomial, i.e. estimating the coefficients of this polynomial in terms of $a, b, c, \alpha, \beta, \gamma$. We code them up in the next part to get the roots of the polynomial.

Part of the proof for P3P is given in class (PnP slides). Please refer to this document for more details in solving P3P. P3P reference paper (you need to use Grunert's solution)

2.3.2 P3P (Programming)

Please complete function in **solve-p3p.py**. Important Points to consider:

- you have 4 corners and are free to use any 3 of them.
- You can use numpy.roots to get the roots of the polynomial
- You will get up to 4 roots for the polynomial.
- Now after getting multiple roots please follow these steps to select the solution.
  - First of all eliminate the roots which are imaginary. Keep the real roots.
  - Then we require roots which are greater than zero ($u > 0$, $v > 0$). (Convention as per P3P paper.)

– Since we know the pixel coordinate and the ground truth 3D position of the 4th unused point, we use the reprojection error to select the solution: we get the R,t from each of the solutions by passing it to the Procrustes function. Then using that estimated extrinsic information we re-project the 4th unused world point to the pixel coordinate and compare the consistency with the known true 4th pixel coordinate. The solution which gives the least re-projection error (squared difference between estimated and correct coordinate) is your final solution.

2.4 Solving Procrustes Problem (14 pts)

You need to use the correspondence of the same 3 points in the world frame and the camera frame and solve for camera pose using the Procrustes method. Use the Procrustes slides from class for your reference.

$$\min_{R,T} \sum_{i}^{N=3} ||Y_i - RX_i + T||^2$$

2.5 Pick Placing Location (6 pts)

Now that you have R and T between the camera and the world coordinate, you can place an object at any location on the table! However, there is one piece missing from interactively selecting a place on the table from a picture - we need to be able to click on a pixel and know its coordinate in the world.

Suppose you have pick a pixel on the table at (u, v), you can find out the 3D location of that point with respect to the world coordinates. Given R_c^w and T_c^w and the camera intrinsic matrix K, please write down mathematically how to calculate the 3D location $X_w \in R^3$ in world coordinates corresponding to pixel $\hat{x} = [u, v, 1]^T$. (**Hint**: points on the table follow $z = 0$) You can use letter and subscripts to represent entries in the matrix, e.g. A_{ij} represents the value at ith row and jth column of A matrix.

Programming: Please complete **est_pixel_world.py**.

2.6 Put Everything Together (14 pts)

At this point, you have all the components needed for simplified augmented reality application based on AprilTags. In this section, you will use the completed functions to generate a video. We provide a series of frames in the **data** folder. For each frame, you are place an arbitrary number of objects in the scene. Now you can place your objects on the table by executing main.py. We have picked two pixel locations in the file. These coordinates correspond to the centers of the virtual objects in the first frame. You should use these coordinates in your submitted video. For each frame, keep the

objects static in the world coordinate and render the image from the current camera view. At the end, you are asked to combine these frames into a **.gif** file. **You should also include the image of the first frame in your PDF solution**.

Deliverables:

1. A gif file of a sequence that contains a set of static virtual objects with real backgrounds, as if the virtual objects were placed in the original scene.

2. The first frame of the video in the submitted PDF file.

If you want to use your own points to project, please add **- -click_points** option when running **main.py**. You will be prompt to click two points on the screen, these points will be selected as the centers of the two objects. Feel free to swap the objects with your favourite models and submit the gif along!

2.7 Files to complete and submit

Note: the auto-grader has totally 56+1 points, 56 for regular coding grades and 1 point for checking whether you upload the gif visualization file, which will count for finally 14pts as suggested in 2.6.

Don't forget to also put the first frame of the gif visualization to your PDF submission.

1. **est_Pw.py**
   This function is responsible for finding world coordinates of a, b, c and d given tag size

2. **solve_pnp.py**
   This function is responsible for recovering R and t from 2D-3D correspondence with coplanar assumption

3. **est_pixel_world.py**
   This function is responsible for solving 3D locations of a pixel on the table.

4. **solve_p3p.py**
   This file has two functions P3P and Procrustes that you need to write. P3P solves the polynomial and calculates the distances of the 3 points from the camera and then uses the corresponding coordinates in the camera frame and the world frame. You need to call Procrustes inside P3P to return R,t.

5. **VR_res.gif**
   This file is generated automatically by executing main.py and needs to be submitted with other $*$.py file.

6. **Visualizations and solutions**
   Please include the first frame of your GIF (saved automatically as vis.png in main.py) and your solution to Section 2.5 in your PDF submission.