CIS 580, Machine Perception, Fall 2022
Homework 1
Due: Monday Sept 26 2022, 8pm ET

Instructions

- This is an individual homework and worth 100 points.
- You must submit your solutions on Gradescope. We recommend that you use LaTeX, but we will accept scanned solutions as well.
- Please complete this homework with Python 3. To run this homework successfully, you will need the following python packages:
  - numpy
  - matplotlib
  - opencv-python
- Start early! Please post your questions on Ed Discussion or come to office hours!

Submission

- You will submit a concise report on Gradescope that includes a discussion on the algorithms implemented as well as results from multiple frames for part 1, and your answers for parts 2 and 3.
- You will also submit all python files you completed as .py files to Gradescope.

1 Virtual Billboard (50 pts)

1.1 Introduction

In this programming assignment, we will use the concepts of projective geometry and homographies to allow us to project an image onto a scene in a natural way that respects perspective. To demonstrate this, we will project our logo onto the goal during a football match. For this assignment, we have provided images from a video sequence of a football match, as well as the corners of the goal in each image and an image of the Penn Engineering logo. Your task is, for each image in the video sequence, compute the homography between the Penn logo and the goal, and then warp the goal points onto the ones in the Penn logo to generate a projection of the logo onto the video frame (e.g. Figure 1).

(a) Original image from video (b) Image with Penn logo projected

Figure 1: Example of logo projection

1.2 Technical Details

The Python script **project_logo.py** will be the main script to run this assignment. In this script, we provide you the following:

- A sequence of images onto which you will project a logo image.
- The corners in each video frame that the logo should project onto.
- The logo image itself.

Your goal will be to complete the functions **est_homography** and **warp_pts** in their respective .py files. **est_homography** estimates the homography that maps the video image points onto the logo points (i.e. $\mathbf{x}_{logo} \sim \mathbf{x}_{video}$), and **warp_pts** then warps the sample points according to this homography, returning the warped positions of the points (that is, the corresponding points in the logo image). We then use these correspondences to copy the points from the logo into the video image. Once you finish these two functions, change your current directory to the directory of "Part_1", and run "python project_logo.py". It will use the two functions you completed to project logo, visualize them in a video, and save sampled results.

1.3 Homography Estimation

To project one image patch onto another, we need, for each point inside the goal in the video frame, to find the corresponding point from the logo image to copy over. In other words, we need to calculate the homography between the two image patches. This homography is a 3x3 matrix that satisfies the following:

$$\mathbf{x}_{logo} \sim H\mathbf{x}_{video} \tag{1}$$

Or, equivalently:

$$\lambda\mathbf{x}_{logo} = H\mathbf{x}_{video} \tag{2}$$

Where $\mathbf{x}_{logo}$ and $\mathbf{x}_{video}$ are homogeneous image coordinates from each patch and λ is some scaling constant. To calculate the homography needed for this projection, we provide, for each image, the corners of the patches that we would like you to warp between in each image. You must calculate the homography using the provided corner points and the technique covered in the lectures and Appendix A. You can then warp each image point using H to find its corresponding point in the logo (note that the homography equation is estimated up to a scalar, so you will need to divide $H\mathbf{x}_{image}$ by the third term, which is λ), and then return the set of corresponding points as a matrix.

1.4 Inverse Warping

You may be wondering that the correct way to calculate homography would be to map the points from the logo image onto the video frames. However, doing this will most likely have the case where multiple logo points project to one video frame pixel (due to rounding of the pixels), while other pixels may have no logo points at all. This results in 'holes' in the video frame where no logo points are mapped. To avoid this, in this assignment, you are calculating the inverse homography to calculate the projection from video frame points to logo points which guarantees that every video frame pixel gets a corresponding pixel from the logo.

We can then replace every point in the video frame ($\mathbf{x}_{video}$) with the corresponding point in the logo ($\mathbf{x}_{logo}$) using the correspondences ($\mathbf{x}_{image}, \mathbf{x}_{logo}$). This is already done for you in **inverse_warping**, and you should not need to change it.

1.5 Files to complete and submit

1. **est_homography.py**
   This function is responsible for computing the homography given correspondence

2. **warp_pts.py**
   This function is responsible for computing the warped points given correspondence and a set of sample points

1.6 Visualizing Results

To play the projected images as a video, run the **project_logo.py** script. First open a terminal window and change the current directory to the directory of "Part_1". Then run command:

python project_logo.py

You can also use an IDE to run the script. If you would like to play with your own data, you can edit **project_logo** and generate your own video with a set of points.

1.7 Appendix A: Calculating Homographies

As we learned in the lectures, a homography H maps a set of points $\mathbf{x} = \begin{pmatrix} x & y & 1 \end{pmatrix}^T$ to another set of points $\mathbf{x'} = \begin{pmatrix} x' & y' & 1 \end{pmatrix}$ up to a scalar:

$$\mathbf{x}' \sim H\mathbf{x} \tag{3}$$

$$\lambda \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{4}$$

$$\lambda x' = h_{11}x + h_{12}y + h_{13} \tag{5}$$

$$\lambda y' = h_{21}x + h_{22}y + h_{23} \tag{6}$$

$$\lambda = h_{31}x + h_{32}y + h_{33} \tag{7}$$

In order to recover x' and y', we can divide equations (5) and (6) by (7):

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \tag{8}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}} \tag{9}$$

Rearranging the terms above, we can get a set of equations that is linear in the terms of H:

$$-h_{11}x - h_{12}y - h_{13} + h_{31}xx' + h_{32}yx' + h_{33}x' = 0 \tag{10}$$

$$-h_{21}x - h_{22}y - h_{23} + h_{31}xy' + h_{32}yy' + h_{33}y' = 0 \tag{11}$$

Finally, we can write the above as a matrix equation:

$$\begin{pmatrix} \mathbf{a_x} \\ \mathbf{a_y} \end{pmatrix} \mathbf{h} = \mathbf{0} \tag{12}$$

Where:

$$a_x = \begin{pmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \end{pmatrix} \tag{13}$$
$$a_y = \begin{pmatrix} 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{pmatrix} \tag{14}$$
$$h = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{21} & h_{22} & h_{23} & h_{31} & h_{32} & h_{33} \end{pmatrix}^T \tag{15}$$

Our matrix H has 8 degrees of freedom, and so, as each point gives 2 sets of equations, we will need 4 points to solve for h uniquely. So, given four points (such as the corners provided for this assignment), we can generate vectors a_x and a_y for each, and concatenate them together:

$$A = \begin{pmatrix} a_{x,1} \\ a_{y_1} \\ \vdots \\ a_{x,n} \\ a_{y,n} \end{pmatrix} \tag{16}$$

Our problem is now:

$$Ah = 0 \tag{17}$$

As A is a 8x9 matrix, there is a unique null space. Normally, we can use MATLAB's **null** function, however, due to noise in our measurements, there may not be an h such that Ah is exactly 0. Instead, we have, for some small $\vec{\epsilon}$:

$$Ah = \vec{\epsilon} \tag{18}$$

To resolve this issue, we can find the vector h that minimizes the norm of this $\vec{\epsilon}$. To do this, we must use the SVD, which we will cover later. For this project, all you need to know is that you need to run the command:

```
[U, S, Vt] = np.linalg.svd(A);
```

Note that Vt is the transpose of the V matrix. The vector h will then be the last column of V, and you can then construct the 3x3 homography matrix by reshaping the 9x1 h vector.

2 Basic Projective Geometry (35pts)

1. (3pts) You capture an image of a real object that is 25m from the camera and has height 6m parallel to the image plane. Your camera takes images of resolution 240(width)×360(height) pixels, with a sensor size of 24mm(width)×36mm(height). The object in your image has a height of 180 pixels. What focal length f (in mm) did you use to take the picture? Hint: To convert from image plane coordinates to pixel coordinates, it is safe to assume that pixels are uniformly sized and packed tightly into the sensor area; this is the case for standard camera sensors.

2. (2pts) Solve the following for the points and lines in $\mathbb{P}^2$:

   - Find an equation for the line passing through the points $[2, 3, 4]$ and $[3, 5, 1]$.
   - Find the intersection point for the lines $2x - 6w = 0$ and $5x - 2y = 0$.

3. (5pts) Find λ such that the three lines in $\mathbb{P}^2$, $w = 0$, $x + \lambda y + \lambda w = 0$, and $\lambda x + y + \lambda w = 0$ have a common intersection. Determine the point of intersection.

4. (4pts) Two circles in homogeneous coordinates actually intersect at *four* points, though only two are visibly apparent. Show that the other two points are $[1, \pm i, 0]$, by defining a circle in homogeneous coordinates and showing that these two points must exist on it.

5. (8pts) Find a projective transformation A that preserves the points $p_1 = (1, 0, 0)$, $p_2 = (0, 1, 0)$, and the origin of the coordinate system O and will map the point $p_3 = (1, 1, 1)$ to the point $p_3' = (3, 2, 1)$?

6. (8pts) A projective transformation B leaves the horizontal lines horizontal but maps all vertical lines to lines going through the point (0, 5). It leaves the points (0, 0) and (1, 1) at the same place. Compute B.

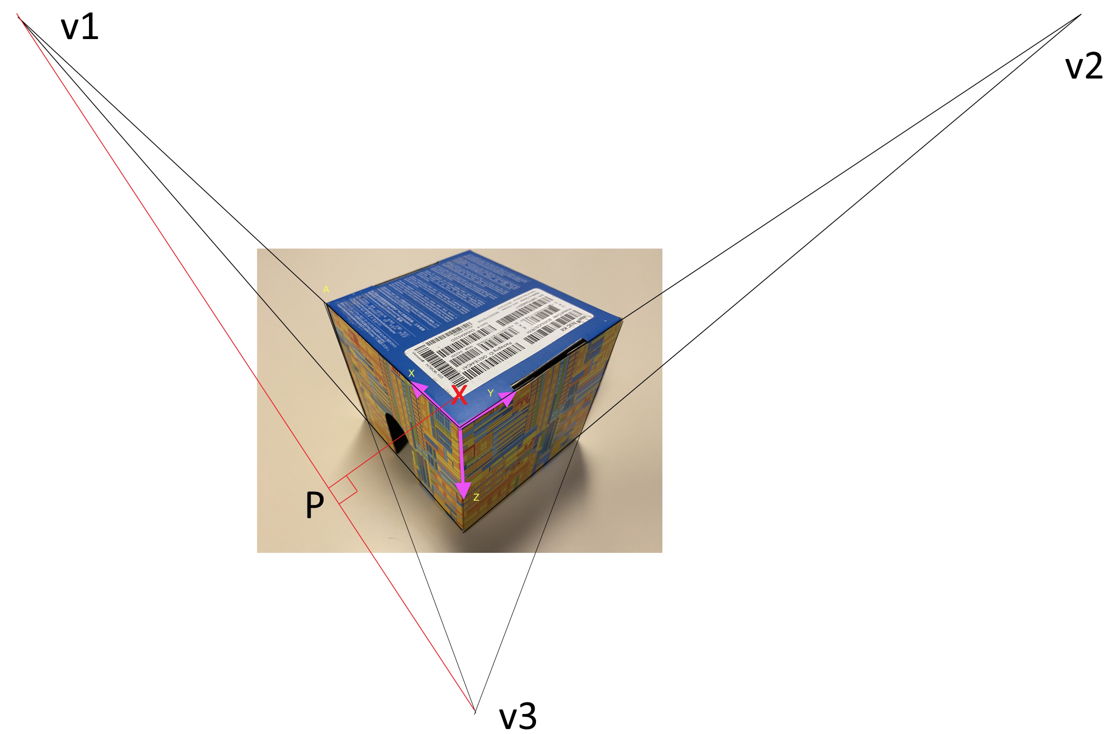
7. (5pts) Points in a world plane p are mapped to points in the image plane q via the projective transformation $q \sim Hp$ with

$$H = \begin{pmatrix} 2 & 1 & 0 \\ 5 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

   Find the equation of the horizon (projection of the line at infinity).

3 Vanishing Points (15pts)

Below is the image of a cube. A world (object) coordinate system is attached to the cube as in the picture. The vanishing points v_1, v_2, v_3 are shown in the image. The point marked with a red **X** is the image center (512, 228) .



1. (10pts) Show how you can compute the intrinsics (focal length) of the camera from these vanishing points.

2. (3pts) Consider a normal drawn from the image centre to the line joining v_1 and v_3, at point P. The distance from P to the image centre measures 4mm. The distance v_1 - P is 16mm and the distance v_2 - P is 10mm. Find the focal length f (in mm) using the formula you derived in the previous question.

3. (2pts) Write the camera intrinsic matrix K (Assume the focal length is same in x and y directions $f_x = f_y = f$).