Aerodynamics Computational Assignment #2: Flow Over Airfoils

Assigned Date: Monday January 31, 2022
Due Date: Sunday February 27, 2022

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Reflection Questions:

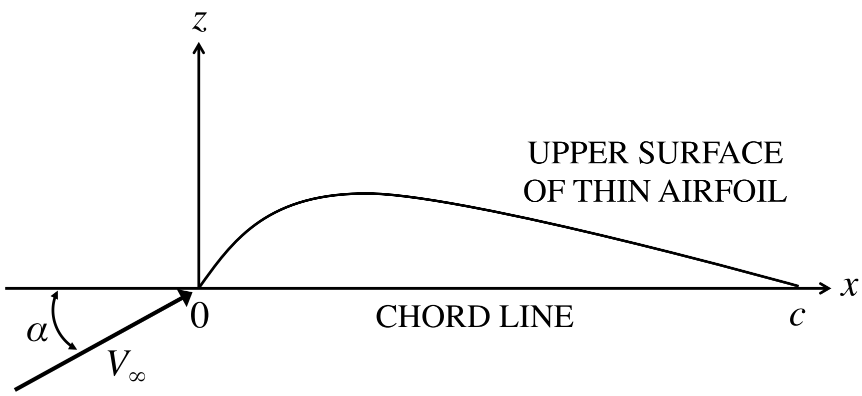
In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Understand how thin airfoil theory is used to approximate aerodynamic forces.
2. Practice using a vortex panel method to calculate aerodynamic forces.
3. Practice using the superposition of elementary flows to complete analysis of an airfoil.
4. Compare the flow fields and forces predicted by thin airfoil theory to that numerically computed for thick airfoils using a vortex panel code.
5. Understand the effect of flow parameters on streamlines, equipotential lines, and pressure contours.
6. Understand the difference between the application and results of thin airfoil theory and the vortex panel method.
7. Understand how the changes in wing section camber and thickness alter the lift slope.

Problem 1: Visualizing Flow Around Thin Airfoils

The flow about a "thin" symmetric airfoil can be approximated by potential flow theory, as elaborated in Chapter 4 of Anderson. Suppose that the chord of the airfoil extends along the x-axis from $x = 0$ to $x = c$ as illustrated in the below figure:

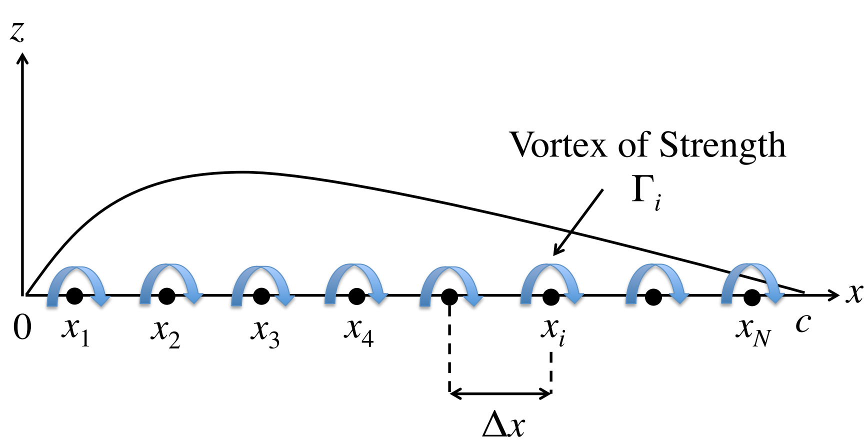


Under the limit that the airfoil thickness goes to zero and it can be approximated as a "thin" airfiol the flow can be represented by a vortex sheet aligned along the chord line whose strength $\gamma(x)$ is given by:

$$\gamma(x) = 2\alpha V_\infty \sqrt{\frac{1 - \frac{x}{c}}{\frac{x}{c}}}$$

where α is the angle of attack of the incoming flow relative to the x-axis (in radians) and V_∞ is the free-stream flow speed. (This formula follows from Anderson 4.24 mapped back to x/c using 4.21.)

Numerically, one can approximate the continuous vortex sheet by a set of N discrete vortices separated by a distance $\Delta x = c/N$ where the i^{th} vortex has strength $\Gamma_i = \gamma(x_i)\Delta x$. This is depicted in the below figure:



Write a MATLAB function which plots the:

1. stream lines, ψ,
2. equipotential lines, ϕ, and
3. pressure contours, c_p,

for flow about a "thin" symmetric airfoil using the approximations detailed above. Your function should take the form:

```
function PlotThinAirfoil(c,alpha,V_inf,p_inf,rho_inf,N)
```

where `c` is the chord length c (in meters), `alpha` is the angle of attack α (in degrees), `V_inf` is the free-stream flow speed V_∞ (in meters per second), `p_inf` is the free-stream pressure p_∞ (in Pascals), `rho_inf` is the free-stream density ρ_∞ (in kilograms per meter cubed), and `N` is the number of discrete vortices N employed to approximate the vortex sheet.

Using your MATLAB function,

- Visualize or generate plots of the stream lines, equipotential lines, and pressure contours for flow about a thin symmetric airfoil with $c = 2$ m, $\alpha = 9°$, $V_\infty = 60$ m/s, $p_\infty = 85.5 \times 10^3$ Pa, and $\rho_\infty = 1$ kg/m^3.
- Conduct a study of the effect of the number of discrete vortices N on the resulting flow and pressure field accuracy for the aforementioned values.

Reflection: Consider how the stream lines, equipotential lines, and pressure contours are affected by changes in: 1) chord length, 2) angle of attack, and 3) free-stream flow speed. What about if the dependent variables x and y are appropriately normalized by the reference chord length? Evaluate the pressure contours, and identify the locations of minimum and maximum pressure. How do these change as the above conditions are altered? Consider the streamlines and equipotential lines; are the fields continuous? What does this imply?

Hint: You will need to use the principle of superposition within your MATLAB function. This principle is illustrated in the MATLAB file `Lifting_Cylinder.m` located on the course website, wherein the stream lines for the flow around a cylinder with circulation are plotted by superposing flow from a uniform flow, a dipole, and a vortex. It is recommended that you follow the logic of `Lifting_Cylinder.m` when building your function `PlotThinAirfoil`.

Problem #2: Visualizing Flow Around Thick Airfoils

As the thickness of an airfoil is increased, it is no longer accurate to approximate the flow around it using thin airfoil theory, with a continuous vortex sheet located on the chord line. Instead a vortex sheet can be "wrapped" around the airfoil surface to better account for the thickness of the body and to directly enforce the boundary conditions on the surface, including no penetration of the surface streamline.

This process can be accomplished with a classical "Vortex Panel Method" as outlined by Anderson in Section 4.10 of the Textbook. The approach discussed by Anderson represents a first order method which approximates the vortex sheet strength as constant along each panel. It can be error prone and unstable. A better, and more complicated, second order method which allows the vortex sheet strength to linearly vary across each panel is defined in Section 5.10 of the textbook by Kuethe and Chow. A pdf copy of this reading is provided on the course website along with a MATLAB function `Vortex_Panel` which implements the FORTRAN code outlined in Kuethe and Chow, along with a few minor modifications. Note that this code computes the flow for any arbitrary two-dimensional body defined by a set of (x, y) coordinates that define its surface.

The provided function has the form:

```
function [c_l, c_p, circ, x_i, y_i] = Vortex_Panel(x_b,y_b,V_inf,alpha, flag)
```

where `c_l` is the section coefficient of lift (to be computed and returned), `c_p` is a vector containing the pressure coefficients computed at the center of each panel, `circ` is a vector of the circulation values computed for each panel (again concentrated at the center of the panel), `x_i` and `y_i` are vectors of the x- and y- coordinates of the center points for each panel where as `x_b` and `y_b` are vectors provided into the method of the x- and y- coordinates of the boundary points for the panels, `V_inf` is the free-stream flow speed, and `alpha` is the angle of attack (in degrees), and `flag` is a control input of either `1` or `0` which enables or disables the plotting of the pressure coefficient within the function. Note that the first and last entries of x and y correspond to the trailing edge point and that the points should be ordered around the surface in a clockwise direction to be consistent with the code from Kuethe and Chow.

Similar to before for "thin" airfoils, write a MATLAB function which plots the:

1. stream lines, ψ,
2. equipotential lines, ϕ, and
3. pressure contours, c_p,

for flow about a "thick" symmetric airfoil using the approximations detailed above. Your function should take the form:

```
function PlotThickAirfoil(c,alpha,V_inf,p_inf,rho_inf,N)
```

where `c` is the chord length c (in meters), `alpha` is the angle of attack α (in degrees), `V_inf` is the free-stream flow speed V_∞ (in meters per second), `p_inf` is the free-stream pressure p_∞ (in Pascals), `rho_inf` is the free-stream density ρ_∞ (in kilograms per meter cubed), and `N` is the number of discrete vortices N employed to approximate the vortex sheet.

Using your MATLAB function,

- Visualize or generate plots of the stream lines, equipotential lines, and pressure contours for flow about a NACA 0012 symmetric airfoil with $c = 2$ m, $\alpha = 9°$, $V_\infty = 60$ m/s, $p_\infty = 85.5 \times 10^3$ Pa, and $\rho_\infty = 1$ kg/m^3.
- Conduct a study of the effect of the number of discrete vortices N on the resulting flow and pressure field accuracy for the aforementioned values.

Reflection: How do the stream lines, equipotential lines, and pressure contours change with increasing angle of attack? How similar is it to thin airfoil theory and what differences exist?

Note: The formula for the shape of a NACA 00xx airfoil, with "xx" being replaced by the percentage of thickness to chord, is:

$$y_t = \frac{t}{0.2}c\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1036\left(\frac{x}{c}\right)^4\right]$$

where c is the chord length, x is the position along the chord from 0 to c, y_t is the half thickness at a given value of x (centerline to surface), and t is the maximum thickness as a fraction of the chord (i.e., $t = \text{xx}/100$).

Problem #3: Comparison of the Lift Generated by Thin and Thick Airfoils

Using the provided vortex panel method MATLAB function, preform a quantitative study of the convergence in the lift as a function of the panel resolution. Specifically for the conditions listed above: comput

- Generate a plot of the lift coefficient, c_l, versus the total number of panels, N, to demonstrate the convergence.
- Find the the number of panels, $N_{1\%}$, needed to converged the lift coefficient to within 1% error, and print both the number of panels and the lift coefficient achieved to the command window.

Following, the convergence analysis of the vortex panel method, use the number of panels you found for convergence, $N_{1\%}$, and obtain plots of the sectional coefficient of lift versus angle of attack for the following airfoils:

- NACA 0006 (Relatively Thin Airfoil)
- NACA 0012 (Moderate Thickness Airfoil)
- NACA 0024 (Relatively Thick Airfoil)

Using these plots, estimate the lift slope and zero-lift angle of attack for each of the airfoils (print these findings to the command window), and compare these results with thin airfoil theory (both in the plot and in the command window). It is recommended that you plot all of these together to provide a clearer comparison.

Reflection: How do changes in the wing section thickness alter the lift slope and the zero lift angle of attack? How accurate is the assumption of thin airfoil theory for each wing section?