Aerodynamics Computational Assignment #3: Flow over Finite Wings

Assigned Date: Monday February 28, 2022
Due Date: Sunday April 03, 2022

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script along with a pdf of your code and its execution (i.e. printed comments and figures) should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Reflection Questions:

In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Understand the difference between the application and results of thin airfoil theory and the vortex panel method.
2. Understand how the changes in wing section camber and thickness alter the lift slope.
3. Practice using Prandtl Lifting Line Theory to calculate lift and drag on a finite airfoil.
4. Understand how the number of terms in PLLT affect the resulting error in the solution.
5. Understand how design factors, like the taper and aspect ratio, affect aerodynamic efficiency and induced drag.

Problem #1: Comparison of the Lift Generated by Cambered Airfoils

Using the vortex panel method MATLAB function (provided for Computational Assignment 02), obtain plots of the sectional coefficient of lift versus angle of attack for the following airfoils:

- NACA 0012 (Symmetric Airfoil)
- NACA 2412 (Less Cambered Airfoil)
- NACA 4412 (More Cambered Airfoil)

Using these plots, estimate the lift slope and zero-lift angle of attack for each of the airfoils (print these findings to the command window), and compare these results with thin airfoil theory (both in the plot and in the command window). It is recommended that you plot all of these together to provide a clearer comparison. For this analysis use the number of panels, N, that you found from Computational Assignment 02 which predicted the lift coefficient, c_l, to within 1%.

Reflection: How do changes in the wing section camber alter the lift slope and the zero lift angle of attack? How accurate is the assumption of thin airfoil theory for each wing section?

Approach:

To help with analyzing the wing geometries, you should write a MATLAB function to construct panels for a given NACA 4-digit airfoil. Your MATLAB function should take the form:

```
[x,y,aLo,dCLda] = function NACA_Airfoils(m,p,t,c,N)
```

where `x` is a vector containing the x-location of the boundary points, `y` is a vector containing the y-location of the boundary points, `aLo` is the zero lift angle of attack ($\alpha_{L=0}$) predicted from thin airfoil theory, `dCLda` is the lift slope ($dc_l/d\alpha$) predicted from thin airfoil theory, `m` is the maximum camber, `p` is the location of maximum camber, `t` is the thickness, `c` is the chord length, and `N` is the number of employed panels.

Note: The formula for the shape of a NACA 4-digit series airfoil with camber is a bit involved. The first ingredient is the thickness distribution of the airfoil from the mean camber line, which is given by:

$$y_t = \frac{t}{0.2}c\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1036\left(\frac{x}{c}\right)^4\right]$$

where c is the chord length, x is the position along the chord from 0 to c, y_t is the half thickness at a given value of x (mean camber line to surface), and t is the maximum thickness aas a fraction of the chord. As with the case of a symmetric NACA airfoil, the last two digits in the NACA XXXX description gives $100t$. The second ingredient is the formula for the mean camber line, which is:

$$y_c = \begin{cases} m\frac{x}{p^2}\left(2p - \frac{x}{c}\right), & 0 \leq x \leq pc \\ m\frac{c-x}{(1-p)^2}\left(1 + \frac{x}{c} - 2p\right), & pc \leq x \leq c \end{cases}$$

where m is the maximum camber and p is the location of maximum camber. The first digit in the NACA XXXX description gives $100m$ while the second digit gives $10p$. Then, the coordinates (x_U, y_U) and (x_L, y_L) of the upper and lower airfoil surface, respectively, become:

$$x_U = x - y_t \sin\xi \qquad y_U = y_c + y_t \cos\xi$$
$$x_L = x + y_t \sin\xi \qquad y_L = y_c - y_t \cos\xi$$

where

$$\xi = \arctan\left(\frac{dy_c}{dx}\right).$$

Note that for the NACA 4415 airfoil, $m = 4/100$, $p = 4/10$, and $t = 15/100$.

Problem #2: Analysis of Approximate Cessna 150 Wing Performance

Write a MATLAB function which solves the fundamental equation of Prandtl Lifting Line Theory for finite wings with thick airfoils:

$$\alpha(\theta) = \frac{4b}{a_0(\theta)c(\theta)} \sum_{n=1}^{\infty} A_n \sin(n\theta) + \alpha_{L=0}(\theta) + \sum_{n=1}^{\infty} nA_n \frac{\sin(n\theta)}{\sin(\theta)}$$

by satisfying the equation at the N prescribed locations:

$$\theta_i = \frac{i\pi}{2N}, \qquad i = 1, \ldots, N$$

and truncating the series expansion for circulation using N odd terms:

$$\Gamma(\theta) = 2bV_\infty \sum_{j=1}^{N} A_{(2j-1)} \sin((2j-1)\theta)$$

Your function should be general enough to work for an arbitrary number of terms in the series expansion for circulation and should allow for a linear spanwise variation of the cross-sectional lift slope, the local chord length, the aerodynamic twist, and the geometric twist. Your function should return as output the span efficiency factor as well as the coefficient of lift and coefficient of induced drag. Consequently, your function should take the form:

```
function [e,c_L,c_Di] = PLLT(b,a0_t,a0_r,c_t,c_r,aero_t,aero_r,geo_t,geo_r,N)
```

where `e` is the span efficiency factor (to be computed and returned), `c_L` is the coefficient of lift (to be computed and returned), `c_Di` is the induced coefficient of drag (to be computed and returned), `b` is the span (in feet), `a0_t` is the cross-sectional lift slope at the tips (per radian), `a0_r` is the cross-sectional lift slope at the root (per radian), `c_t` is the chord at the tips (in feet), `c_r` is the chord at the root (in feet), `aero_t` is the zero-lift angle of attack at the tips (in degrees), `aero_r` is the zero-lift angle of attack at the root (in degrees), `geo_t` is the geometric angle of attack at the tips (in degrees), `geo_r` is the geometric angle of attack at the root (in degrees), and N is the number of odd terms to include in the series expansion for circulation.

Apply your Prandtl Lifting Line Function to a wing with a span of 33 ft 4 in and a straight taper from 5 ft 4 in root chord to 3 ft 8.5 in tip chord. The root airfoil is chosen to be a NACA 2412 while the tip airfoil is chosen to be a NACA 0012. This results in a linear spanwise variation of cross-sectional lift slope and zero-lift angle of attack. The wing is also twisted such that the geometric angle of attack varies linearly from 1° at the root to 0° at the tips.

Determine the lift and induced drag for the wing at a cruise speed of 82 knots and 10,000 ft altitude (standard atmosphere). Moreover, complete the following tasks:

- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with ten percent relative error. Print this value to the command window.
- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with one percent relative error. Print this value to the command window.
- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with 1/10 percent relative error. Print this value to the command window.

Reflection: In this lab only the odd terms were utilized in the PLLT series expansion, why? When would both the odd and even terms be required?

Note: To compute the cross-sectional lift slope and zero-lift angle of attack at the root and tips, use the vortex panel code and your results from Problem #1.

Problem #3: Validation of Span Efficiency Factor

Using the MATLAB function you wrote for Problem #2, make a plot of the span efficiency factor e versus taper ratio c_t/c_r for a *thin* wing with no aerodynamic or geometric twist and aspect ratios $AR = 4, 6, 8, 10$ where c_t is the tip chord and c_r is the root chord. Use at least twenty odd terms in your series expansion for circulation in generating your plot. Your resulting plot should look similar in style to Fig. 5.20 in Anderson's *Fundamental's of Aerodynamics*.

Reflection: Consider the dependence of the span efficiency factor on both the taper ratio and aspect ratio. Under what conditions is the wing the most aerodynamically efficient or under what conditions is the induced drag minimized? How does this compare to the theoretical wing planform with the minimum induced drag?

Note: The aerodynamic twist is defined as the difference in zero-lift angle of attack between a given wing section and the wing section at the root, and the geometric twist is defined as the difference in geometric angle of attack between a given wing section and the wing section at the root. Therefore, a wing with no aerodynamic or geometric twist may still be at some geometric angle of attack, but it is uniform across the spanwise direction.

Hint: For a wing with no aerodynamic or geometric twist, the span efficiency factor is independent of the geometric and zero-lift angles of attack.