multivariable calculus

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1 Dot Product

$x + 2y + 3z = 0$ equation of a plane
Let $\overrightarrow{OP} = (x, y, z)$,$\overrightarrow{A} = (1, 2, 3)$,then $\overrightarrow{OP} \cdot \overrightarrow{A} = 0 \Rightarrow P$ on a plane going through the origin point(O). **Remember to use vectors in multivariable calculus!**

2 Deteminants

2.1 In a plane

Area of a triangle:

$$\pm Area = \frac{1}{2}\left|\overrightarrow{A}\right| \cdot \left|\overrightarrow{B}\right| \sin\theta = \frac{1}{2}\left|\overrightarrow{A'}\right| \cdot \left|\overrightarrow{B}\right| \cos\theta' = \frac{1}{2}\overrightarrow{A'} \cdot \overrightarrow{B} = \frac{1}{2}(-a_2, a_1) \cdot (b_1, b_2)$$
$$= \frac{1}{2}(a_1b_2 - a_2b_1) = det(A, B) = \frac{1}{2}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

($\overrightarrow{A'}$:$\overrightarrow{A}$ rotated by 90° anticlockwise)

2.2 In a space

2.2.1 volume of solids

$\boxed{Theorem}$Geomatrically, $det(\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}) = \pm Volume\, of\, parallelepiped$
cross product of 2 vectors in 3d space:
$\boxed{Def}$

$$\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\hat{\imath} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\hat{k} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

(3×3 only for remembering)
$\boxed{Theorem}$ $\left|\overrightarrow{A} \times \overrightarrow{B}\right|$ is the area of the parallelogram spanned by $\overrightarrow{A}$ and $\overrightarrow{B}$.
$dir(\overrightarrow{A} \times \overrightarrow{B})$ is perpendicular to the plane of the parallelogram.
use **right-hand rule** to determine the direction of the cross product.
For example, $\hat{\imath} \times \hat{\jmath} = \hat{k}$