

## Problem Set 1.1

### 1

Under what conditions on  $a, b, c, d$  is  $\begin{bmatrix} c \\ d \end{bmatrix}$  a multiple  $m$  of  $\begin{bmatrix} a \\ b \end{bmatrix}$ ? Start with the two equations  $c = ma$  and  $d = mb$ . By eliminating  $m$ , find one equation connecting  $a, b, c, d$ . You can assume no zeroes in these numbers.

$$m = \frac{c}{a} = \frac{d}{b} \Rightarrow ad = bc$$

### 2

Going around a triangle from  $(0, 0)$  to  $(5, 0)$  to  $(0, 12)$  to  $(0, 0)$ , what are those three vectors  $\mathbf{u}, \mathbf{v}$  and  $\mathbf{w}$ ? What is  $\mathbf{u} + \mathbf{v} + \mathbf{w}$ ? What are their lengths?

$$\mathbf{u} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -5 \\ 12 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 0 \\ -12 \end{bmatrix}$$

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad |\mathbf{u}| = 5 \quad |\mathbf{v}| = 13 \quad |\mathbf{w}| = 12$$

### 3

Describe geometrically all linear combinations of the given vectors.

(a)  $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  and  $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ : a line. (b)  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ : a plane.

(c)  $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$  and  $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ : all of  $R^3$ .

### 4

Draw  $\mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$  and  $\mathbf{w} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$  and  $\mathbf{v} + \mathbf{w}$  and  $\mathbf{v} - \mathbf{w}$  in a single  $xy$  plane.

