Problem Set 1.1

1

Under what conditions on a, b, c, d is $\begin{bmatrix} c \\ d \end{bmatrix}$ a multiple m of $\begin{bmatrix} a \\ b \end{bmatrix}$? Start with the two equations$c = ma$ and $d = mb$. By eliminating m,find one equation connecting a, b, c, d. You can assume no zeroes in these numbers.

$$m = \frac{c}{a} = \frac{d}{b} \Rightarrow ad = bc$$

2

Going around a triangle from$(0, 0)$to$(5, 0)$to$(0, 12)$to$(0, 0)$, what are those three vectors $\mathbf{u}$,$\mathbf{v}$and$\mathbf{w}$? What is $\mathbf{u} + \mathbf{v} + \mathbf{w}$? What are their lengths?

$$\mathbf{u} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -5 \\ 12 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 0 \\ -12 \end{bmatrix}$$

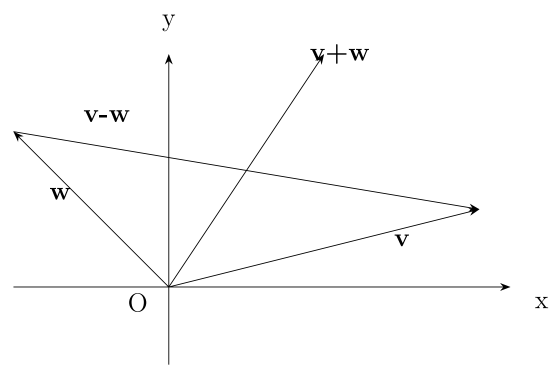
$$\mathbf{u} + \mathbf{v} + \mathbf{w} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad |\mathbf{u}| = 5 \quad |\mathbf{v}| = 13 \quad |\mathbf{w}| = 12$$

3

Describe geometrically all linear combinations of the given vectors.
(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$:a line. (b) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$:a plane.
(c) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$:all ofR^3.

4

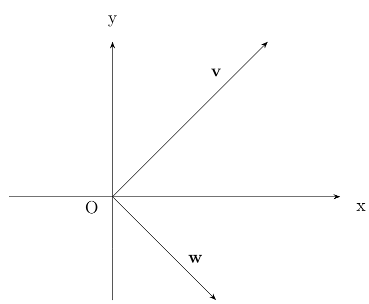
Draw$\mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$and $\mathbf{w} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $\mathbf{v} + \mathbf{w}$and $\mathbf{v} - \mathbf{w}$ in a single xy plane.



5

If $\mathbf{v}+\mathbf{w} = \begin{bmatrix}5\\1\end{bmatrix}$ and $\mathbf{v}-\mathbf{w} = \begin{bmatrix}1\\5\end{bmatrix}$. Compute and draw the vectors $\mathbf{v}$and $\mathbf{w}$.

$$\mathbf{v} = \begin{bmatrix}3\\3\end{bmatrix} \quad \mathbf{w} = \begin{bmatrix}2\\-2\end{bmatrix}$$



6

From$\mathbf{v} = \begin{bmatrix}2\\1\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}1\\2\end{bmatrix}$, find the components of $3\mathbf{v}+\mathbf{w}$ and$c\mathbf{v}+d\mathbf{w}$.

$$3\mathbf{v}+\mathbf{w} = \begin{bmatrix}7\\5\end{bmatrix} \quad c\mathbf{v}+d\mathbf{w} = \begin{bmatrix}2c+d\\c+2d\end{bmatrix}$$

7

$\mathbf{u} = \begin{bmatrix}1\\2\\3\end{bmatrix}$ $\mathbf{v} = \begin{bmatrix}-3\\1\\-2\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}2\\-3\\-1\end{bmatrix}$, compute$\mathbf{u}+\mathbf{v}+\mathbf{w}$ and $2\mathbf{u}+2\mathbf{v}+\mathbf{w}$. How do you know that $\mathbf{u}, \mathbf{v}, \mathbf{w}$lie in a plane?

$$\mathbf{u}+\mathbf{v}+\mathbf{w} = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad 2\mathbf{u}+2\mathbf{v}+\mathbf{w} = \begin{bmatrix}-2\\3\\1\end{bmatrix}$$

$$\mathbf{w} = -\mathbf{u} - \mathbf{v} \Rightarrow rank(\mathbf{u}, \mathbf{v}, \mathbf{w}) = 2 \Rightarrow \mathbf{u}, \mathbf{v}, \mathbf{w}\text{lie in a plane}$$

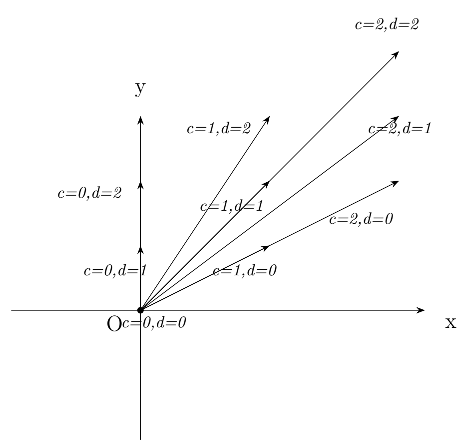
8

Every combination of $\mathbf{v} = (1, -2, 1)$ and $\mathbf{w} = (0, 1, -1)$has components that add to what? Find c and d so that $c\mathbf{v} + d\mathbf{w} = (3, 3-6)$. Why is $(3, 3, 6)$ impossible?

Combination:$a\mathbf{v} + b\mathbf{w} = (a, -2a+b, a-b)$ Their combination adds to 0. $c = 3$,$d = 9$

9

In the xy plane mark all nine of these linear combinations: $c = \begin{bmatrix}2\\1\end{bmatrix}$ and $d = \begin{bmatrix}0\\1\end{bmatrix}$ with $c = 0, 1, 2$ and $d = 0, 1, 2$.



10

How could you decide if the vectors $\mathbf{u} = (1, 1, 0)$ and $\mathbf{v} = (0, 1, 1)$ and $\mathbf{w} = (a, b, c)$ are linearly independent or dependent?

Assume that they are linearly dependent. Then there exist x, y, z not all zero such that $x\mathbf{u} + y\mathbf{v} + z\mathbf{w} = \mathbf{0}$.

$$x\mathbf{u} + y\mathbf{v} + z\mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x(1,1,0) + y(0,1,1) + z(a,b,c) = 0$$

If $z = 0$,then $x = y = 0$.So$z \neq 0$. Let $x_1 = -\frac{x}{z}$,$y_1 = -\frac{y}{z}$,then

$$\begin{cases} a = x_1 \\ b = x_1 + y_1 \\ c = y_1 \end{cases} \quad \Rightarrow b = a + c$$