MAS212 Assignment #3

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Introduction

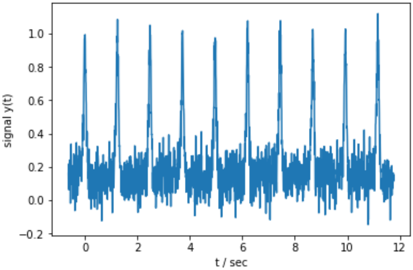
- A **pulsar** is a strongly magnetised neutron star that spins at a very high speed emitting a focused beam of electromagnetic radiation.[1]
- The first pulsars were co-discovered by **Dame Jocelyn Bell Burnell**, an astrophysicist born in Northern Ireland in 1943.
- Although Bell was the first to have observed the pulsars, she was skipped over for the **Nobel Peace Prize** for this discovery as she was a research student and it was instead awarded to her supervisor.

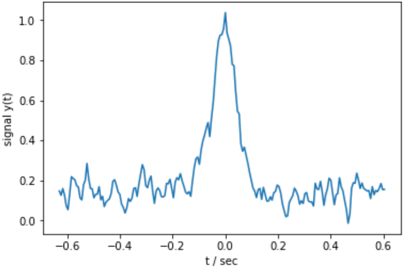
Figure: [2]

- Bell later served as president of **the Royal Astronomical Society** and the **Institute of Physics**. Earlier this year she won **the Special Breakthrough Prize in Fundamental Physics**, donating all the prize money to create scholarships for women, under-represented minorities and refugees wishing to study Physics.[3]

The pulse

- *Upper*: A plot of the astronomers data showing ten pulses coming from the pulsar.
- *Lower*: The averaged pulse profile - a plot of the average signal from the pulses, taken at 189 time intervals.





A linear model: theory

- Derive normal equations from: $f(t,\beta_j) = \sum_j \beta_j \phi_j(t)$, $X_{ij} = \phi_j(t_i)$.
- Find the ith residual and its partial derivative w.r.t β_k:

$$r_i = y_i - f(t_i,\beta_j) = y_i - \sum_j \beta_j \phi_j(t_i)$$

$$\frac{\partial r_i}{\partial \beta_k} = -\sum_j \frac{\partial \beta_j}{\partial \beta_k}\phi_j(t_i) = -\sum_j \delta_{jk}\phi_j(t_i) = -\phi_k(t_i) = -X_{ik}.$$

- Minimize the sum of square residuals S:

$$\frac{\partial S}{\partial \beta_k} = \frac{\partial}{\partial \beta_k}\left(\sum_i r_i^2\right) = 2\sum_i r_i \frac{\partial r_i}{\partial \beta_k} = 0$$

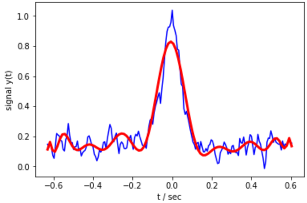
.

- Substitute in $\frac{\partial r_i}{\partial \beta_k} = -X_{ik} = -(X^T)_{ki}$ and $r_i = y_i - \sum_j X_{ij}\beta_j$

$$\Rightarrow -\sum_i (X^T)_{ki}\left(y_i - \sum_j X_{ij}\beta_j\right) = 0.$$

- This is the jth row of the vector in $X^T(y - X\beta) = 0$ rearranging to $(X^TX)\beta = X^Ty$.

A linear model: result



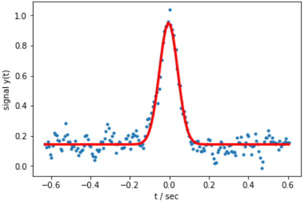
- Fitting the averaged pulse profile with the even polynomial:

$$f(t;\beta_i) = \beta_0 + \beta_1 t^2 + \beta_2 t^4 + \ldots + \beta_n t^{2(m-1)}.$$

- With m set to 19, the **best-fit parameters** β_0 and β_1 equal 0.825417789025 and −1.15096348936 respectively.

- It's hard to know which **value of** m to choose as we don't know whether extra parameters are necessary or a better fit. We have assumed there is no **error in independent variable** t which is probably not a correct assumption.

A non-linear model



- Fitting the averaged pulse profile with the Gaussian model:

$$f(t;\beta_i) = c + Ae^{\left(-\frac{(t-\tau)^2}{2\sigma^2}\right)}.$$

- The **best-fit parameters** for this model are $\beta_0 = c = 0.14302312$, $\beta_1 = A = 0.80901671$, $\beta_2 = \tau = -0.00418574$, and $\beta_3 = \sigma = 0.04622979$.

Which model is best?

- **Parameters.** There were four parameters needed for the non-linear model and I used 19 parameters for the linear model.
- **Root-mean-square deviation.** For the linear model $RMSD = 0.0625252380136$, for the non-linear model $RMSD = 0.0513349772047$.
- The successive **residuals** for the linear model are slightly more strongly correlated than those of the non-linear model as can be seen from the graphs.
- As it has less parameters, a smaller RMSD and slightly less strongly correlated residuals, my conclusion is that the non-linear model is better than the linear model.

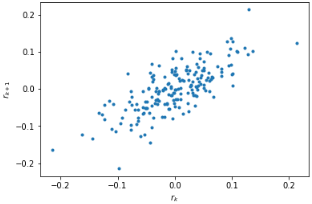


Figure: Linear model

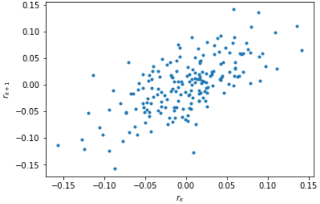


Figure: Non-linear model

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