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Torque demand control by nonlinear MPC with constraints for vehicles with variable valve lift engine

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***Abstract*— In this paper, a Nonlinear Model Predictive Control (NMPC) is applied for a torque demand control for speed tracking of vehicles with a variable valve lift engine. The engine control must satisfy various constraints, e.g., actuator limitations, drivability and A/F ratio, etc. Therefore, we apply NMPC based on a collocation method which can handle the constraints explicitly. Moreover, a fuel injection control is also developed by considering a fuel dynamics of the engine. The validity and robustness of the proposed method are demonstrated by numerical simulations using a SICE engine benchmark simulator.**

I. Introduction

The engine control system development has been complicated, however, the development period is becoming shorter to meet market needs quickly. For such a background, it is difficult to develop products satisfying requirements in the short term by conventional development systems, and the reform of development systems is required. The Model Based Development (MBD), which is a development approach using virtual models and simulators, attracts much attentions as one solution to the problem. To support a research of the engine control through MBD environments, a SICE benchmark problem [1] was provided.

In addition, it is becoming more difficult to capture the entire picture of an automobile development due to specialization of each element. To manage the ever-growing number of interacting vehicle control systems, a torque demand engine control system architecture is often used [2]. The torque demand engine control system uses a torque as an interface, and consists of a torque management part and an engine control part. This leads flexibility and expanding ability of the whole vehicle control system. With the background like this, in this paper, we design the torque demand control system for a vehicle speed tracking control.

The engine system is the nonlinear multi-input system, and the engine control system must satisfy various constraints, e.g., actuator limitations, drivability and A/F ratio, etc. Therefore, we apply NMPC based on a collocation method [3] which can handle the constraints explicitly. Although it is difficult to realize real-time optimization by using this method, we can compute precise control profiles with constraints and compare it to conventional control. Moreover, a fuel injection control is also developed in order to satisfy the A/F ratio specifications.

This paper is organized as follows: Section 2 gives the bench mark model and control specification. In Section 3 we derive the control model of an engine and a car body. Section 4 contains explanations about a procedure to calculate desired torque, nonlinear MPC, and fuel injection control. Numerical simulations show the effectiveness and robustness of these method in Section5. Section 6 is devoted to a conclusion and some ideas of our future work.

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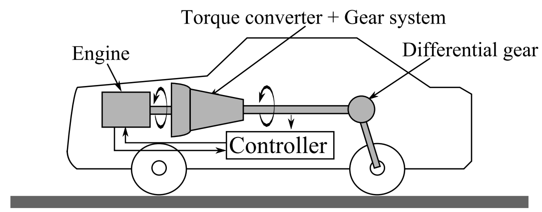


Fig. 1. Benchmark model

TABLE I
INPUTS AND OUTPUTS OF BENCHMARK MODEL

u_{th}	throttle angle	y_{rpm}	engine speed
u_{fi}	fuel amount of cylinder	y_{st}	crank angle
u_{sa}	ignition timing of cylinder	y_{mt}	throttle air mass flow
u_{vl}	command for valve lift	y_{vl}	actual valve position

II. Problem Setting

Fig. 1 shows the benchmark model, which consists of a V6 SI engine with variable valve lift system, an automatic transmission and a car body. We consider the V6 SI engine that has 14-inputs and 4-outputs, and which is described as a continuous and discrete time mixed system. The components of inputs and outputs are shown in TABLE I. u_{th}, u_{vl} can be change at any given timing, but u_{fi} and u_{sa} can be changed only one time in each engine cycle. Moreover, the upper and lower bound constraints, such as $0^\circ \leq u_{th} \leq 90^\circ$ and $0 \leq u_{vl} \leq 1$, exist due to the limits of the actuators.

The control specifications in the benchmark problem are given as follows:

- torque demand control system design:
  The engine control system must realize the torque demand system and the engine control system.
  - The torque demand system coordinates the torque demand variables from all torque requests throughout the vehicle. In this paper, only desired vehicle speed from a driver is considered.
  - The engine control system determines inputs of actuators to achieve the desired torque.
- vehicle speed tracking control:
  The vehicle speed $V(t)$ should follow the desired speed

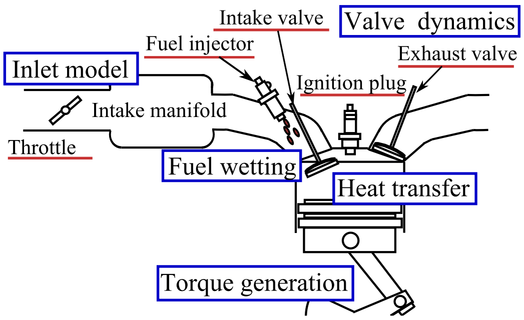


Fig. 2. Engine model

$V_r(t)$. Note that a jerk $\dddot{V}(t)$ (the time rate of change of acceleration) should be smaller than a certain value due to the driving comfort.

- minimization of fuel consumption:
  The fuel consumption u_f should be minimized while achieving other control specifications.

III. Modeling for Controller Design

We will now describe a plant model needed to design the control system.

A. Engine Model

Fig. 2 shows an engine model. It is based on a mean value model which is based on physical first principles and a few experiments [4], [5].

1) Inlet Model: The air mass flow at the throttle $\dot{M}_{th}$ is expressed as

$$\dot{M}_{th} = A_{th}\sqrt{\frac{\kappa+1}{2\kappa}P_a D_a}\ \psi\left(\frac{P_m}{P_a}\right), \tag{1}$$

$$A_{th} = A_{th0}\left(1-\cos u_{th}\right), \tag{2}$$

$$\psi\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) = \begin{cases} \frac{\kappa}{\kappa+1} & \frac{P_{\text{out}}}{P_{\text{in}}} < R_{cr}, \\ \sqrt{(\frac{\kappa}{\kappa+1})^2 - (\frac{P_{\text{out}}}{P_{\text{in}}} - \frac{1}{\kappa+1})^2} & \frac{P_{\text{out}}}{P_{\text{in}}} \geq R_{cr}, \end{cases} \tag{3}$$

where P_a, D_a, P_m is atmosphere pressure, atmosphere density, inlet pressure respectively, and A_{th0} is maximum throttle opening area, R_{cr} is critical pressure ratio.

The air mass flow into the cylinder $\dot{M}_{iv}$ is given by

$$\dot{M}_{iv} = \frac{V_d}{4\pi}\eta_v \omega_e \frac{P_m}{RT_m}, \tag{4}$$

where V_d is the cylinder displacement volume, η_v denotes the volumetric efficiency. Although the value of η_v is effected by many variables, we assume η_v is expressed as a polynomial function of P_m, ω_e and L_v. Coefficients of the polynomial are identified by a least square method.

Since during 1cycle (4π in the crank angle domain) N_{cyl} cylinders complete intake, a mean air mass into the cylinder per cycle M_c is given by

$$M_c = \frac{4\pi}{N_{\text{cyl}}\omega_e}\dot{M}_{iv}. \tag{5}$$

The dynamics of the pressure inside the manifold P_m are derived by the conservation law of energy as

$$\frac{\mathrm{d}}{\mathrm{d}t}P_m = \frac{\kappa R T_a}{V_m}\left(\dot{M}_{th} - \frac{T_m}{T_a}\dot{M}_{iv}\right), \tag{6}$$

where κ, R, V_m, T_a, T_m are ratio of specific heats, gas constant, intake manifold volume, atmosphere temperature and temperature at the intake manifold respectively. Here, we assume $T_m \approx T_a$ for simplicity.

2) Variable Valve Mechanism Model: There is dynamics between command inputs and actual valve positions since the variable valve mechanism is drived by hydraulic pressure. The dynamics of inlet valve lift L_v is expressed as a first order delay system that restricts by engine speed ω_e and oil temperature T_{oil} as

$$\frac{\mathrm{d}}{\mathrm{d}t}L_v = \mathrm{S}\left(\frac{L_v - u_{vl}}{\tau_{L_v}},\ l_b(\omega_e, T_{oil}), u_b(\omega_e, T_{oil})\right), \tag{7}$$

where τ_{L_v} is a time constant of valve response, and $\mathrm{S}\left(x, l_b, u_b\right)$ is saturate function which is given by

$$\mathrm{S}\left(x, l_b, u_b\right) := \begin{cases} l_b & x < l_b, \\ x & l_b \leq x \leq u_b, \\ u_b & x > u_b. \end{cases} \tag{8}$$

3) Fuel Wetting Model: In the fuel model, the amounts of port and valve residual fuel and the amount of the intake fuel into cylinders are determined at crank angle $\theta = 360$[deg] in every cycle and are given by as

$$F_{wp}[z] = P_p F_{wp}[z-1] + R_p u_{fi}[z], \tag{9}$$

$$F_{wv}[z] = P_v F_{wv}[z-1] + R_v u_{fi}[z], \tag{10}$$

$$f_c[z] = (1-P_p)F_{wp}[z-1] + (1-P_v)F_{wv}[z-1] + (1-R_p-R_v)u_{fi}[z], \tag{11}$$

where z denotes engine cycle for each cylinder, F_{wp} is the amount of the port residual fuel, F_{wv} is the amount of the valve residual fuel and f_c is the amount of the admitted fuel. P_p, R_p, P_v and R_v are determined by the engine speed $\omega_e(t)$, the port temperature $T_p(t)$, the valve temperature $T_v(t)$ and the air mass in the previous cycle $M_c[z-1]$.

4) Heat Transfer Model: The dynamics of port temperature T_p, the valve temperature T_v, the oil temperature T_{oil} are derived by heat balance as

$$\frac{\mathrm{d}}{\mathrm{d}t}T_p = N_{\text{cyl}}\frac{q_p}{C_p}, \tag{12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}T_v = \frac{q_v}{C_v}, \tag{13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}T_{oil} = \frac{N_{\text{cyl}}}{2}\frac{q_p}{C_{oil}}, \tag{14}$$

where C_p, C_v, C_{oil} are the heat capacity of port, valve and oil respectively. q_p and q_v are the heat per second that receive port and valve as

$$q_p = H_{cp}q_l - H_{pa}(T_p - T_a), \tag{15}$$

$$q_v = H_{cv}q_l - H_{va}(T_v - T_a), \tag{16}$$

where $H_{cp}, H_{pa}, H_{cv}, H_{va}$ denote coefficients of heat transfer, and q_l is the cylinder heat loss per second. Here, we assume q_l is expressed as the polynomial function of P_m, ω_e and T_v, and identify coefficients by the least square method.

5) Engine Torque Generation Model: The engine speed dynamics is shown by

$$J_e \dot{\omega}_e = \tau_e - \tau_f - \tau_{tr}, \tag{17}$$

where J_e is engine inertia and τ_e, τ_f and τ_{tr} are the engine output torque,the friction torque, the load torque from the torque converter respectively. The friction torque τ_f is given by

$$\tau_f = \frac{a_{f2}}{(\omega_e + a_{f1})^2} + a_{f3}\omega_e^2 + a_{f4}, \tag{18}$$

where a_{f1}, a_{f2}, a_{f3} and a_{f4} are constants. The load torque from the torque converter τ_{tr} is given in the next section.

The engine output torque τ_e is determined by many variables, such as the engine speed ω_e, air mass into the cylinder M_c, A/F ratio λ, ignition timing u_{sa}, etc. In the mean value model, the engine torque generation is described as

$$\tau_e = \frac{H_l}{4\pi\lambda_d} M_c(t - \Delta t)\ \eta_e, \tag{19}$$

where H_l, λ_d and Δt are the lower heating value of gasoline, ideal A/F ratio, and time delay between intake and torque generation. In this model, we assume the efficiency for engine torque generation η_e is expressed by

$$\eta_e \approx \eta_{e0}(M_c, \omega_e) \cdot \eta_{sa}(\tilde{\phi}_{\mathrm{MBT}}) \cdot \eta_\lambda(\lambda), \tag{20}$$

where η_{e0}, η_{sa} and η_λ are the base efficiency, the ignition efficiency, the A/F efficiency respectively, and $\tilde{\phi}_{\mathrm{MBT}}$ denotes the difference from spark angle input to MBT(minimum spark advance for best torque). Here, we assume that values of these efficiencies are expressed using polynomial function, gauss function and sigmoid function. Parameters are identified by the least square method and a nonlinear optimization method.

B. Torque Transfer System and Vehicle Model

We derived the physical model from the input to the engine speed in the previous section. Here, we will derive a physical model from the engine speed to the vehicle speed whose desired value will be given as a target trajectory.

In the engine simulator, an automatic transmission is modeled and the torque transmitted from the torque converter to the engine τ_{tr} and to the shaft τ_s are given as maps as

$$\tau_{tr} = \mathrm{MAP}_C(\omega_s/\omega_e)\omega_e^2, \tag{21}$$

$$\tau_s = \mathrm{MAP}_C(\omega_s/\omega_e)\mathrm{MAP}_T(\omega_s/\omega_e)\omega_e^2, \tag{22}$$

where ω_s denotes the shaft rotational speed.

By ignoring skidding of the tyre and effect of the torsional stiffness in the drive-train, the shaft rotational speed ω_s and the tyre drive force F_t can be described as

$$\omega_s = \frac{G}{r_t} v, \quad F_t = \frac{G}{r_t} \tau_{ts}, \tag{23}$$

where r_t is the tyre radius and G is the gear ratio of the differential gear. Here, we describe a motion equation of the vehicle by modeling only the air resistance as

$$M_{eq}\dot{v} = F_t - Cv^2, \tag{24}$$

where M_{eq} denotes the equivalent mass including rotating part inertia and C denotes the air resistance.

C. State-Space Expression

We need to express the model as a state-space expression for the MPC control approach considered in Section 4. We introduce the following procedure for the efficient calculation and to avoid computational complexity.

- Replacement of all look-up tables with polynomial fits,
- replacement of all saturate functions such as (8) with smooth approximations,
- approximation of all time delay as one order lag, that is $e^{-Ls} \approx 1/(Ls + 1)$ and,
- introduce integrators in front of the actual process because a servo control system is required.

After the above procedure, we obtain a continuous and discrete time mixed model via the following definitions:

$$\begin{aligned} x_c(t) &= [P_m, \omega_e, v, L_v, T_v, T_p, T_{oil}, M_{cd}, \tilde{\phi}_{\mathrm{MBT}d}, \\ &\qquad u_{th}, u_{vl}, u_{sa}]^T \in \mathbb{R}^{12}, \\ v_c(t) &= [\dot{u}_{th}, \dot{u}_{vl}, \dot{u}_{sa}]^T \in \mathbb{R}^3, \\ x_d[k] &= [F_{wv}, F_{wp}]^T \in \mathbb{R}^2, \\ u_d[k] &= u_{fi} \in \mathbb{R}, \end{aligned} \tag{25}$$

where x_c and v_c are the states and the inputs for the continuous time system, x_d and u_d are the states and the inputs for the discrete time. M_{cd} and $\tilde{\phi}_{\mathrm{MBT}d}$ are approximated variables for delayed M_c and $\tilde{\phi}_{\mathrm{MBT}}$, respectively.

IV. Controller Design

In this section, we design the torque demand controller for speed control of vehicles. Fig. 3 shows the structure of the controller. The controller consists of the state estimator, the torque reference generator and two subcontollers. The nonlinear MPC based on the collocation method is applied for the torque tracking control, and the fuel injection controller is designed in order to control the A/F ratio.

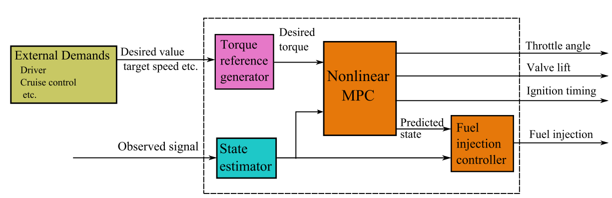


Fig. 3. Structure of the controller

A. Torque Reference Generator

Firstly, we consider the case where the vehicle speed is tracked for desired trajectory, that is, $v = v_d$ and $\dot{v} = \dot{v}_d$. In this case, desired values can be calculated by using (21)-(24) as

$$\begin{aligned}
&\tau_{sd} = a_1 \dot{v}_d + a_2 v_d^2,\ \omega_{sd} = a_3 v_d, \\
&\frac{\tau_{sd}}{\omega_{sd}^2} = \mathrm{MAP_C}\left(\omega_{sd}/\omega_{ed}\right) \cdot \mathrm{MAP_T}\left(\omega_{sd}/\omega_{ed}\right) \cdot \frac{1}{\left(\omega_{sd}/\omega_{ed}\right)^2} \\
&\quad := \mathrm{MAP_{CT}}(\omega_{sd}/\omega_{ed}),
\end{aligned} \tag{26}$$

where $a_1 = M_{eq} r_t / G, a_2 = C r_t / G$ and $c_3 = G / r_t$.

From (26), ω_{ed} is represented by

$$\omega_{ed} = \frac{\omega_{sd}}{\mathrm{MAP_{CT}}^{-1}\left(\tau_{sd}/\omega_{sd}^2\right)}, \tag{27}$$

where $\mathrm{MAP_{CT}}^{-1}:\ \tau_s/\omega_s^2 \to \omega_s/\omega_e$ is inverse function of $\mathrm{MAP_{CT}}$.

By substituting $\omega_{ed},\ \omega_{sd}$ for (21), we can calculate the desired load torque from the torque converter τ_{trd} as

$$\tau_{trd} = \mathrm{MAP_C}\left(\omega_{sd}/\omega_{ed}\right)\omega_{ed}^2. \tag{28}$$

Furthermore, we can obtain the desired engine acceleration $\dot{\omega}_{ed}$ by differencing numerically desired engine speed trajectory (27). By substituting $\dot{\omega}_{ed}$ for (17), we can calculate the desired engine output torque τ_{ed} as

$$\tau_{ed} = \tau_{trd} + \tau_{fd} + J_e \dot{\omega}_{ed}, \tag{29}$$

where τ_{fd} is obtained by substituting ω_{ed} for (18).

However, (28) is considered only about the steady state, and the transient torque tracking error will remain as a steady state offset vehicle speed error. In order to compensate the offset error, we modify τ_{sd} using vehicle speed error $e = v - v_d$ as

$$\tilde{\tau}_{sd} = a_1\left[\dot{v}_d + \frac{a_2}{a_1}v^2 - \left(k_p e + k_i \int e\ \mathrm{d}t\right)\right]. \tag{30}$$

where k_p and k_i is the positive constants. By assuming perfect torque tracking is achieved, that is $\tau_s = \tilde{\tau}_{sd}$, the vehicle speed error dynamics will be given as

$$0 = \dot{e} + k_p e + k_i \int e\ \mathrm{d}t. \tag{31}$$

From (31), vehicle speed error $e \to 0$ if $\tau_s = \tilde{\tau}_{sd}$, and we can calculate torque reference with error compensation $\tilde{\tau}_{ed}$ by substituting $\tilde{\tau}_{sd}$ for (27)-(29).

B. Nonlinear MPC

In this section, We will now describe the nonlinear MPC method for a torque tracking control.

At first, we describe the general dynamic optimization problem. We consider the general dynamic optimization problem can be stated as follows:

$$\begin{aligned}
&\min_{u(t)}\ \varphi(x(t_f)) + \int_{t_0}^{t_f} L(x(t), y(t), u(t))\ \mathrm{d}t \qquad (32)\\
&\mathrm{subject.\ to.} \\
&\quad \frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t), y(t), u(t)),\ x(0) = x_0, \\
&\quad g(x(t), y(t), u(t)) = 0, \quad g_f(x(t_f)) = 0, \\
&\quad u_L \le u(t) \le u_U,\ y_L \le y(t) \le y_U, x_L \le x(t) \le x_U.
\end{aligned}$$

where $x(t), u(t)$ and $y(t)$ are differential state profile vectors, control profile vectors and algebraic state profile vectors respectively. $g(\cdot)$ denotes stage constraints, and $g_f(\cdot)$ denotes terminal constraints.

Because it is very difficult to calculate the input value analytically from the problem (32), some level of discretization that converts the original continuous time problem into a discrete problem is often applied. As one of these discretization, there is a method that discretize only control profiles. However, it is difficult to handle state constraints explicitly, because this method treat problem through only input $u(t)$. In this paper, we apply NMPC based on collocation method[3] that discretize the state and control profiles.

1) Collocation Method: The original optimization problem is converted into an nonlinear programming (NLP) problem by approximating state and control profiles by a family of polynomials. For simplicity, we consider here only about a single stage discretization. More details are in [3]. By applying polynomial approximations, differential-algebraic equations are converted into algebraic equations. In the following, the number is assumed to be K.

Here, we use a monomial representation for the differential state profiles as

$$x(t) = x_0 + h\sum_{q=1}^{K}\left[\Omega_q\left(\frac{t - t_0}{h}\right)\frac{\mathrm{d}x}{\mathrm{d}t}_q\right], \tag{33}$$

where $h = t_f - t_0$, $\mathrm{d}x/\mathrm{d}t_q$ is the value of its first derivative at the collocation point q and Ω_q is the polynomial of order K, satisfying

$$\Omega_q(0) = 0,\ q = 1, \ldots, K, \tag{34}$$

$$\frac{\mathrm{d}\Omega_q}{\mathrm{d}t}(\rho_r) = \begin{cases} 1 & q, r = 1, \ldots, K, \\ 0 & q \ne r \end{cases} \tag{35}$$

where ρ_r is the location of the r-th collocation point. In addition, the control and algebraic profiles are approximated using a similar monomial basis representation as

$$u(t) = \sum_{q=1}^{K} l_q\left(\frac{t - t_0}{h}\right)u_q,\ y(t) = \sum_{q=1}^{K} l_q\left(\frac{t - t_0}{h}\right)y_q, \tag{36}$$

where, u_q, y_q represent the values of the algebraic and control variables, respectively. $l_q(\cdot)$ is the Lagrange polynomial of order K satisfying

$$l_q(\tau) = \Pi_{k=0,\ne q}^{K}\frac{\tau - \tau_k}{\tau_q - \tau_k}. \tag{37}$$

By substituting (33)-(37) into (32), the original optimal control problem is converted to the following NLP.

$$\min_{x_q,u_q} \ \varphi(x_N) + \sum_{q=1}^{K} w_q L(x_q, u_q) \tag{38}$$

subject. to.

$$\frac{\mathrm{d}x}{\mathrm{d}t}_q = f(x_q, y_q, u_q), \quad x_q = x_0 + h \sum_{q'=1}^{K} \Omega_{q'}(\rho_q) x_q,$$
$$g(x_q, y_q, u_q) = 0, \quad g_f(x_N) = 0,$$
$$u_L \le u_q \le u_U, \ y_L \le y_q \le y_U, \ x_L \le x_q \le x_U.$$

We can obtain an approximate solution of the original optimal control problem (32) by solving NLP problem (38). In this paper, we use IPOPT[6] as a NLP solver.

C. Fuel Injection Controller

The A/F ratio in the cylinder is defined as $\lambda = M_c/f_c$ where M_c and f_c are air mass and fuel mass into the cylinder, and the ideal A/F ratio is 14.5. With the ideal A/F ratio, not only efficient engine combustion is realized but also three-way catalystic converters work best to avoid the undesirable exhaust.

Here, to avoid the complexity of controller design, we design the torque tracking controller and the fuel injection controller in a cascade way separately.

1) As a solution of collocation method for the continuous-time system, we can obtain following predicted state sequence.
   - air mass into the cylinder M_c,
   - The fuel residual rate at the intake valve and port P_v, P_p,
   - The fuel adhesion rate at the intake valve and port R_v, R_p.
2) Considering M_c, P_p, R_p, P_v and R_v as parameters, we can reformulate the following discrete-time optimal control problem into quadratic programming problem.

$$\min_{U_{fi}} \sum_{k=0}^{N} \left[\frac{1}{2} Q_{F/A} \left(\frac{f_c[k]}{M_c[k]} - \frac{1}{\lambda_d} \right)^2 + \frac{1}{2} R_{fi} u_{fi}[k]^2 \right] \tag{39}$$

subject. to.

$$x_d[k+1] = A_k x[k] + B_k u_{fi}[k],$$
$$u_{fi}[k] \ge 0, \quad \lambda_{\min} \le \frac{M_c[k]}{f_c[k]} \le \lambda_{\max},$$

where k denotes the count of the engine cycle.

   - When above problem is infeasible, we resolve the problem after relaxing the A/F ratio constraint.

D. State Estimator

The proposed control method in previous section can handle the constraints explicitly, in addition, the engine system has many state constraints. Therefore, it is expected that the state estimation method which consider the constraints is more effective. In this study, we apply Interval constrained Unscented Kalman Filter (IUKF) [7] for intake pressure estimation.

V. Simulation

In this section, we apply the proposed method to the SICE engine benchmark problem. Here we consider the torque demand control for speed tracking of vehicles. A target vehicle speed is a first step of 10-15 driving mode.

In order to apply the nonlinear MPC method, we define the cost function in (32) as

$$\varphi = \frac{1}{2}(y(t_f) - y_d(t_f))^{\mathrm{T}} Q_f (y(t_f) - y_d(t_f)), \tag{40}$$

$$L = \frac{1}{2}(y(t) - y_d(t))^{\mathrm{T}} Q (y(t) - y_d(t)) + \frac{1}{2} v(t)^{\mathrm{T}} R v(t), \tag{41}$$

In order to achieve torque demand control while considering also the drivability and the fuel consumption, we choose four output variables as follows:

- the engine speed ω_e: when the gear is 'Neutral', we consider the engine speed control. To avoid the overshoot of the engine speed, the engine speed is regulated to desired value ω_{ed} by the nonlinear MPC.
- the engine torque τ_e: the torque demand control for the engine torque τ_e is considered when the gear is 'Drive'. The desired value is calculated by the torque reference generator, and the torque tracking is achieved by the nonlinear MPC.
- the intake air mass into the cylinder M_c: here, the fuel injection input is calculated based on (39). Thus, we can reduce the fuel consumption by regulating the intake air mass M_c.
- the spark angle u_{sa}: to achieve high efficiency torque generation, we evaluate the difference between the spark angle input and an optimal(MBT) spark angle.

According to the definition above, output and its desired are defined as

$$y := [\ \omega_e, \ \tau_e, \ M_{\mathrm{cyl}}, \ u_{sa}\ ]^{\mathrm{T}},$$
$$y_d := [\ \omega_{ed}, \ \tau_{ed}, \ 0, \ \phi_{MBT}\ ]^{\mathrm{T}}.$$

The constraint is given as follows:

$$u_{\min} := [\ 10^\circ, \ 0.01, -40^\circ\ ]^{\mathrm{T}}, \ u_{\max} := [\ 90^\circ, \ 1.0, \ 40^\circ\ ]^{\mathrm{T}}.$$
$$v_{\min} := [\ -10, \ -10, -30\ ]^{\mathrm{T}}, \ v_{\max} := [\ 10, \ 10, \ 30\ ]^{\mathrm{T}}.$$

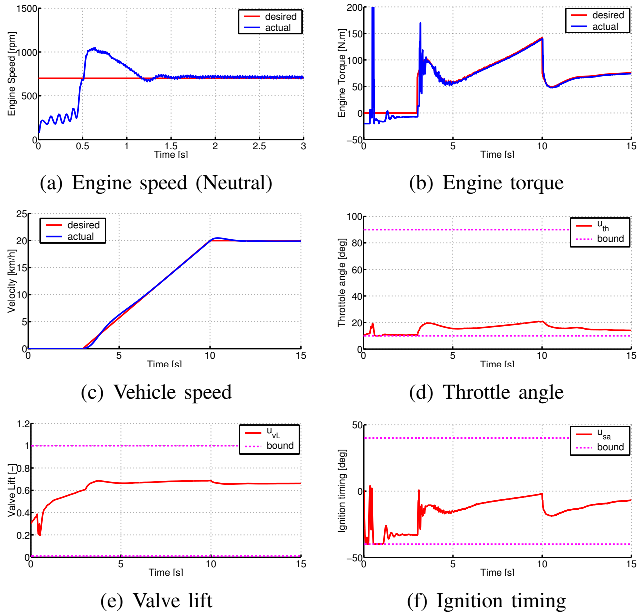
The jerk constraint and the A/F constraint are given as

$$-2.0 \le \ddot{v} \le 2.0, \ \lambda_{\min} = 13.5, \ \lambda_{\max} = 15.5. \tag{42}$$

Simulation results are shown in Fig. 4 - Fig. 7.

In Fig. 4(a), we plot the result of engine speed control. The engine speed is regulated to $\omega_{ed} = 700$[rpm] while the gear is 'Neutral'($t \le 3$[sec]), and it prevents the drastic change of vehicle acceleration.

Fig. 4(b)-Fig. 4(f) shows the result of torque control. At first, from Fig. 4(b), we can see that torque tracking is achieved by nonlinear MPC, and torque reference is modified when vehicle speed error is occurred. As the results, the

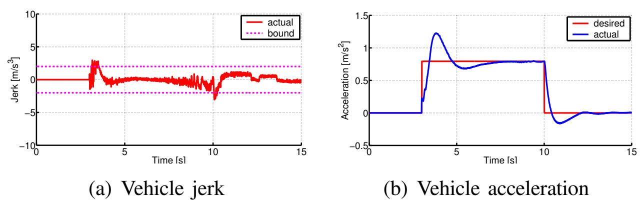


(a) Engine speed (Neutral) (b) Engine torque

(c) Vehicle speed (d) Throttle angle

(e) Valve lift (f) Ignition timing

Fig. 4. Simulation result: Torque demand control



(a) Vehicle jerk (b) Vehicle acceleration

Fig. 5. Simulation result: Jerk constraint

vehicle speed follows a target speed closely as shown in Fig. 4(c). Furthermore, in Fig. 4(d)-Fig. 4(f), it also be seen that input constraints are satisfied.

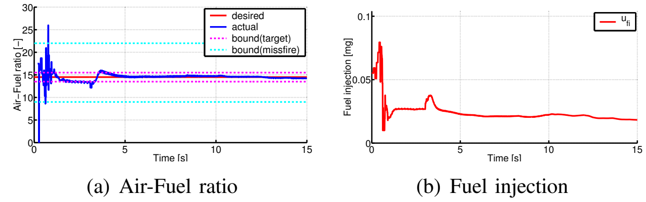
Fig. 5 shows the simulation result of vehicle jerk and acceleration. From Fig. 5(a), it is seen that the jerk constraint is satisfied at almost time. Thus the vehicle acceleration in Fig. 5(b) becomes smooth profile, which implies the driving comfort.

The A/F control result are shown in Fig. 6. Although the misfire occurs in the engine startup, the A/F ratio stays within the target A/F constraint at the steady state.

Fig. 7 shows a simulation result with $\pm 20\%$ modeling errors in coefficients of the base torque efficiency η_{e0} in (20). Although input signals show different behavior, the vehicle speed doesn't show much difference.

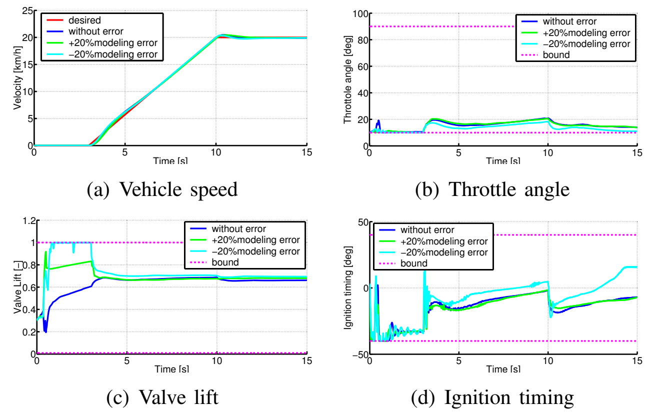
VI. Conclusion

In this paper, torque demand control method by nonlinear MPC has been proposed. To deal with torque demand control problem with constraints, we designed the controller which consists of the torque reference generator and the nonlinear MPC based on the collocation method. The validity of the proposed method was demonstrated by numerical simulations using the SICE engine benchmark simulator. From the simulation results, it is shown that the proposed method satisfy the design specification are satisfied.



(a) Air-Fuel ratio (b) Fuel injection

Fig. 6. Simulation result: A/F control



(a) Vehicle speed (b) Throttle angle

(c) Valve lift (d) Ignition timing

Fig. 7. Simulation result: effect of the modeling error

Future work will be focussed on both the NMPC controller improvement and the online optimization. About the former, we think the combination design of the NMPC controller and the fuel injection controller. For the online optimization, we think the combination with collocation method and other fast calculation method such as C/GMRES[8].

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