# Motion planning

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# Motion planning

# Context

industrial robot

aerial vehicle

autonomous vehicle


Autonomous mobile systems

- moving in an environment cluttered by obstacles
- possibly subject to kinematic or dynamic constraints

Motion planning : automatically compute a feasible collision-free path between two given configurations.

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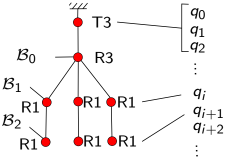
# Robot

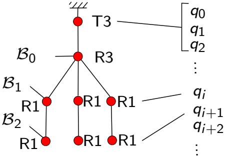
Set of rigib bodies $\mathcal{B}_0, \cdots \mathcal{B}_m$, linked to one another by *joints*.




Joint : mobility of a body in the reference frame of its parent, parameterized by one or several numbers.

# Robot configuration

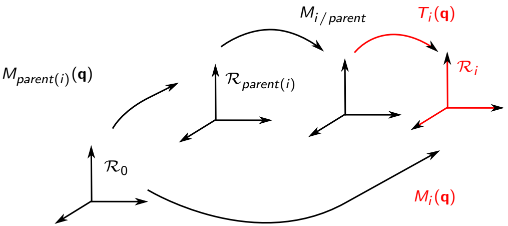
The configuration **q** of a robot is represented by the concatenation of the parameters of each joint.




# Forward kinematics

Computation of the position of each joint in world frame.

$$M_i(\mathbf{q}) = M_{parent(i)}(\mathbf{q})\ M_{i/parent}\ \ T_i(\mathbf{q})$$




# Definitions

- Workspace : $\mathcal{W} = \mathbb{R}^2$ or $\mathbb{R}^3$ : space in which the robot moves
- Workspace obstacle : compact subset of $\mathcal{W}$, denoted by $\mathcal{O}$.
- Configuration space : $\mathcal{C}$.
- Position in configuration $\mathbf{q}$ of a point $M \in \mathcal{B}_i$ : $\mathbf{x}_i(M, \mathbf{q})$.
- Configuration space obstacle :

$$\begin{aligned} \mathcal{C}_{obst} \quad = \{\mathbf{q} \in \mathcal{C}, \quad & \exists i \in \{1, \cdots, m\},\ \exists M \in \mathcal{B}_i,\ \mathbf{x}_i(M, \mathbf{q}) \in \mathcal{O} \text{ ou} \\ & \exists i, j \in \{1, \cdots, m\},\ \exists M_i \in \mathcal{B}_i,\ \exists M_j \in \mathcal{B}_j, \\ & \mathbf{x}_i(M_i, \mathbf{q}) = x_j(M_j, \mathbf{q})\} \end{aligned}$$

- Free configuration space : $\mathcal{C}_{free} = \mathcal{C} \setminus \mathcal{C}_{obst}$.

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# Motion

- Motion :
  - continuous mapping from $[0, 1]$ into $\mathcal{C}$.
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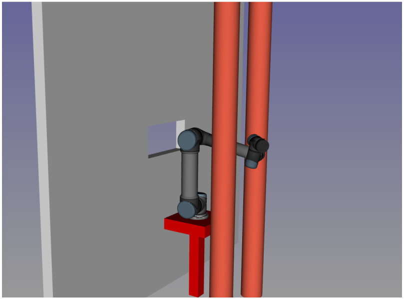
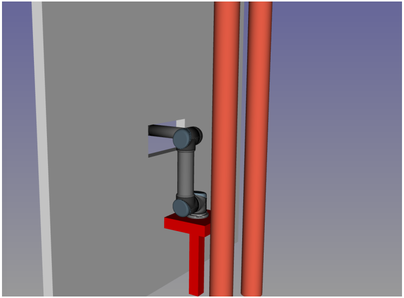
# Motion planning problem



initial configuration



goal configuration

$$\mathcal{C} = [-2\pi, 2\pi]^6$$

# History

- before the 1990's : mainly a mathematical problem
  - Real algbraic geometry
  - Decidability : Schwartz and Sharir 1982
    - Tarski theorem, Collins decomposition
  - Approximate cell decomposition
- from the 1990's : an algorithmic problem
  - random sampling (1993)
  - asymptotically optimal random sampling (2011)

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# Random sampling

- Random sampling motion planning methods appeared in the early 1990's
- Principle
  - sample random configurations
  - test whether they are collision-free
  - build a roadmap the nodes of which are the free configurations,
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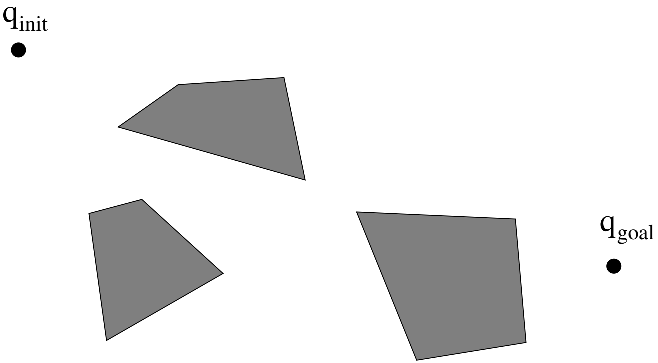

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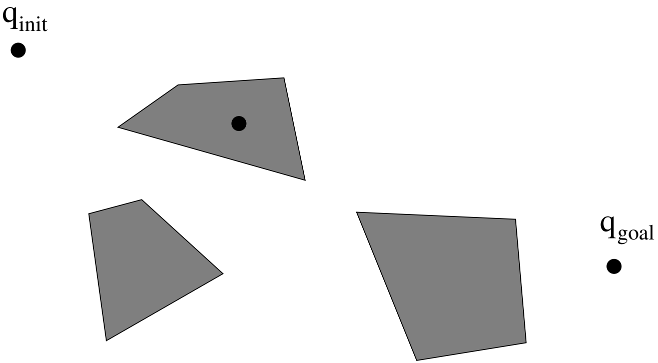

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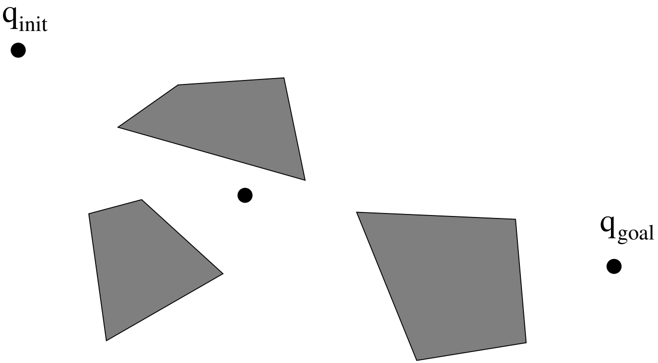

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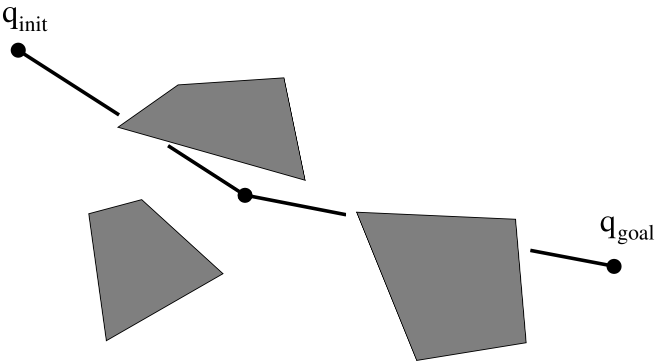

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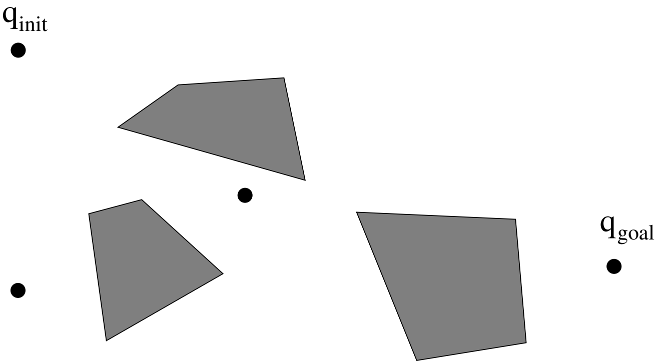

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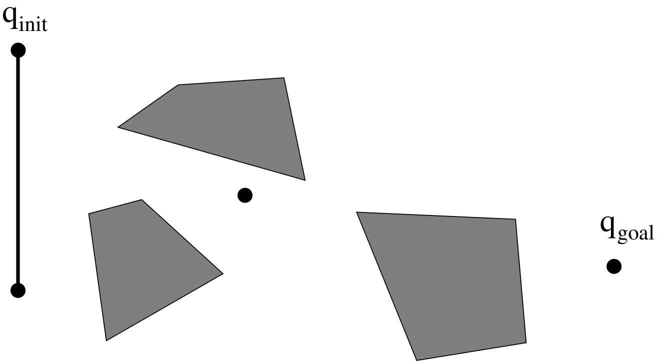

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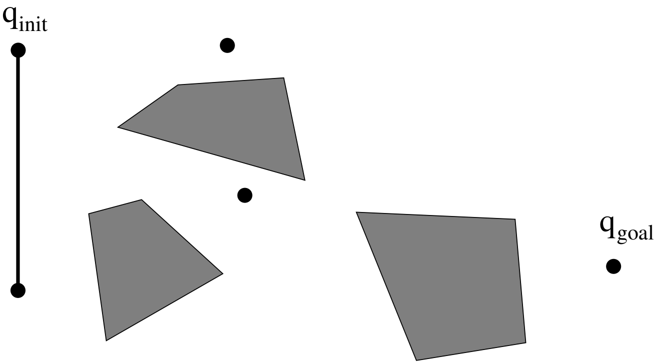

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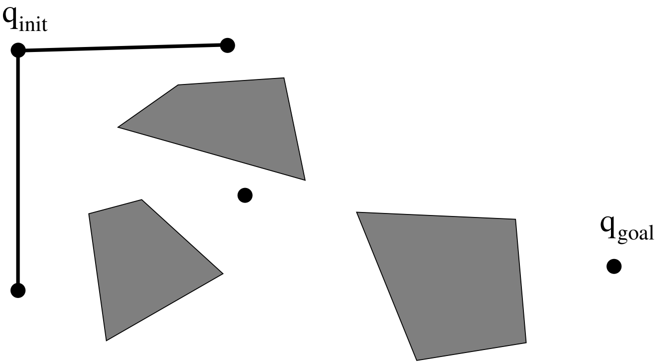

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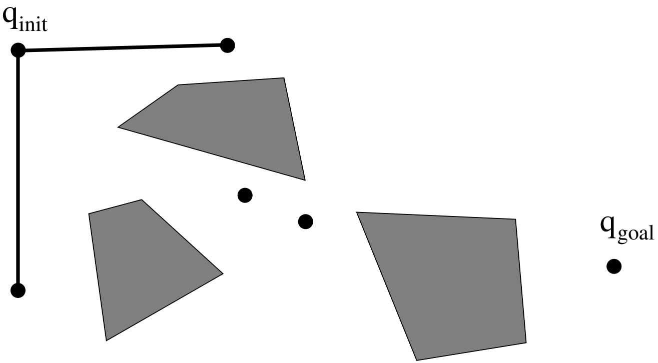

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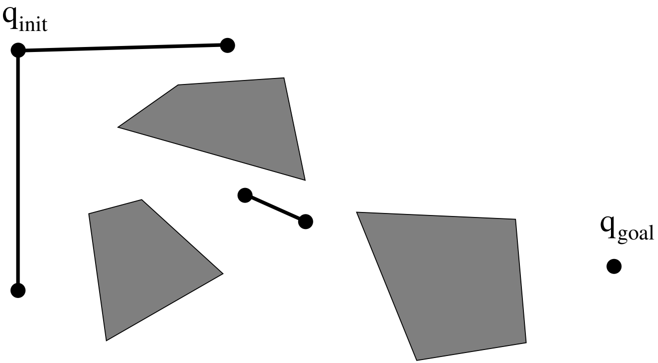

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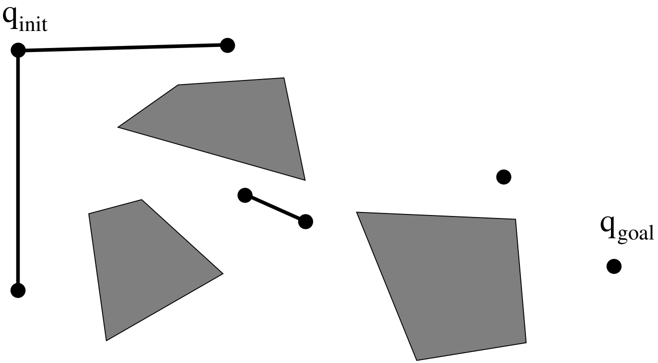

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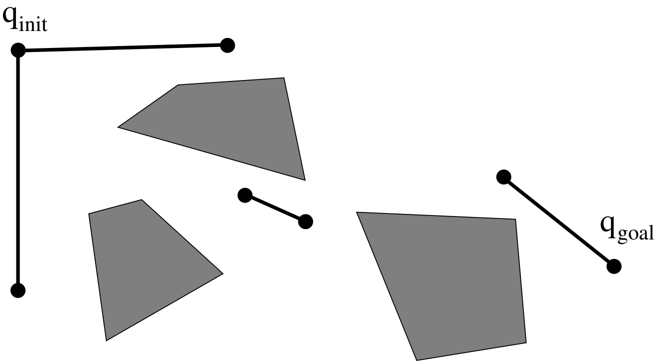

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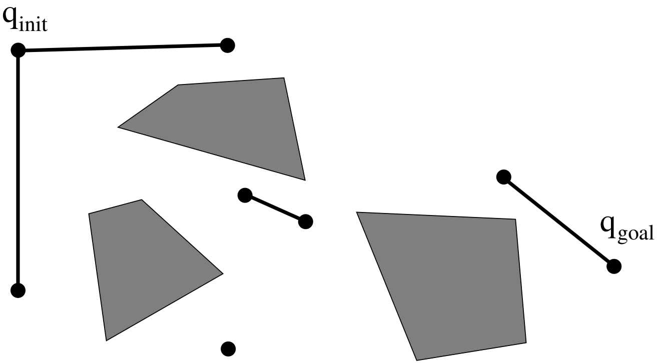

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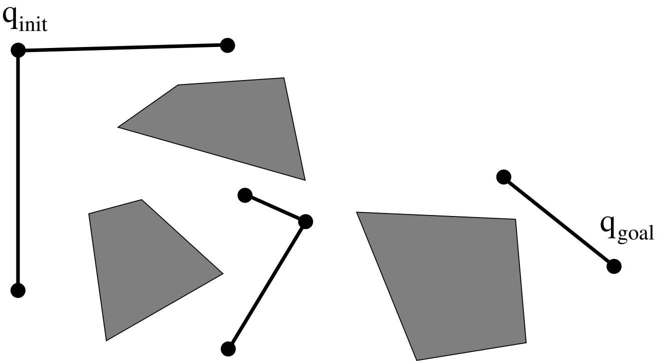

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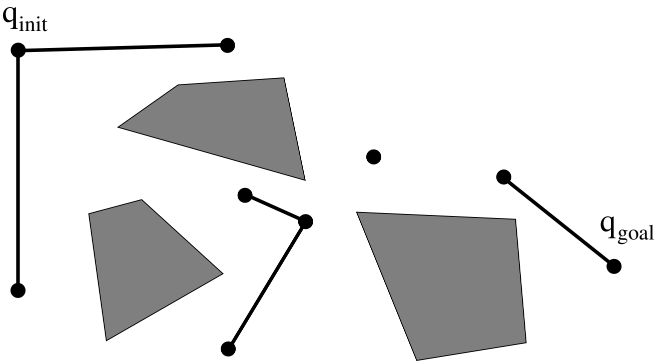

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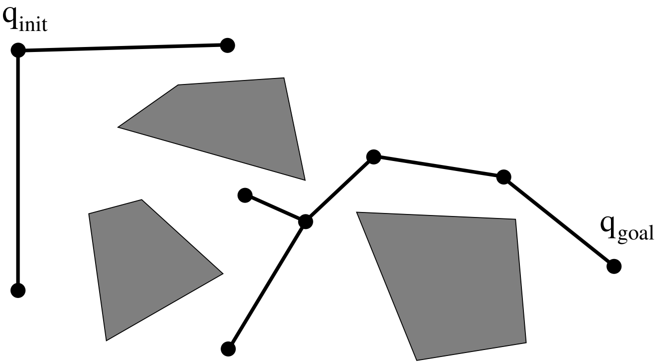

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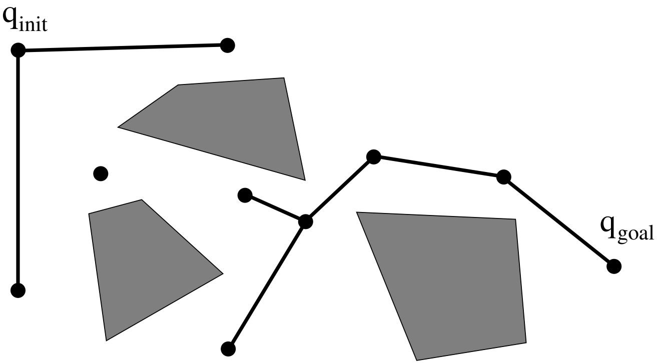

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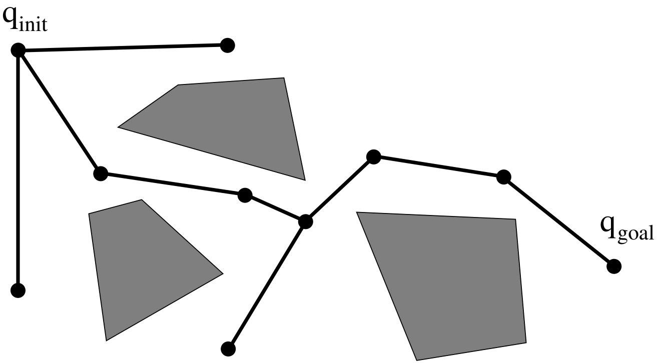
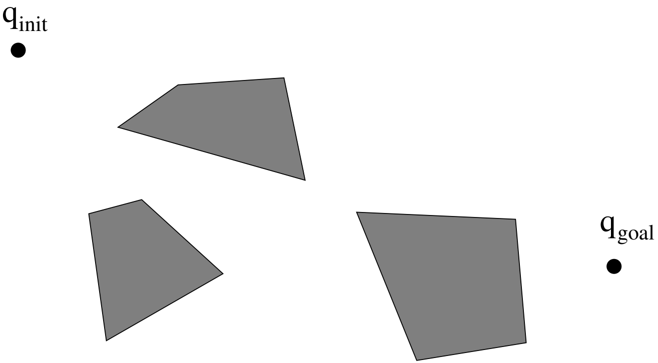


# Probabilistic roadmap (PRM)

- Numerous useless nodes are created
  - this makes the connection of new nodes more time consuming
- Variant : visibility-based PRM
  - only *interesting* nodes are kept.

# Visibility-based probabilistic roadmap (Visi-PRM) 1999



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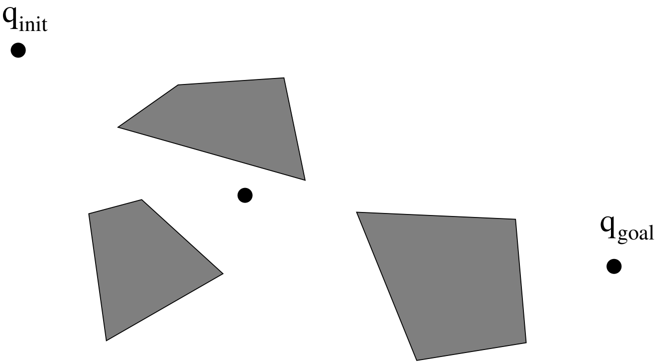

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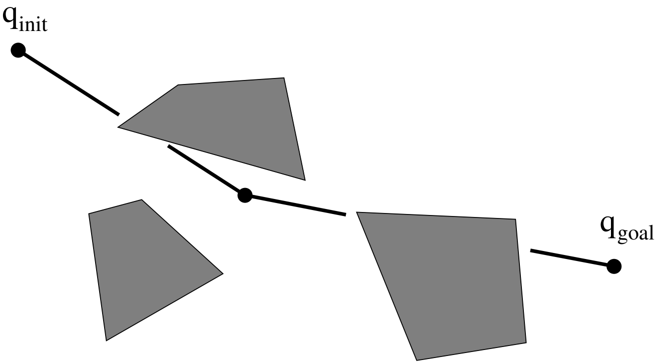

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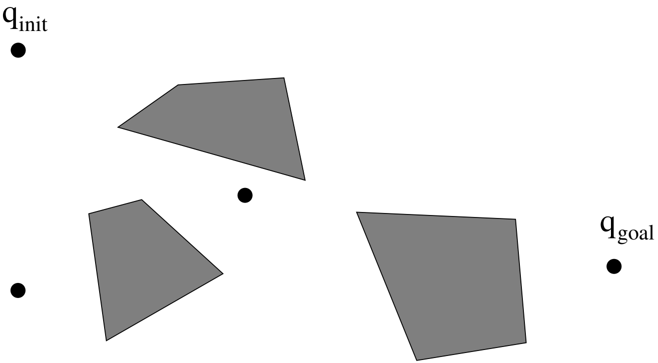

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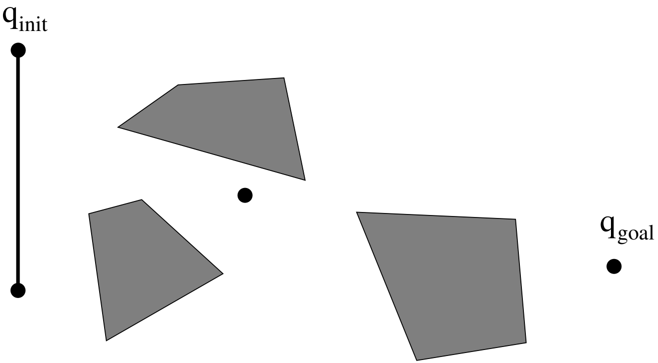

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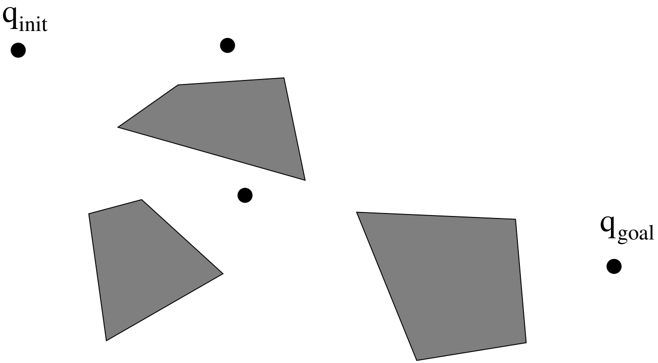

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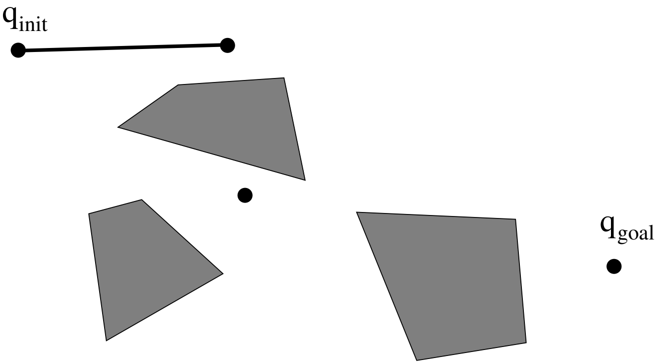

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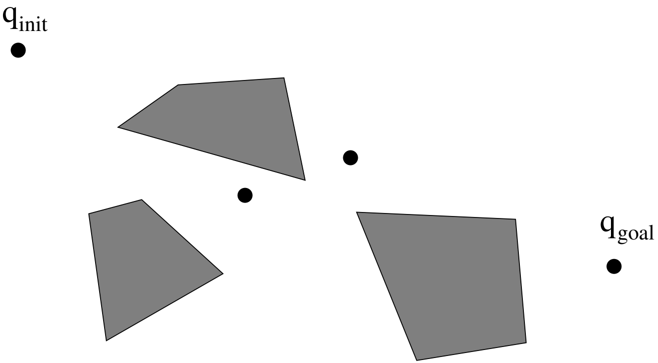

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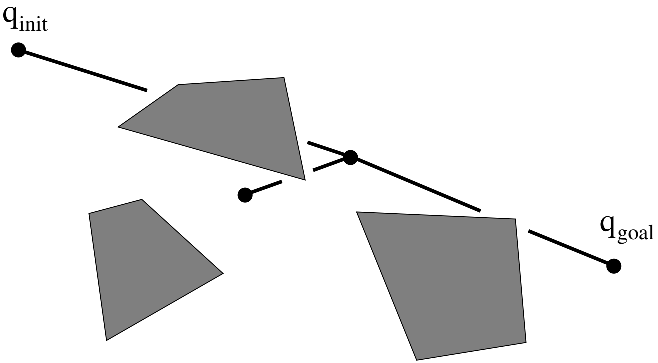

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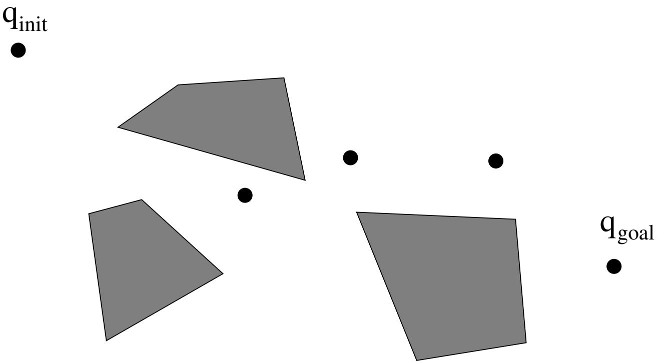

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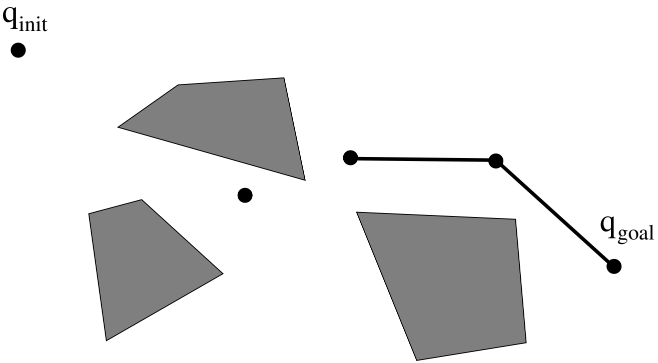

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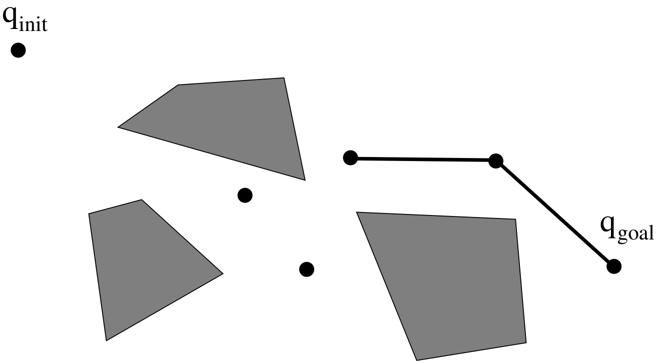

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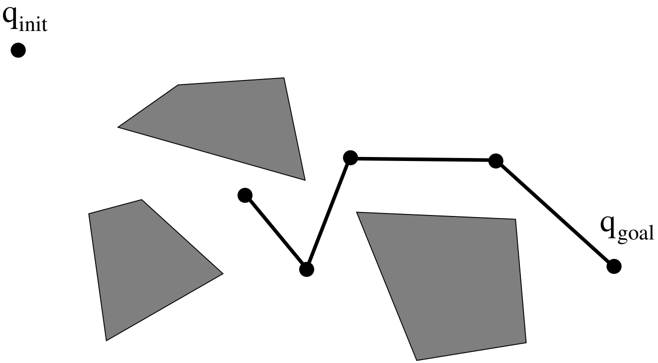

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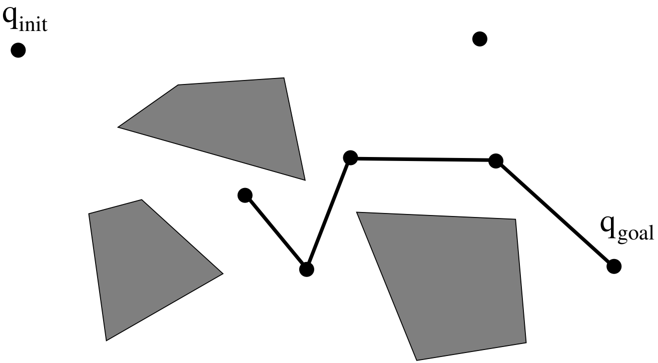

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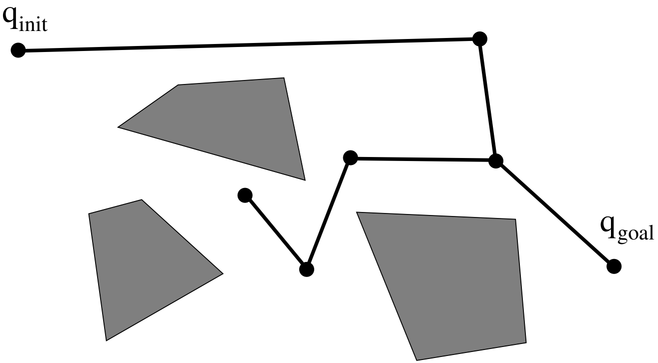
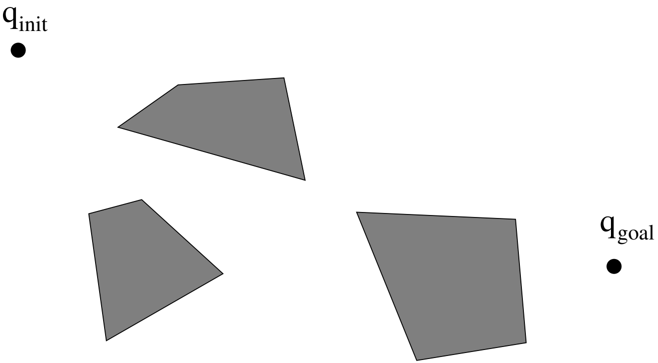


# Rapidly exploring Random Tree (RRT) 2000



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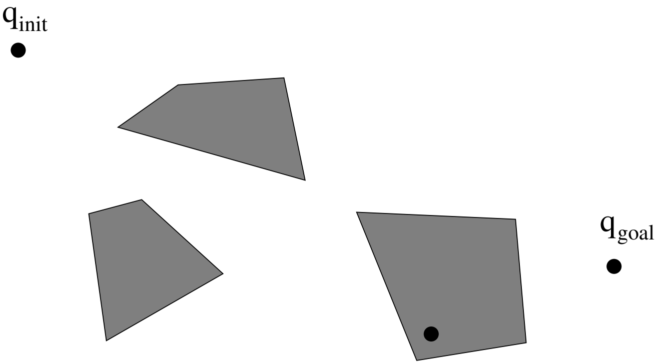

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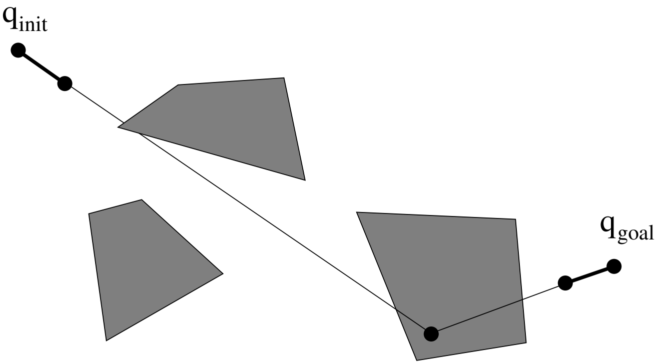

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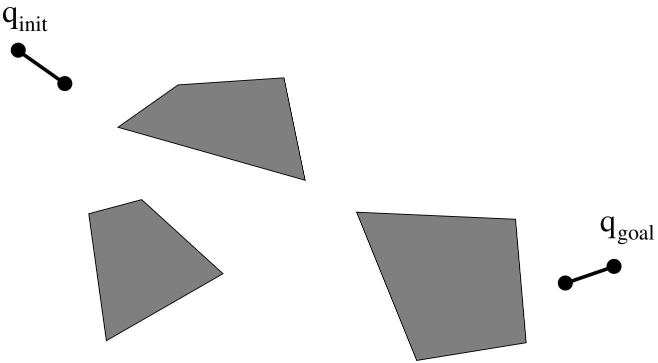

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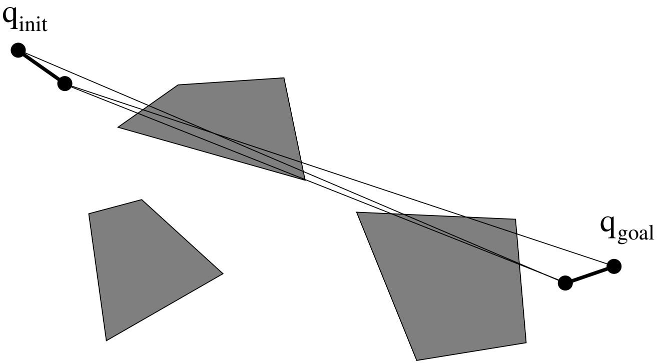

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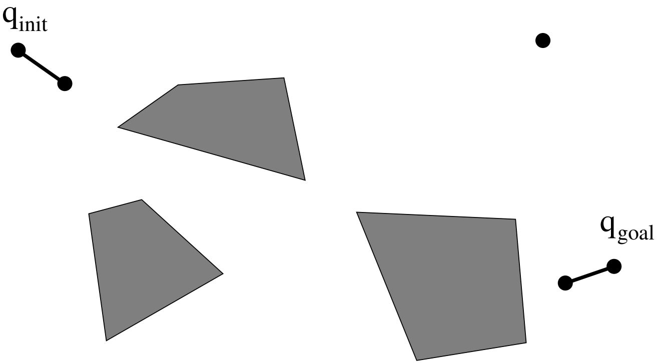

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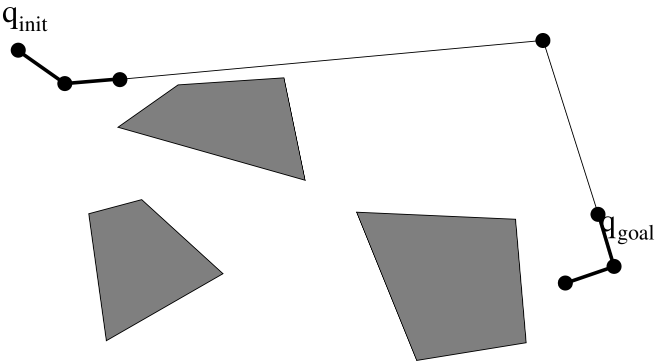

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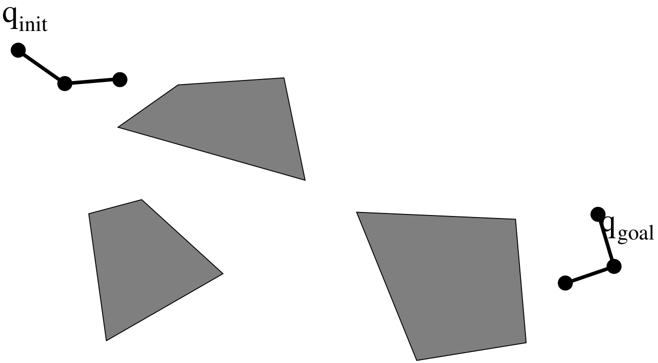

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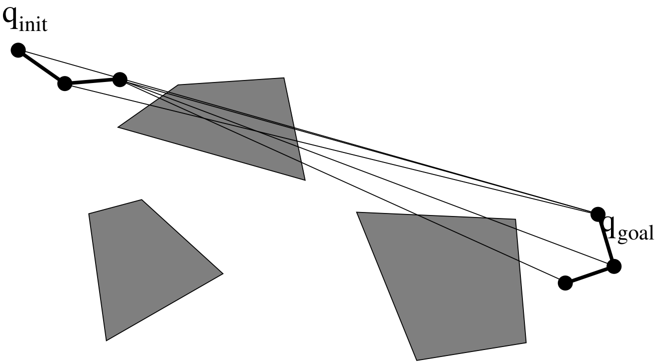

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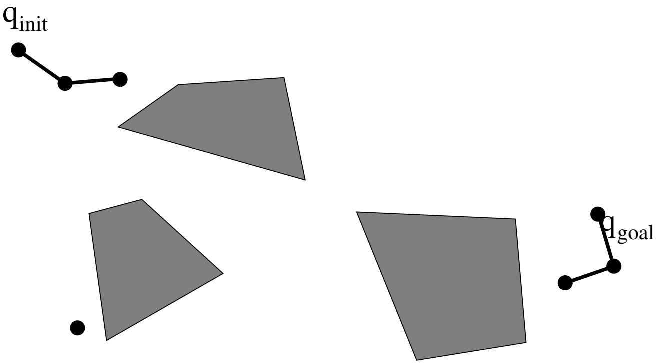

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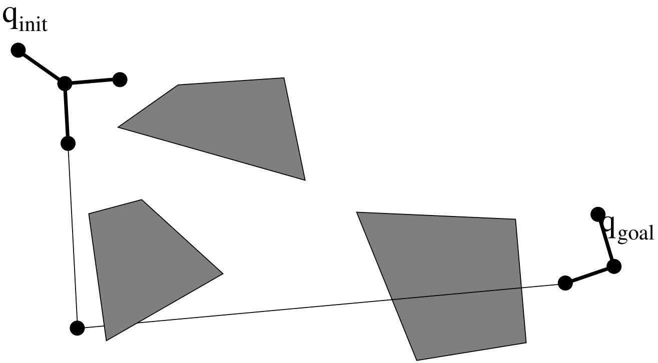

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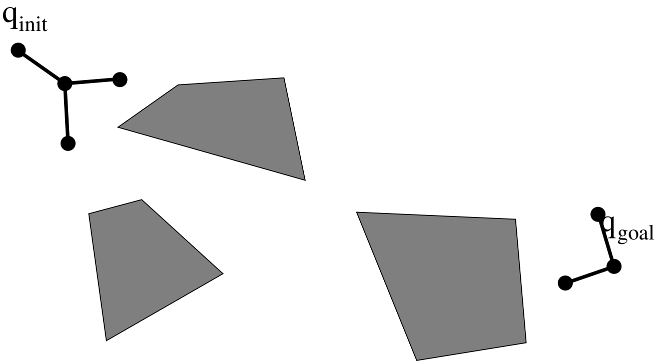

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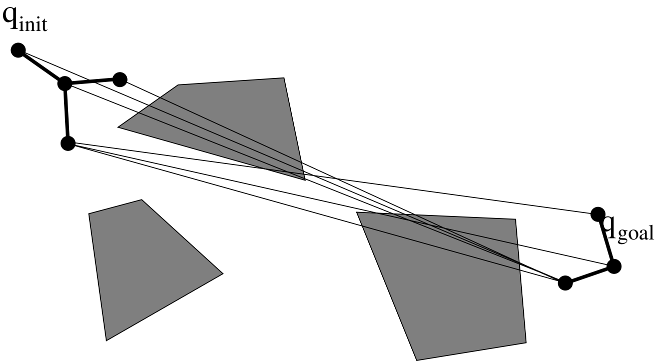

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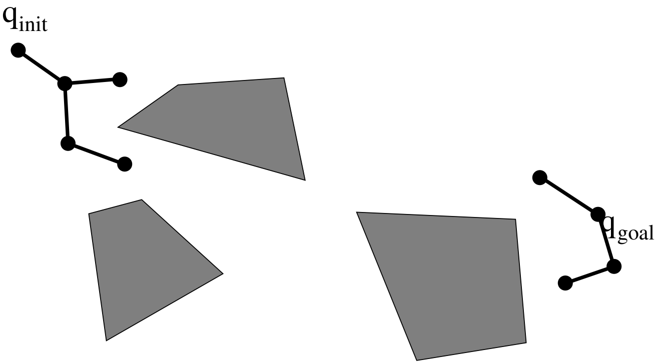

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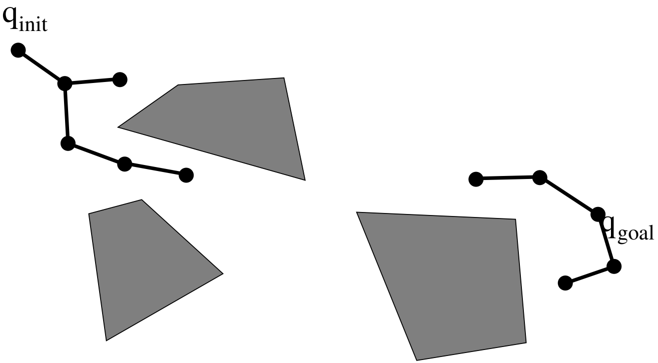

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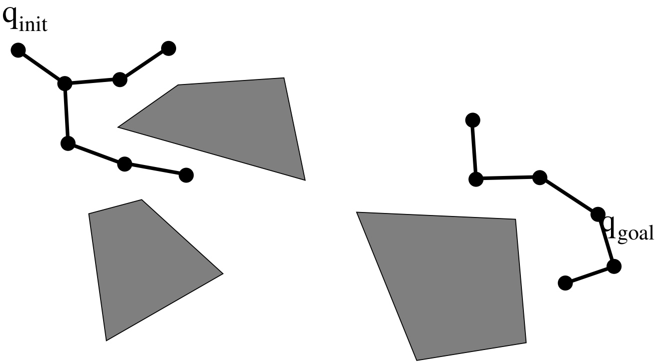

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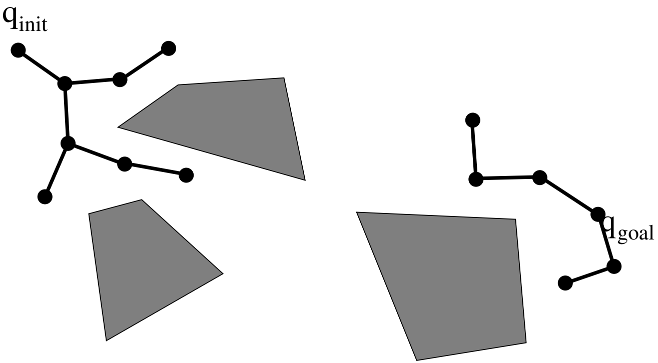

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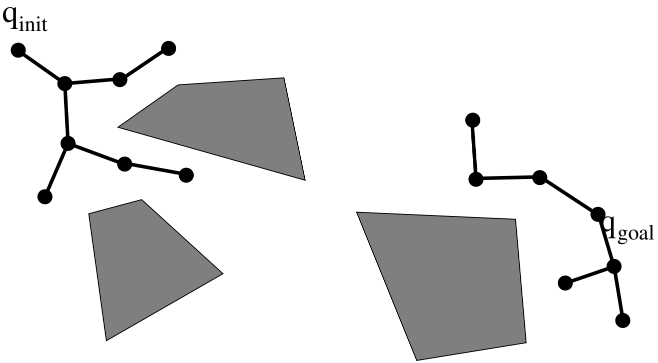

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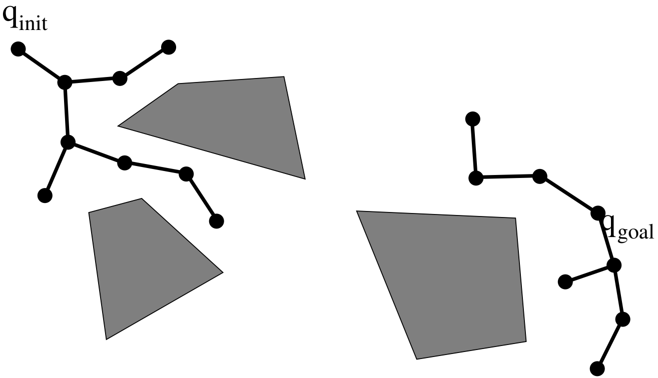

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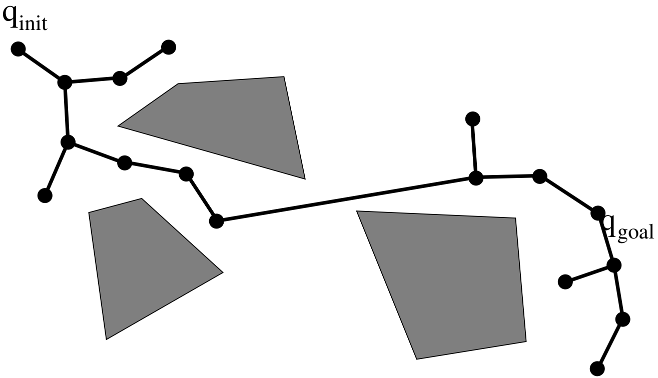

# Random sampling

- Pros :
  - no explicit computation of the configuration space
  - easy to implement,
  - robust.
- Cons :
  - no completeness, only *probrabilistic* completeness
  - difficult to find narrow passages.
- required operators :
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# Asymptotically optimal random sampling

Variants of PRM and RRT exist, and are asymptotically optimal :

- when the number of nodes tends to infinity,
- the solution computed by the algorithm tends to the optimal collision-free path.

# PRM*

**PRM**

$V \leftarrow \emptyset$, $E \leftarrow \emptyset$
**for** $i \in \{0, \cdots, n\}$ **do**
  $x_{rand} \leftarrow \text{SampleFree}_i$
  $U \leftarrow G.\text{Near}(x_{rand}, r)$
  **for all** $u \in U$ in order of increasing $\|u - x_{rand}\|$ **do**
    **if** $x_{rand}$ and $u$ in different connected components **then**
      TryConnect $(x_{rand}, u)$
    **end if**
  **end for**
**end for**

**PRM***

$V \leftarrow \text{SampleFree}_{i=1,\cdots n}$, $E \leftarrow \emptyset$
**for** $v \in V$ **do**
  $U \leftarrow G.\text{Near}(v, r^*) \setminus v$
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# PRM*, kPRM*

Note that :

- PRM*, kPRM* are not iterative anymore,
- making them iterative is not trivial.

# Rapidly Exploring Random trees

There exists also asymptotically optimal variants of RRT

- RRG, RRT*

but they are specific to a given problem $(\mathbf{q}_{init}, \mathbf{q}_{goal})$.

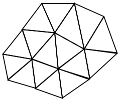
# Collision tests

- static : for configurations
  - problem : given
    - two rigid objects made of triangles
    - the relative position of one with respect to the other one

    determine whether they are colliding.

# Bounding volume hierarchies

- binary tree of bounding volumes such that
  - each node has two children,
  - leaves are triangles.



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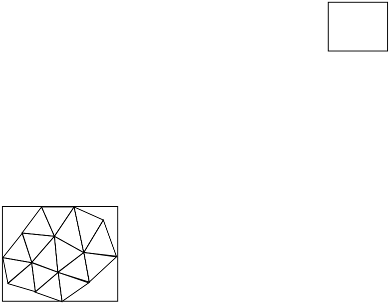

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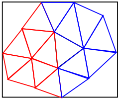

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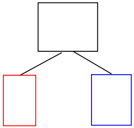

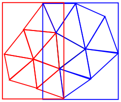

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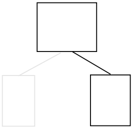

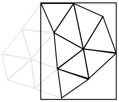

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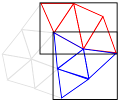

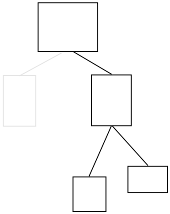

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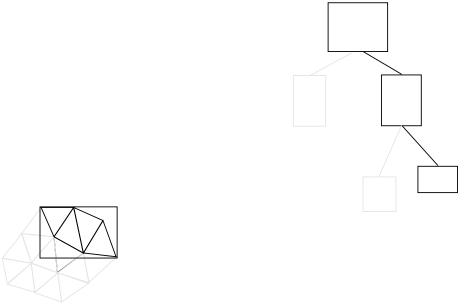

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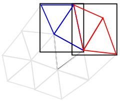

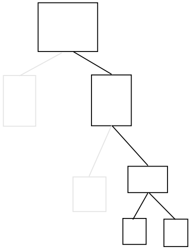

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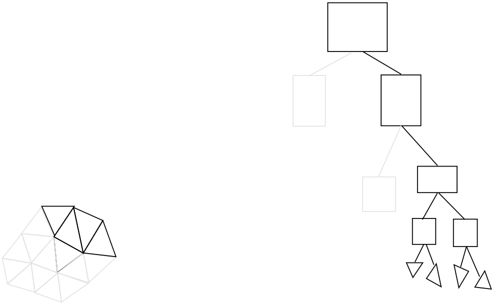
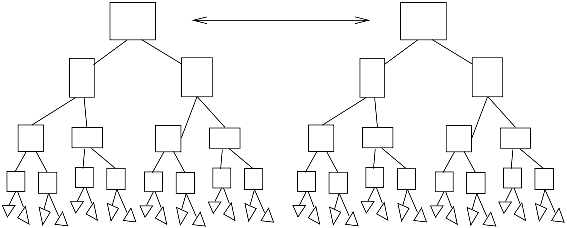


# Collision testing for configurations

- Algorithm
  - test root nodes of each tree,
  - if two bounding volumes collide, test one with the children of the other one.



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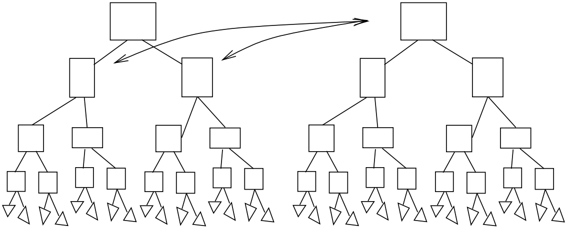

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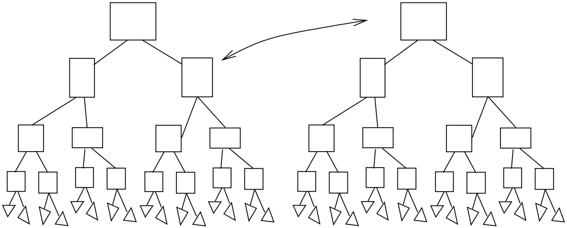

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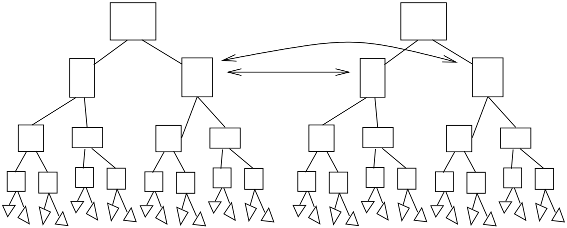

# Open source software platform

Several open-source platforms for motion planning are available

- OMPL (Rice University)
  - no kinematic chain,
  - no collision checking.
- Openrave (CMU)
- MoveIt (ROS)
  - Integration in ROS of
  - fcl (collision checking), KDL (kinematic chain)
- Humanoid Path Planner
  - numerical constraints (quasi-static equilibrium)
  - advanced manipulation planning

# Humanoid Path Planner

https ://humanoid-path-planner.github.io/hpp-doc