Robust adaptive deadzone compensation of DC servo system

W. Zhonghua, Y. Bo, C. Lin and Z. Shusheng

Abstract: Deadzone in DC servo system is often poorly known and it can severely limit system performance. A robust adaptive deadzone compensation scheme is presented for general DC servo system with deadzone of unknown width. The tuning algorithms were rigorously shown to guarantee small tracking errors as well as bounded deadzone parameter estimates using Lyapunov stability. The other prominent feature of the controller is that it can deal with the uncertainty in both inertial and friction parameters. The simulation results are presented to demonstrate its efficacy.

1 Introduction

High accuracy motion control is widely used in modern mechanical systems, such as machine tools, semiconductor manufacturing equipment and medical robot and so on. In the actuators of these systems, there exist some non-smooth nonlinearities, including deadzone, backlash, saturation and so on. In the cases where deadzone nonlinearities are not very severe and control specifications are not very strict, they can be neglected. However, there are many cases where ignoring these nonlinearities will give unacceptable control performance such as excessive steady state error and poor transient system response. Proportional-derivative (PD) controllers have been observed to result in limit cycles if the actuators have deadzones. Therefore the study of how to deal with deadzone nonlinearities is an important topic for the control design engineer.

The most straightforward way to cope with deadzone nonlinearities is to cancel them by employing their inverses. However, this can be done only when the deadzone nonlinearities are exactly known. In practice, even though deadzone nonlinearities can be parameterised, the parameters are often unknown and time varying. Hence, the constant inverse approach can hardly be applied.

Various deadzone compensation techniques have been recently proposed for high-precision control, such as dithering [1], variable structure control [2], neural networks [3] and fuzzy logic deadzone compensation [4–8]. Adaptive control is considered as a good solution to the control of plants with unknown parameters. The study of adaptive control for systems with unknown deadzone was initiated by Recker *et al.* [9], where an adaptive scheme was proposed for the case of full-state measurement. Tao and Kokotović [10] designed a model reference adaptive controller for plants with unknown deadzones by considering the more realistic situation where only a single output measurement is available. Adaptive output feedback deadzone compensation scheme for systems with an unknown deadzone at the input of an nth-order smooth nonlinear dynamics in the output-feedback canonical form has been presented by Tian and Tao [11], which employs an adaptive deadzone inverse to cancel the deadzone. In Taware *et al.* [12], for sandwich nonlinear systems having an unknown sandwiched deadzone between the linear dynamic blocks, an adaptive hybrid control scheme is developed. Most recently, Wang *et al.* [13] proposed a robust adaptive control scheme for a class of nonlinear systems without using the deadzone inverse, which ensures global stability of the adaptive system and achieves desired tracking precision.

In this paper, we present a robust adaptive deadzone compensation method in a DC servo-motor control system. The general case of non-symmetrical deadzones is treated. The designed controller provides robustness not only to deadzones, but also to external disturbances. Simulation results show its efficacy in cancelling the deleterious effects of actuator deadzones and external disturbances.

2 Dynamics of DC servo system

In this section, the DC servo system is considered. First, the adaptive precompensator is designed for the nonsymmetrical deadzone nonlinearity. Secondly, the dynamic model of the DC servo system is constructed.

2.1 Deadzone nonlinearity

If u and T are scalars, the non-symmetrical deadzone nonlinearity, shown in Fig. 1, is given by

$$T = D_{\mathrm{d}}(u) = \begin{cases} u - d_+, & u > d_+ \\ 0, & d_- < u \le d_+ \\ u - d_-, & d_- \ge u \end{cases} \qquad (1)$$

The parameter vector $\boldsymbol{d} = [d_+ \quad d_-]^{\mathrm{T}}$ characterises the width of the motion deadband. In practical motion-control systems, the width of the deadzone is unknown, so that

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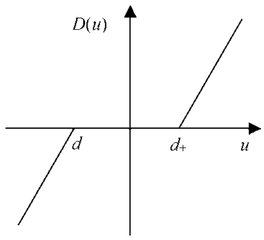

Fig. 1 *Non-symmetrical deadzone nonlinearity*

the compensation is difficult. Most compensation schemes cover only the case of symmetric deadzone where $d_- = d_+$.

The non-symmetrical deadzone may be written as

$$T = D_d(u) = u - \text{sat}_d(u) \tag{2}$$

where the non-symmetrical saturation function is defined as

$$\text{sat}_d(u) = \begin{cases} d_+, & u > d_+ \\ u, & d_- < u \le d_+ \\ d_-, & d_- \ge u \end{cases} \tag{3}$$

To compensate the deleterious effects of the deadzone, one can place a precompensator as illustrated in Fig. 2. There, the desired function of the precompensator is to cause the composite throughput from w to T to be unity. As the width $[d_+ \quad d_-]^T$ of deadzone is not known, the learning or adaptation method is considered intuitively. An estimate $\hat{\boldsymbol{d}} = [\hat{d}_+ \quad \hat{d}_-]^T$ of the deadzone width parameter vector $\boldsymbol{d}$ is as the precompensator to offset the effects of deadzone. The function from w to u with the precompensator can be expressed by

$$u = w + \mu \cdot \hat{d}_+ + (1 - \mu) \cdot \hat{d}_- \tag{4}$$

where

$$\begin{matrix} \mu = 1 & (w \ge 0) \\ \mu = 0 & (w < 0) \end{matrix}.$$

The composite throughput from w to T of the adaptive compensator plus the deadzone is

$$\begin{aligned} T = D_d(u) &= w + \mu \cdot \hat{d}_+ + (1 - \mu) \cdot \hat{d}_- \\ &- \text{sat}_d\left(w + \mu \cdot \hat{d}_+ + (1 - \mu) \cdot \hat{d}\right) \end{aligned} \tag{5}$$

If the deadzone width estimation error is defined as

$$\tilde{\boldsymbol{d}} = \boldsymbol{d} - \hat{\boldsymbol{d}} \tag{6}$$

then by simply deriving, the throughput of the compensator plus deadzone can be written as

$$T = w - \tilde{\boldsymbol{d}}^T \bar{\mu} + \tilde{\boldsymbol{d}}^T \delta \tag{7}$$

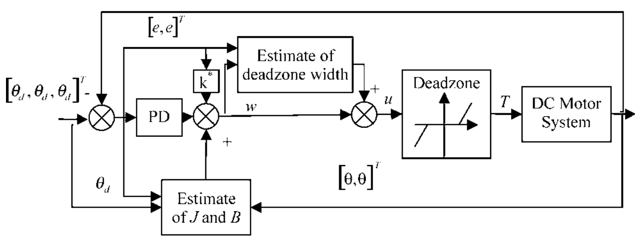

Fig. 2 *Tracking controller with robust adaptive deadzone compensation*

where $\bar{\mu}^T = [\mu \quad 1 - \mu]$, δ is the modelling mismatch term and satisfies the bound $\|\delta\| \le 1$ [6].

Equation (7) shows that, as the estimate $\hat{\boldsymbol{d}}$ approaches the actual deadzone parameter vector $\boldsymbol{d}$, the adaptive precompensator effectively provides an inverse for the deadzone nonlinearity.

2.2 Model of DC servo system

The dynamics of DC servo systems with no vibratory modes can be written as

$$J\ddot{\theta} + B\dot{\theta} + T_d = T \tag{8}$$

where $\theta(t)$ is the motor angular position, J the inertia, B the coefficient of viscous friction, T_d represents the composite throughput of the nonlinear friction plus external disturbance (e.g. variable load). The motor torque $T(t)$ is related to the control input $u(t)$ through the deadzone

$$T = D_d(u) = u - \text{sat}_d(u) \tag{9}$$

The unknown deadzone widths are bounded so that

$$\|\boldsymbol{d}\| < d_M \tag{10}$$

for some scalar d_M. It is assumed that the deadzone widths are constant so that

$$\dot{d} = 0 \tag{11}$$

To design a motion controller that causes the servo system to track a prescribed trajectory $\theta_d(t)$, define the tracking error by

$$e = \theta_d - \theta, \quad \dot{e} = \dot{\theta}_d - \dot{\theta} \tag{12}$$

And the tracking error metric by

$$s(t) = \dot{e} + \lambda e \quad \text{with } \lambda > 0 \tag{13}$$

The equation $s(t) = 0$ defines a time-varying hyperplane in R^2 on which the tracking-error vector decays exponentially to zero, so that perfect tracking can be asymptotically obtained by maintaining this condition. Similarly, if the magnitude of s can be shown to be bounded by a constant Φ, then the actual tracking errors can be shown to be ultimately bounded by [14].

$$\begin{aligned} |e| &\le \lambda^{-1}\Phi \\ |\dot{e}| &\le 2\Phi \end{aligned} \tag{14}$$

Φ can be incorporated into this error metric by defining a continuous function s_Δ as

$$s_\Delta(t) = s(t) - \Phi\,\text{sat}\left(\frac{s(t)}{\Phi}\right) \tag{15}$$

The function s_Δ has several properties useful in the design of adaptive laws: if $|s| < \Phi$, then $\dot{s}_\Delta = s_\Delta = 0$, whereas if $|s| > \Phi$, then $\dot{s}_\Delta = \dot{s}$ and $|s_\Delta| = |s| - \Phi$.

Differentiating (13) and invoking (7) and (8), it is seen that the closed loop dynamics DC of the servo system is expressed by

$$J\dot{s} = -w + J\ddot{\theta}_d + \tilde{d}^T \bar{\boldsymbol{\mu}} - \tilde{d}^T \delta + B\dot{\theta} + T_d + J\lambda\dot{e} \tag{16}$$

3 Robust adaptive controller design

In practice, the inertia-J and the coefficient of viscous friction-B are always unknown. They can be identified offline using a standard identification method, but they are varying depending on many factors such as lubrication, temperature, load and so on. In this paper, the adaptive laws are designed to estimate them online. Before designing adaptive controller, define some error variables

$$\begin{aligned} \tilde{J} &= J - \hat{J} \\ \tilde{B} &= B - \hat{B} \end{aligned} \tag{17}$$

where $\hat{J}$ and $\hat{B}$ are the estimate values of J and B, respectively. J and B are assumed to be constants. We make the following assumption.

Assumption 1 (Known disturbance bound)*:* It is assumed that the disturbance term T_d in (8) satisfies

$$|T_\mathrm{d}| \le T_M(\boldsymbol{x}) \tag{18}$$

for some known bounding function $T_M(\boldsymbol{x})$. Vector $\boldsymbol{x}$ contains all the signals related to T_d. In practice, this assumption is realistic.

Assumption 2 (Known extent of the parametric uncertainties)*:* The parameters J and B satisfy

$$\begin{aligned} J_{\min} &\le J \le J_{\max} \\ B_{\min} &\le B \le B_{\max} \end{aligned} \tag{19}$$

where $J_{\min}$, $J_{\max}$, $B_{\min}$ and $B_{\max}$ are known. In practice, this assumption is also realistic.

The problem is to design a control law $w(t)$ that ensures that the tracking-error metric $s(t)$ lies in the predetermined bound ultimately. The following controller is constructed for the nonlinear system (16)

$$w = k_\mathrm{d} s + \hat{J}\left(\ddot{\theta}_\mathrm{d} + \lambda\dot{e}\right) + \hat{B}\dot{\theta} + k^* \ \mathrm{sat}\frac{s}{\Phi} \tag{20}$$

where $k_\mathrm{d} > 0$ and $k^* > |T_M|$. We now specify the parameter update laws

$$\dot{\hat{J}} = \begin{cases} 0 & \text{if } \hat{J} = J_{\max} \text{ and } k_J(\ddot{\theta}_\mathrm{d} + \lambda\dot{e})s_\Delta > 0 \\ 0 & \text{if } \hat{J} = J_{\min} \text{ and } k_J(\ddot{\theta}_\mathrm{d} + \lambda\dot{e})s_\Delta < 0 \\ k_J(\ddot{\theta}_\mathrm{d} + \lambda\dot{e})s_\Delta & \text{otherwise.} \end{cases}$$

$$\dot{\hat{B}} = \begin{cases} 0 & \text{if } \hat{B} = B_{\max} \text{ and } k_B\dot{\theta}s_\Delta > 0 \\ 0 & \text{if } \hat{B} = B_{\min} \text{ and } k_B\dot{\theta}s_\Delta < 0 \\ k_B\dot{\theta}s_\Delta & \text{otherwise.} \end{cases} \tag{21}$$

$$\dot{\hat{d}} = \boldsymbol{\Gamma}\bar{\mu}s_\Delta - k\boldsymbol{\Gamma}\hat{d}|s_\Delta|$$

where the scalars $k_J > 0$, $k_B > 0$, $k > 0$ and the matrix $\Gamma > 0$ are design parameters.

Remark 1: Note that $k_\mathrm{d}s$ is actually a PD control with gains $k_\mathrm{d}s = k_\mathrm{d}e + k_\mathrm{d}\lambda\dot{e}$. The deadzone effect is ameliorated by a feedforward compensator.

Remark 2: The adaptive laws of J and B in (21) are modified by projection that guarantees

$$\begin{aligned} J_{\min} &\le \hat{J} \le J_{\max} \\ B_{\min} &\le \hat{B} \le B_{\max} \end{aligned} \tag{22}$$

Theorem 1: Consider the system described by (8) and (9) and the control objective of tracking desired trajectories defined by θ_d, $\dot{\theta}_\mathrm{d}$ and $\ddot{\theta}_\mathrm{d}$ (they are bounded). The control law given by (20) with (21) ensures that the system states and parameters are uniformly ultimately bounded (UUB). Furthermore, tracking error metric $s(t)$ satisfies $|s(t)| \le \Phi + (1 + kd_M)^2/4kk_\mathrm{d}$ for large enough $t \gg 0$, which can be made as small as desired by increasing the PD gain k_d and adjusting Φ.

Proof: We first define a Lyapunov function candidate $V(t)$ as

$$V(t) = \frac{1}{2}Js_\Delta^2 + \frac{1}{2k_J}\tilde{J}^2 + \frac{1}{2k_B}\tilde{B}^2 + \frac{1}{2}\tilde{d}^\mathrm{T}\boldsymbol{\Gamma}^{-1}\tilde{d} \tag{23}$$

Noting that $\dot{s}_\Delta = \dot{s}$ and $s_\Delta\,\mathrm{sat}(s/\Phi) = |s_\Delta|$ outside the boundary layer ($|s| > \Phi$), whereas $s_\Delta = 0$ and $\dot{V}(t) = 0$ inside the boundary layer ($|s| < \Phi$), differentiating $V(t)$ and substituting from (16), (20) and (21) yields

$$\begin{aligned} \dot{V} &= Js_\Delta\dot{s} + \frac{1}{k_J}\tilde{J}\dot{\tilde{J}} + \frac{1}{k_B}\tilde{B}\dot{\tilde{B}} + \tilde{d}^\mathrm{T}\boldsymbol{\Gamma}^{-1}\dot{\tilde{d}} \\ &= -k_\mathrm{d}s_\Delta^2 - k_\mathrm{d}\Phi|s_\Delta| - (k^* - |T_\mathrm{d}|)|s_\Delta| - \tilde{d}^\mathrm{T}\delta s_\Delta \\ &\quad + \tilde{J}\left[(\ddot{\theta}_\mathrm{d} + \lambda\dot{e})s_\Delta - k_J^{-1}\dot{\hat{J}}\right] + \tilde{B}\left(\dot{\theta}s_\Delta - k_B^{-1}\dot{\hat{B}}\right) \\ &\quad + \tilde{d}^\mathrm{T}\left(\bar{\mu}s_\Delta - \boldsymbol{\Gamma}^{-1}\dot{\hat{d}}\right) \end{aligned} \tag{24}$$

where $\dot{\tilde{J}} = -\dot{\hat{J}}$ and $\dot{\tilde{B}} = -\dot{\hat{B}}$ from (17) and $\dot{\tilde{d}} = -\dot{\hat{d}}$ from (6) and (11) are used in the deriving process. By using the update law (21), $\tilde{J}[(\ddot{\theta}_\mathrm{d} + \lambda\dot{e})s_\Delta - k_J^{-1}\dot{\hat{J}}] \le 0$ and $\tilde{B}(\dot{\theta}s_\Delta - k_B^{-1}\dot{\hat{B}}) \ge 0$ are ensured, so we have

$$\begin{aligned} \dot{V} &\le -k_\mathrm{d}s_\Delta^2 - k_\mathrm{d}\Phi|s_\Delta| - (k^* - |T_\mathrm{d}|)|s_\Delta| - \tilde{d}^\mathrm{T}\delta s_\Delta \\ &\quad + \tilde{d}^\mathrm{T}k\hat{d}|s_\Delta| \end{aligned} \tag{25}$$

Note that $\|\delta\| \le 1$

$$\dot{V} \le -|s_\Delta|[k_\mathrm{d}|s_\Delta| - (1 + kd_M)\|\tilde{d}\| + k\|\tilde{d}\|^2] \tag{26}$$

This is negative as long as the quantity in the brackets is positive. To determine conditions for this, complete the square to see that $\dot{V}$ is negative as long as either

$$|s_\Delta| > \frac{(1 + kd_M)^2}{4kk_\mathrm{d}} \tag{27}$$

or

$$\|\tilde{d}\| > \frac{1}{k} + d_M \tag{28}$$

According to the standard Lyapunov theorem, the tracking error decreases as long as the error $|s_\Delta|$ is bigger than the right-hand side of (27). This implies $|s(t)| \le \Phi + (1 + \kappa d_M)^2/4\kappa k_\mathrm{d}$ for large enough $t \gg 0$, which shows that, as the PD gain k_d increasing (within the system maintains stable) and Φ being adjusted, the tracking error may be made as small as desired. Also, Lyapunov extension shows that $\|\tilde{d}\|$ is bounded to a neighbourhood of the right-hand side of (28). Therefore both $s(t)$ and $\tilde{d}$ are UUB. Following Assumption 2 and Remark 2, $\hat{J}$ and $\hat{B}$ are also UUB. □

Remark 3: The last term in (20) and the second term of the third equation in (21) are used to provide robustness to disturbances.

Remark 4: To achieve high-accuracy tracking, Φ should be chosen to be small. However, a small Φ may cause control chatter. Therefore there should be a tradeoff between the desired tracking error and the discontinuity of the input tolerable.

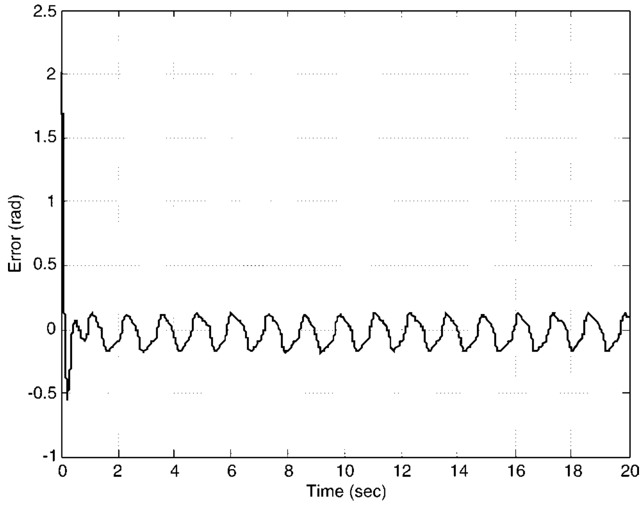

Fig. 3 *Tracking error with PD controller*

Remark 5: Equation (27) provides a practical bound on the tracking error $s_\Delta(t)$. After an initial transient period, $|s_\Delta|$ is restricted to a neighbourhood of with radius given by the right-hand side of (27). If the PD gain k_d are increased, the stability radius may be decreased.

4 Simulation results

In this section, simulation results are provided to illustrate the effectiveness of the proposed method. We simulated a DC motor with deadzone nonlinearity under feedback, using PD without deadzone precompensation and using the PD controller with robust adaptive compensation. The exact parameters used in the simulation such as J, B, d_+ and d_- are identified using standard identification method on the X-axis of X–Y table installed at Southeast University. They are $k_d = 0.1$, $\lambda = 20$, $J = 7.876 \times 10^{-3}\ \mathrm{V/rad \cdot s^{-2}}$, $B = 8.666 \times 10^{-3}\ \mathrm{V/rad \cdot s^{-1}}$, $d_+ = 0.08$ V, $d_- = -0.2$ V, $k_J = 0.001$, $k_B = 0.1$, $\boldsymbol{\Gamma} = \mathrm{diag}(10, 10)$, $k = 50$, $k^* = 0.5$ and $\Phi = 0.002$. The desired tracking trajectory is a sinusoidal reference signal, $\theta_d = 2 + \sin t + 0.5 \sin 5t$. The sample time is 0.0001 s. The initial values are all zero, that is, $\hat{J}(0) = \hat{B}(0) = \hat{d}_+(0) = \hat{d}_-(0) = 0$. The extent of parameters J and B are all selected as $[5.0 \times 10^{-3}, 1.1 \times 10^{-2}]$. For the sake of equity, the proposed controller's PD gains

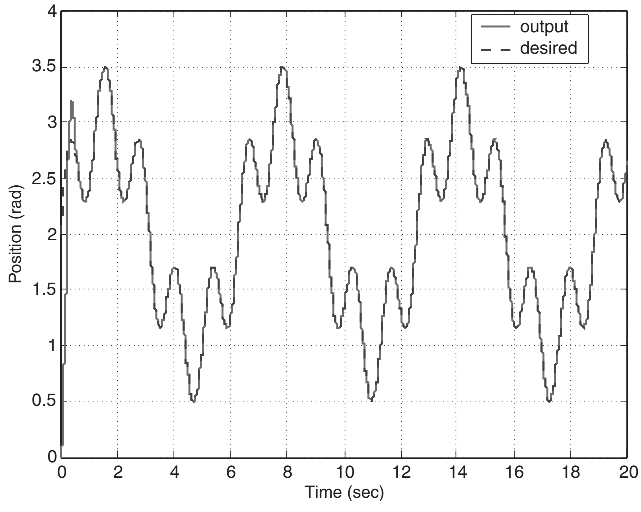

Fig. 4 *Output and the desired input*

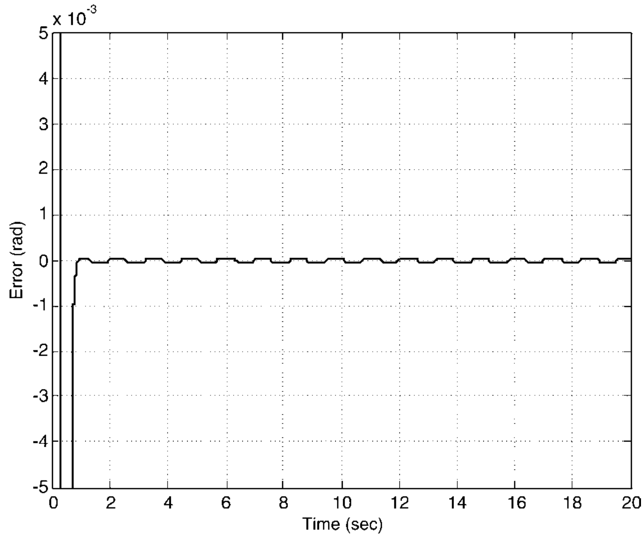

Fig. 5 *Tracking error with robust adaptive deadzone compensation*

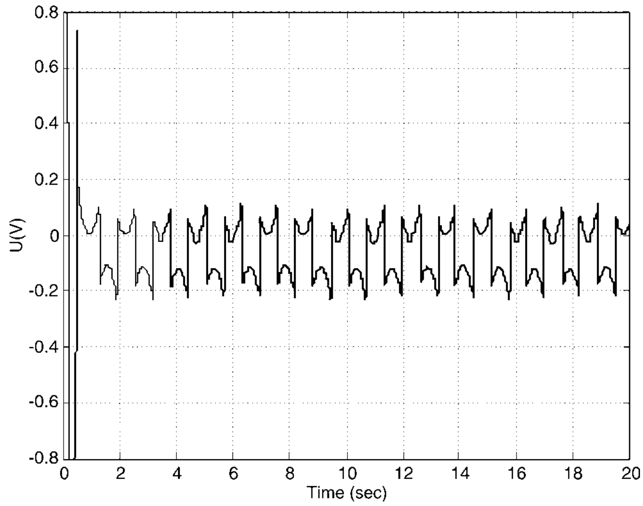

Fig. 6 *Input signal u with robust adaptive deadzone compensation*

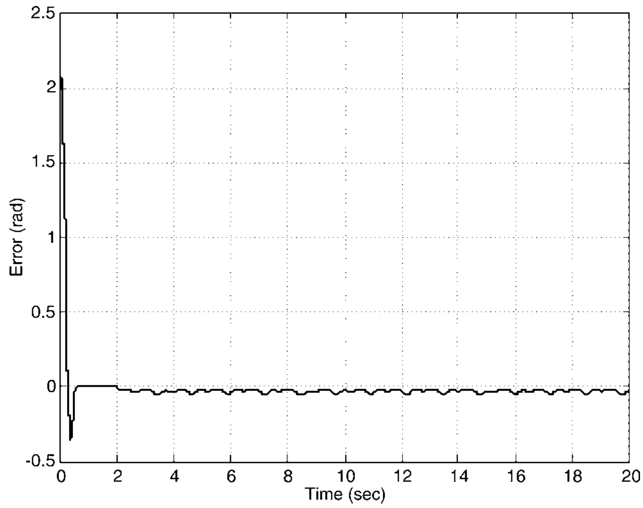

Fig. 7 *Tracking error with the robust adaptive compensation when −0.4 V disturbance is added to DC system at 2 s*

are selected to be the same as the PD controller for the adaptive scheme. Fig. 3 shows that a steady-state error or limit cycles exist when a PD controller without any deadzone compensation is applied to the DC motor system without deadzone. Figs 4 and 5 show that the tracking error is reduced and convergence rate is fast when the proposed robust adaptive controller is applied to the same system, and the tracking performance is superior when compared with the preceding PD case. Fig. 6 shows the input signal u with the proposed robust adaptive deadzone compensation. Fig. 7 shows the robust performance to disturbance when -0.4 V disturbance is added to the DC system after the system runs 2 s. The 0.4 V disturbance is large when compared with 0.8 V input saturation voltage of the linear amplifier.

5 Conclusion

In practical control systems, deadzone with unknown widths in physical components severely limit the tracking performance of control. In this paper, a robust adaptive deadzone compensator is proposed for a DC motor servo system. The deadzone parameters are learned online, resulting in a robust adaptive compensator. The designed controller also can compensate for uncertainty in both inertial and friction parameters. Because of the direct compensation of the deadzone, high precision control can be achieved using relatively low PD gains, thus resulting in high robustness. Using nonlinear stability techniques, the tuning algorithms were rigorously shown to guarantee small tracking errors as well as bounded deadzone parameter estimates. Simulation results illustrate and clarify the approach.

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