Published in IET Control Theory and Applications
Received on 21st November 2007
Revised on 6th May 2008
doi: 10.1049/iet-cta:20070435

ISSN 1751-8644

Anti-windup design: an overview of some recent advances and open problems

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Abstract: The anti-windup technique which can be used to tackle the problems of stability and performance degradation for linear systems with saturated inputs is dealt with. The anti-windup techniques which can be found in the literature today have evolved from many sources and, even now, are diverse and somewhat disconnected from one another. In this survey, an overview of many recent anti-windup techniques is provided and their connections with each other are stated. The anti-windup technique is also explained within the context of its historical emergence and the likely future directions of the field are speculated. The focus is on so-called 'modern' anti-windup techniques which began to emerge during the end of the 20th century and which allow *a priori* guarantees on stability to be made. The survey attempts to provide constructive LMI conditions for the synthesis of anti-windup compensators in both global and local contexts. Finally, some interesting extensions and open problems are discussed, such as nested saturations, the presence of time delays in the state or the input, and anti-windup for non-linear systems.

Nomenclature

For any vector $x \in \Re^n$, $x \succeq 0$ means that all the components of x, denoted $x_{(i)}$, are non-negative. For two vectors x, y of $\Re^n$, the notation $x \succeq y$ means that $x_{(i)} - y_{(i)} \geq 0$, $\forall i = 1, \ldots, n$. The elements of a matrix $A \in \Re^{m\times n}$ are denoted by $A_{(i,l)}$, $i = 1, \ldots, m$, $l = 1, \ldots, n$. $A_{(i)}$ denotes the ith row of matrix A. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A. $\mathrm{diag}(x)$ denotes a diagonal matrix obtained from vector x. I denotes the identity matrix of appropriate dimensions. $\mathrm{Co}\{.\}$ denotes a convex hull.

1 Introduction – Philosophy

Most control engineers are acutely aware of the simultaneous blessing and curse of linearity. On the one hand, linear techniques allow powerful mathematical results to be stated by combining a convenient and tractable framework for controller design. On the other hand, it is well known that linear systems are only crude approximations of the process they are purported to represent. It is thus a common practice for control engineers to design controllers which attempt to keep signals small so that deviation from the linear operating point is also small.

However, in practice, it is not always possible to ensure that all signals are small and, particularly for high-performance applications, significant control activity is often necessary. Unfortunately the actuators which deliver the control signal in physical applications are always subject to limits in their magnitude or rate. Common examples of such limits are the deflection limits in aircraft actuators, the voltage limits in electrical actuators and the limits on flow volume or rate in hydraulic actuators. Although such limits obviously restrict the performance achievable in the systems of which they are part, if these limits are not treated carefully and if the relevant controllers do not account for these limits appropriately, peculiar and pernicious behaviour may be observed. In particular, actuator saturation or rate limits have been implicated in various aircraft crashes [1] and the meltdown of the Chernobyl nuclear power station [2].

Roughly speaking, there are two approaches which one could adopt to avoid saturation problems in systems which are known to have actuator limits (i.e. the vast majority of practical systems). One approach, which we shall refer to as the *one-step* approach is, as its name implies, an approach to controller design where a (possibly non-linear) controller is designed 'from scratch'. This controller then attempts to ensure that all nominal performance specifications are met and also handles the saturation constraints imposed by the actuators. Although this approach is satisfactory in principle, and has a significant portion of the literature devoted to it, it has often been criticized because of its conservatism, lack of intuition (in terms of tuning rules etc) and lack of applicability to some practical problems.

An alternative approach to the one-step approach is to perform some separation in the controller such that one part is devoted to achieving nominal performance and the other part is devoted to constraint handling. This is the approach taken in *anti-windup compensation* which is the subject of this survey. In anti-windup compensation, a linear controller which does not *explicitly* take into account the saturation constraints is first designed, usually using standard linear design tools. Then, after this controller has been designed, a so-called *anti-windup* compensator is designed to handle the saturation constraints. The anti-windup compensator is designed to ensure that stability is maintained (at least in some region near the origin) and that less performance degradation occurs than when no anti-windup is used. Such an approach is considered attractive in practice because no restriction is placed upon the *nominal linear* controller design and, assuming no saturation is encountered, this controller alone dictates the behaviour of the linear closed loop. It is only when saturation is encountered that the anti-windup compensator becomes active and acts to modify the closed-loop's behaviour such that it is more resilient to saturation. The implication is that anti-windup techniques can be retro-fitted to existing controllers which may function very well except during saturation, making them a popular choice with practicing engineers. Let us note that there are several other methods which can be used to handle saturation non-linearities in control system design and, furthermore, that in some situations these techniques maybe more suitable than the anti-windup strategy. There is a vast literature on this more general topic (see for example [3–7] and references therein). Here we concentrate on anti-windup, and especially 'modern' anti-windup, to make our survey reasonably brief and up-to-date.

One of the problems with the emergence of anti-windup techniques is that they were developed from many sources (some unpublished) and they began to evolve in an almost organic way. Although periodically, there have been attempts to unify the results, since 2000 no real effort to continue this unification has been performed. Despite the existence of several book chapters and technical papers devoted to the presentation of new theoretical advances, there appears to be no concise overview of the existing literature allowing the reader to connect and assess the various techniques available. Thus, this survey aims to summarise the more modern anti-windup compensation techniques that have emerged, with a bias towards those which give rigorous stability guarantees. The paper is structured as follows. Section 1.1 provides a general description of anti-windup architecture, and some historical elements are given in Section 1.2. The problem is stated in Section 2. Then, some solutions in regional (local) and global contexts are presented in Section 3. In Section 3.4, some criteria and associate trade-offs relative to the optimisation issues are discussed, followed by Section 4 which presents some extensions and challenging problems to pursue in the future.

1.1 General anti-windup architecture

The basic idea underlying anti-windup designs for linear systems with saturating actuators is to introduce control modifications in order to recover, as much as possible, the performance induced by a previous design carried out on the basis of the unsaturated system. Thus, the general principle of the anti-windup scheme can be depicted in Fig. 1. In this figure, the (unconstrained) signal produced by the controller is compared with that which is actually fed into the plant (the constrained signal). This difference is then used to adjust the control strategy in a manner conducive to stability and performance preservation.

Note that the depiction in Fig. 1 is very general and over the years has been refined into the form depicted in Fig. 2. In this figure, there is a separation between the so-called 'unconstrained controller' and the 'anti-windup' compensator. As with Fig. 1, the anti-windup compensator is driven by the difference between the constrained and unconstrained control signals. To do this, the knowledge of both signals (i.e. the output produced by the linear controller and the saturated version of this) are assumed. In some case, it is not realistic because only the saturated version of the signal is known. This may be problematic for the two-stage (anti-windup) approach, and hence an observer can be used to overcome this difficulty [7]. Some discussion of this will be provided in Section 4.

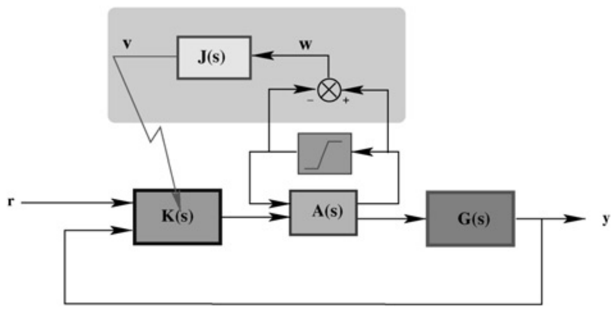

Figure 1 *General principle of anti-windup*

$G(s)$, $K(s)$, $A(s)$ and $J(s)$ are the plant, the controller, the actuator and the anti-windup controller, respectively

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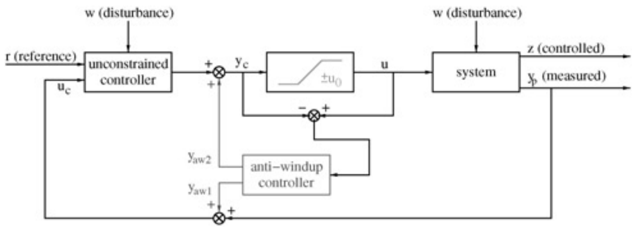

Figure 2 *Principle of anti-windup*

The anti-windup compensator itself emits two signals, one which is fed directly into the constrained control signal and one which may be used to drive the controller state equation directly. Virtually all anti-windup compensators which are present in the literature can be represented in this form and thus the anti-windup compensators discussed herein will be assumed to have this form.

Remark 1: It is noted that many anti-windup strategies inject the signal y_{aw2} directly into the controller state equation, rather than additively with the measured outputs. Although the former strategy may give more freedom in the anti-windup design, the stability and optimality conditions remain roughly similar with both strategies. Thus, not much attention is devoted to this issue.

1.2 Historical development

The study of anti-windup probably began in industry where practitioners noticed performance degradation in systems where saturation occurred. In their book, Glattfelder and Schaufelberger [8] cite boiler regulators as sources of saturation problems, although similar problems were probably found elsewhere too. Many authors note that the term 'windup' was a phenomenon associated with saturation in systems with integral controllers and alluded to the build up of charge on the integrator's capacitor during saturation. The subsequent dissipation of this charge would then cause long settling times and excessive overshoot, thereby degrading the system's performance. Modifications to the controller which avoided this charge build-up were often termed 'anti-windup' modifications and hence the term anti-windup was born. Since then however, the term 'anti-windup' has evolved and it now means the generic two-step procedure for controller design which was described earlier.

It is hard to pin-point exactly the origins of anti-windup compensation because of lack of published work on the subject in the early years of control. Teel and co-authors, in their ACC03 Workshop T-1: Modern Anti-Windup Synthesis, trace the discovery back to the 1930s and in particular cites the paper of Lozier as being one of the key early academic papers in identifying the windup problem. Teel then goes on to describe the evolution of anti-windup compensation on a time-line spanning the last 80 years or so. In a similar manner, we view the development of anti-windup as that depicted in Fig. 3 where several stages of anti-windup development are identified as described below:

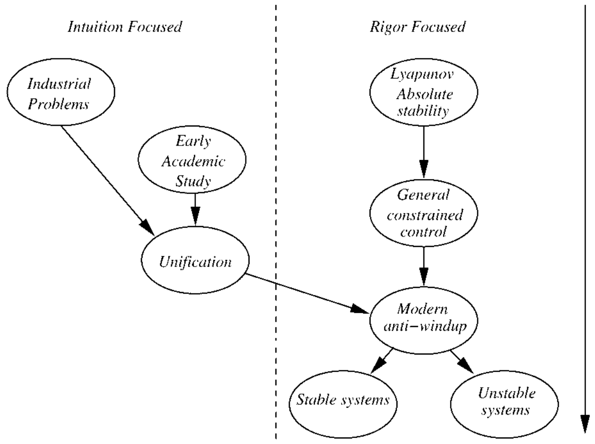

Figure 3 *The development of anti-windup compensation*

- *The first stages in anti-windup development.* It seems certain that practitioners were aware of the problems which saturation caused from the early days of control and that they adopted *ad hoc* solutions to the problem, although there appears little evidence of formal treatment. This awareness also manifests itself in some of the early work on absolute stability which was published from the 1940s onwards, and where formal treatment of the saturation problem seems to have begun. Note however, there is a separation from the anti-windup problem and the work on absolute stability, which was more general than just anti-windup, but arguably less practical.

- *Early academic study.* Some years after this, academics began to study the problem of saturation in control systems. Teel traces this back to Lozier [9] who was able to explain saturation problems in the integrator portion of PI controllers. Somewhat later, Fertik and Ross [10] proposed perhaps the first properly documented anti-windup methodology. This was also accompanied by various work on intelligent integrators [11] and later the celebrated conditioning technique of Hanus *et al.* [12–14]. This early literature heralded the beginnings of more formal studies of the saturation problem but, at this point, most papers still focused on developing *ad hoc* techniques which seemed to overcome certain practical problems, but provided no guarantees. Several years after Hanus' conditioning technique was proposed, several researchers [15, 16] provided a unification of many anti-windup schemes and it was Kothare *et al.* who [17] related this to modern robust control ideas. This unification was later continued in the work of Edwards and Postlethwaite [18] where a state-space interpretation was given in 'generic' form for many compensators.

- *Constrained input control.* Although anti-windup was being developed from a very applied perspective, new results were beginning to be developed in the more general constrained input control field, particularly from the mid-1980s onwards. Such work was focused on using Lyapunov's second method to develop *one-step* controllers which could guarantee stability of the *non-linear*

doi: 10.1049/iet-cta:20070435

closed-loop systems. Although this work was not anti-windup *per se*, this line of work provided important theoretical results which anti-windup would later benefit from. Some useful references on this subject are [19–22] and the survey [3] contains an excellent overview.

- *Modern anti-windup*. It is difficult to define exactly what constitutes a modern anti-windup technique but we view it as a *systematic* method which can be used to design an anti-windup compensator which provides rigorous guarantees of stability and or performance. Most of these techniques were developed from the late 1990s [23–28] onwards and were developed in part, thanks to the constrained input control research which preceeded them. At about this time there was also a split in the development of anti-windup controllers. Some researchers chose to investigate the problem of enforcing global stability and performance properties for anti-windup compensators [28–30] whereas others began to look at local stability and performance properties, which was necessary for unstable plants [26, 31, 32].

Note that the above description of the development of anti-windup is only a broad approximation and does not include every nuance in the subject's history. Nevertheless, it does provide a useful overview of how anti-windup developed into its present form. It is important to note that it has developed from the problem-specific *ad hoc* solutions [10, 15] aimed particularly at PID controllers, to sophisticated synthesis methods which populate today's anti-windup literature. In particular, major improvements in this field have been achieved in the last decade, as can be observed in [23, 24, 33–38] among others.

Many LMI-based approaches now exist to adjust the anti-windup gains in a systematic way (see, for example, [39] for a quick overview). Most often, these are based on the optimisation of either a stability domain [31, 32] or a non-linear $\mathcal{L}_2$-induced performance level [40–42]. More recently, based on the linear fractional transformation/linear parameter varying (LFT/LPV)-framework, extended anti-windup schemes were proposed (see [41, 43, 44]). In these contributions, the saturations are viewed as sector non-linearities and anti-windup controller design is recast into a convex optimisation problem under LMI constraints. Following a similar path, alternative techniques using less conservative representations of the saturation non-linearities, yet with sector non-linearities, are proposed in [31, 42, 45, 46]. Moreover, during the last phases previously evoked, we can point out several papers dealing with practical experiments with application of anti-windup strategies in various fields like aeronautical or spatial domains [46, 47–50], mechanical domains [51, 52], open water channels [53], nuclear fusion [54], telecommunication networks [55, 56] and hard disk drive control [57].

Hence, it is clear that the use of anti-windup strategies is of a real interest for the engineers in various domains because of its potential to provide, for example:

1. Reduction of validation costs of control laws

2. Better use of actuator (and/or sensor) capacity

3. The possibility for control engineers to become involved in the design process at a much earlier stage in order to help choose actuators/sensors which have a reduced size, mass etc.

2 Problem statement

Consider the continuous-time linear plant as shown in Fig. 2 (For simplicity, the time dependence in the vector will be omitted.)

$$\begin{aligned} \dot{x}_{\mathrm{p}} &= A_{\mathrm{p}}x_{\mathrm{p}} + B_{\mathrm{pu}}u + B_{\mathrm{pw}}w \\ y_{\mathrm{p}} &= C_{\mathrm{p}}x_{\mathrm{p}} + D_{\mathrm{pu}}u + D_{\mathrm{pw}}w \\ z &= C_{z}x_{\mathrm{p}} + D_{\mathrm{zu}}u + D_{\mathrm{zw}}w \end{aligned} \tag{1}$$

where $x_{\mathrm{p}} \in \Re^{n}$, $u \in \Re^{m}$, $w \in \Re^{q}$ and $y_{\mathrm{p}} \in \Re^{p}$ are the state, the input, the exogenous input and the measured output vectors of the plant, respectively. $z \in \Re^{l}$ is the regulated output vector used for performance purposes. Matrices A_{p}, B_{pu}, B_{pw}, C_{p}, C_{z}, D_{pu}, D_{pw}, D_{zu} and D_{zw} are real constant matrices of appropriate dimensions. Pairs $(A_{\mathrm{p}}, B_{\mathrm{pu}})$ and $(C_{\mathrm{p}}, A_{\mathrm{p}})$ are assumed to be controllable and observable, respectively.

Considering system (1), we assume that an n_{c}'th-order dynamic output stabilising compensator

$$\begin{aligned} \dot{x}_{\mathrm{c}} &= A_{\mathrm{c}}x_{\mathrm{c}} + B_{\mathrm{c}}u_{\mathrm{c}} + B_{\mathrm{cw}}w \\ y_{\mathrm{c}} &= C_{\mathrm{c}}x_{\mathrm{c}} + D_{\mathrm{c}}u_{\mathrm{c}} + D_{\mathrm{cw}}w \\ u_{\mathrm{c}} &= y_{\mathrm{p}} \end{aligned} \tag{2}$$

where $x_{\mathrm{c}} \in \Re^{n_{\mathrm{c}}}$ is the controller state, $u_{\mathrm{c}} \in \Re^{p}$ is the controller input and $y_{\mathrm{c}} \in \Re^{m}$ is the controller output, has been designed in order to guarantee some performance requirements and the stability of the closed-loop system in the absence of control saturation.

Indeed, it is important to note that the interconnection considered to compute the stabilising controller (2) is the linear interconnection defined as follows:

$$u = y_{\mathrm{c}} = C_{\mathrm{c}}x_{\mathrm{c}} + D_{\mathrm{c}}y_{\mathrm{p}} + D_{\mathrm{cw}}w \tag{3}$$

Remark 2: Systems (1) and (2) are assumed to be well-posed. Hence, the interconnection (3) is defined from

$$y_{\mathrm{c}} = \Delta C_{\mathrm{c}}x_{\mathrm{c}} + \Delta D_{\mathrm{c}}C_{\mathrm{p}}x_{\mathrm{p}} + \Delta(D_{\mathrm{c}}D_{\mathrm{pw}} + D_{\mathrm{cw}})w \tag{4}$$

with $\Delta = (I - D_{\mathrm{c}}D_{\mathrm{pu}})^{-1}$.

Remark 3: By assumption the closed-loop without saturation (with connection (3)) is supposed internally

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doi: 10.1049/iet-cta:20070435

stable and well-posed. In other words, the closed-loop matrix $\mathbb{A}$ defined as

$$\mathbb{A} = \begin{bmatrix} A_p + B_{pu}\Delta D_c C_p & B_{pu}\Delta C_c \\ B_c(I + D_{pu}\Delta D_c)C_p & A_c + B_c D_{pu}\Delta C_c \end{bmatrix} \quad (5)$$

with Δ defined in Remark 2, is supposed to be Hurwitz, i.e. in the absence of control bounds, the closed-loop system would be globally stable.

Suppose now that the input vector u is subject to amplitude limitations as follows:

$$-u_{0(i)} \leq u_{(i)} \leq u_{0(i)}, \quad u_{0(i)} > 0, \quad i = 1, \ldots, m \quad (6)$$

As a consequence of the control bounds, the actual control signal to be injected in the system is a saturated one, that is, the real interconnection between the plant (1) and the controller (2) is a non-linear one described by

$$\begin{aligned} u &= \mathrm{sat}_{u_0}(y_c) \\ &= \mathrm{sat}_{u_0}(\Delta C_c x_c + \Delta D_c C_p x_p + \Delta(D_c D_{pw} + D_{cw})w) \end{aligned} \quad (7)$$

In (7), each component of the saturation term $\mathrm{sat}_{u_0}(y_c)$ is classically defined $\forall i = 1, \ldots, m$ by

$$\mathrm{sat}_{u_0}(y_{c(i)}) = \mathrm{sign}(y_{c(i)}) \min(|y_{c(i)}|, u_{0(i)}) \quad (8)$$

In order to mitigate the undesirable effects of windup, caused by input saturation [58], one can consider an anti-windup controller defined as follows

$$\begin{aligned} \dot{x}_{aw} &= A_{aw}x_{aw} + B_{aw}(\mathrm{sat}_{u_0}(y_c) - y_c) \\ y_{aw} &= C_{aw}x_{aw} + D_{aw}(\mathrm{sat}_{u_0}(y_c) - y_c) \end{aligned} \quad (9)$$

where $x_{aw} \in \Re^{n_{aw}}$ is the anti-windup state, $u_{aw} = (\mathrm{sat}_{u_0}(y_c) - y_c)$ is the anti-windup input and $y_{aw} \in \Re^{n_c+m}$ is the anti-windup output. This anti-windup output is more precisely defined as follows

$$\begin{aligned} y_{aw} &= \begin{bmatrix} y_{aw1} \\ y_{aw2} \end{bmatrix} \in \Re^{n_c+m} \\ y_{aw1} &= \begin{bmatrix} I_{n_c} & 0 \end{bmatrix} y_{aw} \\ y_{aw2} &= \begin{bmatrix} 0 & I_m \end{bmatrix} y_{aw} \end{aligned} \quad (10)$$

Such an anti-windup controller can be added to the controller through y_{aw} which can act both on the dynamics of the dynamic controller (through y_{aw1}) and on its output (through y_{aw2}) [42, 59, 60]. Thus, considering the dynamic controller and this anti-windup strategy, the closed-loop system reads

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_{pu}\mathrm{sat}_{u_0}(y_c) + B_{pw}w \\ y_p &= C_p x_p + D_{pu}\mathrm{sat}_{u_0}(y_c) + D_{pw}w \\ z &= C_z x_p + D_{zu}\mathrm{sat}_{u_0}(y_c) + D_{zw}w \\ \dot{x}_c &= A_c x_c + B_c y_p + B_{cw}w + y_{aw1} \\ y_c &= C_c x_c + D_c y_p + D_{cw}w + y_{aw2} \\ \dot{x}_{aw} &= A_{aw}x_{aw} + B_{aw}(\mathrm{sat}_{u_0}(y_c) - y_c) \\ y_{aw1} &= \begin{bmatrix} I_{n_c} & 0 \end{bmatrix}(C_{aw}x_{aw} + D_{aw}(\mathrm{sat}_{u_0}(y_c) - y_c)) \\ y_{aw2} &= \begin{bmatrix} 0 & I_m \end{bmatrix}(C_{aw}x_{aw} + D_{aw}(\mathrm{sat}_{u_0}(y_c) - y_c)) \end{aligned} \quad (11)$$

Remark 4: To remove the presence of implicit loop in the closed-loop system because of y_{aw2}, a simplified anti-windup controller can be considered by injecting only the anti-windup output in the dynamics of x_c. In this case, the building y_{aw} is such that $y_{aw} \in \Re^{n_c}$.

The system (11) can be more concisely written as

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \sim \begin{cases} \dot{\xi} = \mathcal{A}\xi + \mathcal{B}_1 \mathrm{sat}_{u_0}(y_c) + \mathcal{B}_2 w \\ z = \mathcal{C}_1\xi + \mathcal{D}_{11}\mathrm{sat}_{u_0}(y_c) + \mathcal{D}_{12}w \\ y_c = \mathcal{C}_2\xi + \mathcal{D}_{21}\mathrm{sat}_{u_0}(y_c) + \mathcal{D}_{22}w \end{cases} \quad (12)$$

where $P(s)$ represents the linear part of the system, and the extended state vector ξ is defined as

$$\xi = \begin{bmatrix} x_p \\ x_c \\ x_{aw} \end{bmatrix} \in \Re^{n+n_c+n_{aw}} \quad (13)$$

The state-space matrices $\mathcal{A}$, $\mathcal{B}_1$ etc. are constructed from the plant, controller and anti-windup state-space matrices. A schematic view of this representation is shown in Fig. 4.

It is well known within the literature on constrained control that any system containing a saturation nonlinearity can be re-written as one containing a dead-zone non-linearity using the identity (Note that other authors define

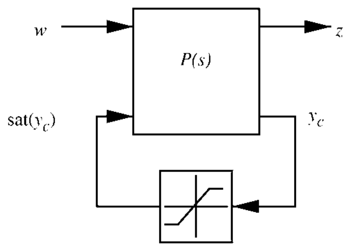

Figure 4 *Closed-loop system*

a dead-zone non-linearity as $Dz(v) = v - \mathrm{sat}_{u_0}(v)$.)

$$\mathrm{sat}_{u_0}(v) = v + \phi(v) \quad v \in \Re^m \tag{14}$$

Thus, using this identity an equivalent state-space realisation of system (11) (and equivalently system (12)) is

$$\tilde{P}(s) = \begin{bmatrix} \tilde{P}_{11}(s) & \tilde{P}_{12}(s) \\ \tilde{P}_{21}(s) & \tilde{P}_{22}(s) \end{bmatrix} \sim \begin{cases} \dot{\xi} = \tilde{\mathcal{A}}\xi + \tilde{\mathcal{B}}_1\phi(y_c) + \tilde{\mathcal{B}}_2 w \\ z = \tilde{\mathcal{C}}_1\xi + \tilde{\mathcal{D}}_{11}\phi(y_c) + \tilde{\mathcal{D}}_{12}w \\ y_c = \tilde{\mathcal{C}}_2\xi + \tilde{\mathcal{D}}_{21}\phi(y_c) + \tilde{\mathcal{D}}_{22}w \end{cases} \tag{15}$$

Again the state-space matrices $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}_1$ etc. are constructed from the plant, controller and anti-windup state-space matrices; they can also be determined directly from the state-space matrices given in equation (12). A schematic view of this representation is shown in Fig. 5. It is now possible to identify two issues with the system (12) (and therefore the system (15)): the stability and performance problems. When $w = 0$, it is of interest to estimate the basin of attraction of system (11), denoted $\mathcal{B}_a$ which is defined as the set of all $\xi \in \Re^{n+n_c+n_{aw}}$ such that for any $\xi(0)$ belonging to $\mathcal{B}_a$, the corresponding trajectory converges asymptotically to the origin. In particular, when, global stability of the system is ensured the basin of attraction corresponds to the whole state space. However, more generally, the exact characterisation of the basin of attraction is not possible. In this case, it is important to obtain estimates of the basin of attraction. In this sense, regions of asymptotic stability can be used to estimate the basin of attraction [61]. On the other hand, in some practical applications one can be interested in ensuring the stability for a given set of admissible initial conditions. This set can be seen as a practical operating region for the system, or a region where the states of the system can be brought by the action of temporary disturbances.

In the case where $w = 0$, one of the problems of interest with respect to the closed-loop system (11) modified by the addition of the two static anti-windup loops y_{aw1} and y_{aw2} consists of computing the anti-windup gains in order to enlarge the basin of attraction of the resulting closed-loop system. In the case where $w \neq 0$, the problem of interest is

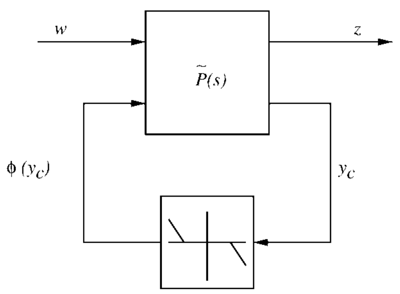

Figure 5 *Closed-loop system*

to ensure a certain level of performance which can be measured from the finite $\mathcal{L}_2$ gain from the exogenous input w to the performance output z. In this case, the problem can then be formulated as follows.

Problem 1: Determine the anti-windup matrices A_{aw}, B_{aw}, C_{aw} and D_{aw} and a region of asymptotic stability, denoted $\mathcal{E}_0$, as large as possible, such that

1. The closed-loop system (11) with $w = 0$ is asymptotically stable for any initial condition belonging to the set $\mathcal{E}_0$.

2. The map from w to z is finite $\mathcal{L}_2$ gain stable with gain $\gamma > 0$.

Note that the implicit objective in the first item of Problem 1 is to optimise the size of the basin of attraction for the closed-loop system (11) (with $w = 0$) over the choice of matrices A_{aw}, B_{aw}, C_{aw} and D_{aw}. This can be accomplished indirectly by searching for an anti-windup compensator defined from A_{aw}, B_{aw}, C_{aw} and D_{aw} that leads to a region of stability for the closed-loop system as large as possible. Considering quadratic Lyapunov functions and ellipsoidal regions of stability, the maximisation of the region of stability can be accomplished by using some well known size optimisation criteria for ellipsoidal sets, such as: minor-axis maximisation, volume maximisation, or even the maximisation of the ellipsoid in certain given directions. On the other hand, when the open-loop system is asymptotically stable, it can be possible to search for the controller matrices in order to guarantee the global asymptotic stability of the origin of the closed-loop system.

Remark 5: Problem 1 can be studied in the context of a static anti-windup gain by considering $n_{aw} = 0, A_{aw} = 0$, $B_{aw} = 0$, $C_{aw} = 0$ and by computing the gain D_{aw}.

Remark 6: Points 1 and 2 of Problem 1 can be studied in an analysis context by considering the saturated closed-loop system without anti-windup loop i.e. $n_{aw} = 0, A_{aw} = 0$, $B_{aw} = 0$, $C_{aw} = 0$ and $D_{aw} = 0$. This allows one to measure the improvement because of the anti-windup loop.

In the sequel, some results to address Problem 1 will be presented in the case of full-order or low- (or reduced-) order anti-windup controller. At this stage, it is very important to underline that the notion of full-order anti-windup has different meanings depending on the authors in the literature. For example, in [44, 62, 63], the authors use full order to mean plant order (i.e. $n_{aw} = n$). In contrast, in [60, 64], the authors use full order to mean $n_{aw} = n + n_c$. Both cases will be discussed.

Throughout the paper the class of disturbance vector under consideration is assumed to be limited in energy, that is

IET Control Theory Appl., 2009, Vol. 3, No. 1, pp. 1–19
doi: 10.1049/iet-cta:20070435

$w \in \mathcal{L}_2^q$ and for some scalar δ, $0 < (1/\delta) < \infty$, one gets

$$\|w\|_2^2 = \int_0^\infty w(s)' w(s) \mathrm{d}s \le \delta^{-1} \tag{16}$$

Note that a particular case can be studied by considering that the exogenous signal (which can represent reference signals or disturbance) is generated by a linear equation as follows (see, for example, [46])

$$\tau \dot{w} + w = 0, \quad w(0) = w_0 \in \Re^q \tag{17}$$

Thus, it is easily verified for small values of τ, that the exogenous signals w can be interpreted as bounded step inputs. This bound is clearly fixed by $\|w_0\|$. Furthermore, w is also $\mathcal{L}_2$ bounded as according to (16) one gets

$$\|w\|_2^2 = \int_0^\infty w(s)' w(s) \mathrm{d}s = \frac{w_0}{2\tau} \tag{18}$$

3 Some solutions

3.1 Preliminary results

Let us first consider the non-linear operator $\phi(.)$ in $\Re^m$ which is characterised as follows

$$\phi(y) = \begin{bmatrix} \phi(y_{(1)}) & \ldots & \phi(y_{(m)}) \end{bmatrix}' \tag{19}$$

where $\phi(.)$ is a dead-zone non-linearity. Furthermore, each element $\phi(y_{(i)})$, $i = 1, \ldots, m$, is defined by

$$\phi(y_{(i)}) = \begin{cases} 0 & \text{if } |y_{(i)}| \le u_{0(i)} \\ u_{0(i)} - y_{(i)} & \text{if } y_{(i)} > u_{0(i)} \\ -u_{0(i)} - y_{(i)} & \text{if } y_{(i)} < -u_{0(i)} \end{cases} \tag{20}$$

By definition, $\phi(.)$ is a decentralised and memoryless operator.

As noted earlier, it is important to underline that every system, which involves saturation-type non-linearities, may be easily rewritten with dead-zone non-linearities. Indeed, considering a saturation function $\mathrm{sat}_{u_0}(y)$, the resulting dead-zone non-linearity $\phi(y)$ is obtained from $\phi(y) = \mathrm{sat}_{u_0}(y) - y$, which can be depicted in Fig. 6.

Several ways to mathematically represent the saturation can be considered to derive constructive conditions (in the sense of being able to associate to them numerical procedures) of stability/stabilisation based on the use of Lyapunov functions. Hence, the exact representation through regions of saturation consists of dividing the state space into 3^m regions [65, 66]. Such a representation is mainly used for stability analysis purpose, and in general in the case of systems with only few inputs, because of the complexity of the resulting conditions. A modelling

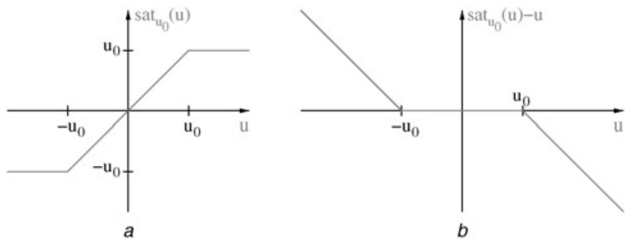

Figure 6 *Saturation function and its associated dead-zone non-linearity*

a Saturation function
b Associate dead-zone non-linearity

approach based on linear differential inclusion (LDI) and leading to polytopic models can also be used [5, 67, 68]. The main drawback of using LDI models is that the conditions allowing the computation of the anti-windup gains typically lead to bilinear matrix inequalities (BMIs) [32].

Let us then focus on the representation through modified sector conditions. Hence, in this context, let us define the following polyhedral set

$$S(u_0) = \{y \in \Re^m, \omega \in \Re^m; \; -u_{0(i)} \le y_{(i)} - \omega_{(i)} \le u_{0(i)}, \; i = 1, \ldots, m\} \tag{21}$$

Lemma 1: [69]: If y and ω are elements of $S(u_0)$ then the non-linearity $\phi(y)$ satisfies the following inequality

$$\phi(y)' S^{-1}(\phi(y) + \omega) \le 0 \tag{22}$$

for any diagonal positive definite matrix $S \in \Re^{m \times m}$.

Remark 7: Particular formulations of Lemma 1 can be found in [31] (concerning the case of systems with a single saturation function) and in [45, 50] (concerning systems presenting both amplitude- and dynamics-restricted actuators).

It should be pointed out that $\omega = \Lambda y$, where Λ is a diagonal matrix such that $0 < \Lambda \le I$, (see for instance [61, 70]), is a particular case of the generic formulation (22). A key advantage of condition (22) is that, contrary to the classical case with $\omega = \Lambda y$, it allows the formulation of conditions directly in LMI form. Moreover, Lemma 1 caters easily for nested saturations.

Remark 8: Particular formulations of Lemma 1 can be stated by considering

$$S_\omega(u_0) = \{\omega \in \Re^m; \; -u_{0(i)} \le \omega_{(i)} \le u_{0(i)}, \; i = 1, \ldots, m\} \tag{23}$$

$$-\phi(y)' S^{-1}(\mathrm{sat}(y) + \omega) \ge 0 \tag{24}$$

instead of (21) and (22), respectively. Such a formulation is used in [42] and [71] and gives simplified conditions in the case when y depends on $\phi(y)$ leading to implicit function and nested conditions.

Lemma 1, as written, is rather dedicated to the regional case. It can be considered in a global context. For this, it suffices to consider $\omega = y$ and therefore set $S(u_0)$ is the state space. Hence, the sector condition (22) is globally satisfied. Note, however, that the global stability of the closed-loop system subject to such a non-linearity will be obtained only if some assumptions on the stability of the open-loop system are verified.

3.2 Description of the conditions in the local case

Only a few papers have been dedicated to anti-windup strategy for exponentially unstable systems, that is in a local context, until around 2000. The majority of those papers presented algorithms for computing anti-windup compensators but without any guarantees about stability: see, for example [72] and [24]. However, a key element in the local case is the ability to guarantee the stability and therefore to characterise the region of stability for the closed-loop system (11). One of the first paper addressing clearly the local case with a guarantee of stability was Teel's paper [35]. In [35], an algorithm, which requires measurement of the exponentially unstable modes, was proposed. The results provided an anti-windup compensator extending those presented in [24] by removing some restrictions on the transient behavior of the unsaturated feedback loop. In [35], the conditions were not however in an LMI form. Indeed, one of the first applications of LMI to the anti-windup synthesis problem in the local was given in [27, 31] by considering only static anti-windup loop (A_{aw}, B_{aw}, C_{aw} were all matrices of zero row and/or column dimension and only D_{aw} was sought). Gomes da Silva Tarbouriech [31] followed in particular the papers [32, 73] in which the conditions proposed are not into LMI form but in BMI form due mainly to the way chosen to model the saturation terms based on LDIs or classical sector conditions.

In terms of the notation used by us, perhaps the main result derived from [31] to solve Problem 1 in the context of static anti-windup case with $w = 0$ can be stated as follows. With this aim, let us write the closed-loop system (11) in this case

$$\begin{aligned} \dot{\bar{\xi}} &= \mathbb{A}\bar{\xi} + (\mathbb{B}_1 + \mathbb{R}_1 D_{aw})\phi(y_c) \\ y_c &= \mathbb{C}_2\bar{\xi} + (\mathbb{D}_{21} + \mathbb{R}_2 D_{aw})\phi(y_c) \end{aligned} \tag{25}$$

with $\mathbb{A}$ defined in (5), Δ defined in Remark 2 and

$$\begin{aligned} &\bar{\xi} = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}; \mathbb{B}_1 = \begin{bmatrix} B_{pu}(I_m + \Delta D_c D_{pu}) \\ B_C D_{pu}(I_m + \Delta D_c D_{pu}) \end{bmatrix}; \\ &\mathbb{R}_1 = \begin{bmatrix} B_{pu}\Delta[0 \quad I_m] \\ D_c D_{pu}\Delta[0 \quad I_m] + [I_{n_c} \quad 0] \end{bmatrix} \\ &\mathbb{C}_2 = [\Delta D_c C_p \quad \Delta C_c]; \mathbb{D}_{21} = \Delta D_c D_{pu}; \\ &\mathbb{R}_2 = \Delta[0 \quad I_m] \end{aligned} \tag{26}$$

Theorem 1: If there exist a symmetric positive definite matrix $W \in \Re^{(n+n_c)\times(n+n_c)}$, a matrix $Y \in \Re^{m\times(n+n_c)}$, a matrix $Z \in \Re^{n_c\times m}$ and a diagonal positive definite matrix $S \in \Re^{m\times m}$ satisfying

$$\begin{bmatrix} W\mathbb{A}' + \mathbb{A}W & \mathbb{B}_1 S + \mathbb{R}_1 Z - W\mathbb{C}_2' - Y' \\ \star & -2S - \mathbb{D}_{21}S - S\mathbb{D}_{21}' - \mathbb{R}_2 Z - Z'\mathbb{R}_2' \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix} W & Y'_{(i)} \\ \star & u^2_{0(i)} \end{bmatrix} \geq 0, \quad i = 1, \ldots, m \tag{28}$$

then the gain matrix $D_{aw} = ZS^{-1}$ is such that the ellipsoid $\mathcal{E}(W) = \{\bar{\xi} \in \Re^{n+n_c}; \bar{\xi}' W^{-1}\bar{\xi} \leq 1\}$ is an asymptotic stability region for system (25), or equivalently for system (11) with $w = 0$.

Remark 9: LMI conditions stated in Theorem 1 are obtained by using the particular formulation of Lemma 1 described in Remark 8with $\omega = YW^{-1}\bar{\xi}$.

Theorem 1 can be easily extended to the case of the complete system (11), i.e. with $w \neq 0$, in order to design static anti-windup compensator solution to Problem 1. Furthermore, in [74], an extension of [63] is proposed allowing the computation of dynamic anti-windup compensators. Nevertheless, contrary to [31], the region in which the stability of the closed-system is guaranteed is not clearly described. Recently, several papers dealing with performance, like $\mathcal{L}_2$ performance, have been published mainly in the context of dynamic anti-windup compensator design: see, for example, [7] in which the first 6 chapters are dedicated to anti-windup strategies and their applications. See also [42, 64, 71].

Let us express a general result addressing Problem 1 with respect to system (11) using notation given in (15). For simplicity we consider the case $\xi(0) = 0$.

Theorem 2: If there exist a symmetric positive definite matrix $W \in \Re^{(n+n_c+n_{aw})\times(n+n_c+n_{aw})}$, a matrix $Y \in \Re^{m\times(n+n_c+n_{aw})}$, a diagonal positive definite matrix $S \in \Re^{m\times m}$ and two

doi: 10.1049/iet-cta:20070435

positive scalars δ and γ satisfying

$$\begin{bmatrix} W\tilde{\mathcal{A}}' + \tilde{\mathcal{A}}W & \tilde{\mathcal{B}}_1 S - W\tilde{\mathcal{C}}_2' - Y' & \tilde{\mathcal{B}}_2' & W\tilde{\mathcal{C}}_1' \\ \star & -2S - \tilde{\mathcal{D}}_{21}S - S\tilde{\mathcal{D}}_{21}' & -\tilde{\mathcal{D}}_{22} & S\tilde{\mathcal{D}}_{11}' \\ \star & \star & -I_q & \tilde{\mathcal{D}}_{12}' \\ \star & \star & \star & -\gamma I_l \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} W & Y'_{(i)} \\ \star & \delta u^2_{0(i)} \end{bmatrix} \geq 0, \quad i = 1, \ldots, m \tag{30}$$

then there exists an anti-windup compensator (9) which solves Problem 1, that is, such that

1. when $w(t) \neq 0$, the closed-loop trajectories remain bounded in the set $\mathcal{E}(W) = \{\xi \in \Re^{n+n_c+n_{aw}};\ \xi' W^{-1}\xi \leq \delta^{-1}\}$ and $\|z\|_2^2 \leq \gamma \|w\|_2^2$.

2. when $w = 0$, the closed-loop trajectories asymptotically converge to the origin.

Theorem 2 is not constructive; it does not give information on how to obtain a suitable anti-windup compensator. At this stage, results can be derived, for example, from [42, 59, 60, 64, 71], in both full- and reduced-order cases. Let us underline that in the full-order case, the conditions are LMIs (in both cases $n_{aw} = n + n_c$ or $n_{aw} = n$) . In the reduced-order case, it is interesting to note that the computation of matrices A_{aw} and C_{aw} amounts to solving a problem similar to the static output gain design, whereas the computation of matrices B_{aw} and D_{aw} amounts to solving a problem similar to the static state feedback gain design. Indeed, the condition (29) is non-convex in the decision variables A_{aw} and C_{aw} (BMI), and convex in the the decision variables B_{aw} and D_{aw} (LMI). Condition (29) becomes convex as soon the matrices A_{aw} and C_{aw} are fixed. In particular, in the context of disturbances satisfying (17), detailed procedures are provided in [64] and have been applied to the on-ground aircraft control design in the context of AIRBUS transport aircraft [75].

3.3 Description of the conditions in the global case

Solutions to the anti-windup problem in the local case are typically of somewhat higher complexity than the global anti-windup problem because a key element of their solution requires some sort of description of the saturated system's region of attraction. The geometry of this region is generally not easy to describe exactly and thus, as mentioned previously, normally the region is estimated using an ellipsoid or a polyhedral. For stable linear plants (i.e. $\Re e\lambda_i(A_p) < 0\,\forall i$) it is possible to develop anti-windup compensators which go beyond local guarantees and provide stability for all $\xi \in \Re^{n+n_c+n_{aw}}$. This simplifies the anti-windup problem somewhat as now a description of the region of attraction is not required, as it is the whole state-space. This section will discuss the global anti-windup problem.

Problem 1 began to become numerically tractable for stable linear systems when LMI's became established in the control literature. Although in principle the design of anti-windup compensators for stable linear systems subject to input saturation could be achieved using absolute stability tools such as the Circle and Popov criteria [61], these were most useful for single-loop systems and analysis. Design was somewhat harder until LMIs began to emerge, although useful classical tools are reported in [76, 77].

3.3.1 Early LMI-based methods: One of the first applications of LMIs to the anti-windup synthesis problem was given by [78] which considered the anti-windup synthesis problem as an application of absolute stability theory involving common Lyapunov functions. Marcopoli and Philips [78] followed Kothare *et al.* [17] by considering only static anti-windup compensators, namely A_{aw}, B_{aw}, C_{aw} were all matrices of zero row and/or column dimension and only D_{aw} was sought. Although this is somewhat restrictive, it was a common assumption at the time. Instead of using the standard sector bound to represent the dead-zone non-linearity, Marcopoli and Philips [78] chose instead to represent the relationships between the input and outputs of each element of the dead-zone as a non-linear gain follows

$$\frac{\phi_i(y_{c,i})}{y_{c,i}} = k_i(y_{c,i}) \quad i \in \{1, \ldots, m\} \tag{31}$$

Note that $k_i(.) \in [0, 1]$ and that, as the dead-zone is, by definition, decentralised, the operator $\phi(y_c)$ can therefore be represented as a nonlinear matrix gain

$$K(y_c) = \begin{bmatrix} k_1(y_{c,1}) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_m(y_{c,m}) \end{bmatrix}$$
$$k_i(.) \in [0, 1] \quad i = \{1, \ldots, m\} \tag{32}$$

Note that $K(y_c)$ defines a polytope of matrices and [78] noted that in order to check stability of the non-linear system, it was sufficient to check the stability of every vertex in the polytope. The polytope vertices are given by the 2^m diagonal matrices, K^j with either 0 or 1 as their diagonal entries. Thus [78] replaced system (12) with the following systems (assuming $\mathcal{D}_{21} = 0$ to avoid implicit equations)

$$\begin{aligned} \dot{\xi} &= (\tilde{\mathcal{A}} + \tilde{\mathcal{B}}_1 K^j \tilde{\mathcal{C}}_2)\xi + (\tilde{\mathcal{B}}_2 + \tilde{\mathcal{B}}_1 K^j \tilde{\mathcal{D}}_{22})w \\ &\quad j \in \{1, \ldots, 2^m\} \\ z &= (\tilde{\mathcal{C}}_1 + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{C}}_2)\xi + (\tilde{\mathcal{D}}_{12} + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{D}}_{22})w \end{aligned} \tag{33}$$

Thus [78] stated the following theorem which could be used to analyse the stability of system (33) and obtain an upper bound on its $\mathcal{L}_2$ gain. This can be re-stated as follows.

Theorem 3: The system (33) is globally asymptotically (exponentially) stable when $w = 0$ and has $\mathcal{L}_2$ gain less than γ if the following LMIs in $P = P' > 0$, $\mu = \gamma^2 > 0$ is satisfied.

$$\begin{bmatrix} (\tilde{\mathcal{A}} + \tilde{\mathcal{B}}_1 K^j \tilde{\mathcal{C}}_2)'P + P(\tilde{\mathcal{A}} + \tilde{\mathcal{B}}_1 K^j \tilde{\mathcal{C}}_2) + (\tilde{\mathcal{C}}_1 + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{C}}_2)'(\tilde{\mathcal{C}}_1 + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{C}}_2) & \star \\ (\tilde{\mathcal{B}}_2 + \tilde{\mathcal{B}}_1 K^j \tilde{\mathcal{D}}_2 2)'P + (\tilde{\mathcal{D}}_{12} + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{D}}_{22})'(\tilde{\mathcal{C}}_1 + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{C}}_2) & -\mu I + (\tilde{\mathcal{D}}_{12} + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{D}}_{22})'(\tilde{\mathcal{D}}_{12} + \tilde{\mathcal{D}}_{11} K^j \tilde{\mathcal{D}}_{22}) \end{bmatrix} < 0 \tag{34}$$

The main problem with Theorem 3 is that it is useful mainly for analysis, that is if a compensator D_{aw} is assumed to be given. Unfortunately as the matrices $\mathcal{A}, \mathcal{B}_1$ etc are affine functions of the matrix D_{aw}, if this is allowed to be a variable, the above matrix inequality becomes *bilinear* and therefore difficult to solve. To overcome this [78] suggested that the anti-windup compensator design process to be split into an *analysis* stage in which D_{aw} is fixed and P, γ which solve the (linear) matrix inequality (34) are found; then in the *synthesis* stage $P > 0$ is fixed and D_{aw}, $\gamma > 0$ are sought instead. Thus the process is a so-called iterative LMI solution (I-LMI). Although no guarantees of convergence of this procedure are given, it appeared to work well in examples given in [78] although the procedure in general is not numerically sound.

Similar ideas to the above were also exploited in [79] where the observer-based structure of anti-windup compensators was used (i.e. again the anti-windup compensator was static). In Romanchuk's paper however, the 2^m matrix inequalities were replaced by simply two inequalities corresponding to all $k_i = 0$ and all $k_i = 1$, which have the interpretation of the nominal open-loop and nominal closed-loop systems without saturation. The inequalities were given for stability only and, in our terminology can be stated as

$$\tilde{\mathcal{A}}'P + P\tilde{\mathcal{A}} < 0 \tag{35}$$

$$(\tilde{\mathcal{A}} + \tilde{\mathcal{B}}_1 \tilde{\mathcal{C}}_2)'P + P(\tilde{\mathcal{A}} + \tilde{\mathcal{B}}_1 \tilde{\mathcal{C}}_2) < 0 \tag{36}$$

However, the paper's emphasis on LMIs is again slightly misleading as again the matrix $\mathcal{B}_1\mathcal{C}_2$ is an affine function of the anti-windup compensator gain D_{aw}, leading in general to bilinear matrix inequalities. Romanchuk thus suggested searching for the anti-windup parameter, D_{aw}, and then using the above (linear) matrix inequalities to check for quadratic stability. The anti-windup parameter search is done using incremental gain ideas, and appears most suitable for single loop anti-windup synthesis.

3.3.2 True LMI design methods:

Although the papers [78] and [79] were important steps in anti-windup design and they both used the LMI framework as part of the anti-windup synthesis procedure, the design methods were not *wholly* LMI-based as the inequalities were really bilinear matrix inequalities which were linearised by fixing one of the free variables. The late part of the 20th century and early part of the 21st century saw the development of two anti-windup synthesis methods which were wholly LMI-based.

The first method, discussed in detail in [80] continues the static anti-windup theme but effectively uses the Circle Criterion with an $\mathcal{L}_2$ gain constraint to devise a purely LMI based synthesis method. In terms of the notation used in this paper, perhaps the main result of [80] can be stated as

Theorem 4: [80] There exists an anti-windup compensator which solves Problem 1 if there exist matrices $Q = Q' > 0$, $U = \mathrm{diag}(\nu_1, \ldots, \nu_m) > 0$, L and scalar $\gamma > 0$ such that the following LMI is solved

$$\begin{bmatrix} Q\tilde{\mathcal{A}}' + \tilde{\mathcal{A}}Q & \mathcal{B}_1^0 U + Q\tilde{\mathcal{C}}_1 - \bar{\mathcal{B}}_1 L & \tilde{\mathcal{B}}_2 & Q\tilde{\mathcal{C}}_2' \\ \star & -2U - \bar{\mathcal{D}}_{21}L - L'\bar{\mathcal{D}}_{21}' + \mathcal{D}_{21}^0 U + U\mathcal{D}_{21}^{0'} & \tilde{\mathcal{D}}_{22} & U\mathcal{D}_{11}^0 + L'\bar{\mathcal{D}}_{11}' \\ \star & \star & -\gamma I & \tilde{\mathcal{D}}_{12}' \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0 \tag{37}$$

If the above LMI is satisfied, a suitable compensator can be constructed as $D_{\mathrm{aw}} = LU^{-1}$.

In the above LMI, the following matrices have been partitioned into affine forms: $\tilde{\mathcal{B}}_1 = \mathcal{B}_1^0 + \bar{\mathcal{B}}_1 D_{\mathrm{aw}}$, $\tilde{\mathcal{D}}_{11} = \mathcal{D}_{11}^0 + \bar{\mathcal{D}}_{11} D_{\mathrm{aw}}$ and $\tilde{\mathcal{D}}_{21} = \mathcal{D}_{21}^0 + \bar{\mathcal{B}}_{21} D_{\mathrm{aw}}$. With these affine partitions, the matrix inequality in the Theorem is linear, thus allowing standard LMI design tools to be used in the synthesis technique. The work in [80] is similar to that in [43] where a small gain approach is used to synthesize an anti-windup compensator. Arguably the results of [80] are somewhat more elegant than [43] and they are convex in the general multivariable case. The results in [43] are generally stated in terms of bilinear matrix inequalities which transpire to be linear in the special cases of single-input–single-output systems and also when the static multiplier is fixed. However the advantage of the results of [43] is that they can be applied to unstable

doi: 10.1049/iet-cta:20070435

systems, although no estimate of the region of attraction is provided.

A similar result to Theorem 4 was reported in [81] except that the results were improved in two ways. Firstly provision for *low-order* anti-windup synthesis was made, which significantly enlarges the class of compensators which can be designed and also allows compensators with superior performance (in terms of their $\mathcal{L}_2$ gains) to be obtained. Secondly, [81] proposed a performance map which allowed minimisation from linear performance to be minimised explicitly via LMIs – this concept is revisited in the next section.

The main problem with the results of [80] was, because the anti-windup compensator was limited to having a static structure, it was not always possible to construct an anti-windup compensator for any given plant-controller combination; that is, the LMI (37) is not always feasible for arbitrary systems. Mulder and Kothare [82] tried to relax this by using piecewise linear Lyapunov functions to decrease the conservatism introduced by the Circle Criterion, but this did not lead to *linear* matrix inequalities and resulted in some rather complicated constructions for the anti-windup compensator.

However in [28] and [63] conditions were given which allowed a general n_{aw}th order anti-windup compensator to be constructed using 'almost' LMI conditions. In the general case these were non-convex but under certain conditions could be relaxed to be linear. The main results of [63] can be stated as follows.

Theorem 5: [63] There exists an anti-windup compensator of order n_{aw} which solves Problem 1 if there exist matrices $R > 0$ and $S > 0$ and a scalar $\gamma > 0$ such that the following conditions hold

$$\begin{bmatrix} R_{11}A_p' + A_pR_{11} & B_{pw} & R_{11}C_z' \\ \star & -\gamma I & D_{zw} \\ \star & \star & -\gamma I \end{bmatrix} < 0 \tag{38}$$

$$\begin{bmatrix} S\tilde{A}' + \tilde{A}S & \tilde{B}_2 & S\tilde{C}_1' \\ \star & -\gamma I & \tilde{D}_{12} \\ \star & \star & -\gamma I \end{bmatrix} < 0 \tag{39}$$

$$R = R' = \begin{bmatrix} R_{11} & R_{12} \\ \star & R_{22} \end{bmatrix} > 0 \tag{40}$$

$$S = S' > 0 \tag{41}$$

$$R - S \geq 0 \tag{42}$$

$$\operatorname{rank}(R - S) \leq n_{aw} \tag{43}$$

The above theorem is an existence condition and, in general is difficult to satisfy because of the non-convex rank constraint. In fact the above result is similar to the LMI $\mathcal{H}_\infty$ synthesis problem which is generally a number of LMIs coupled with a rank constraint [83]. However, it transpires that, for the special cases of $n_{aw} = 0$ and $n_{aw} \geq n_p$, that the rank constraint vanishes leaving simply a set of linear matrix inequalities. Although the above theorem is an existence condition, an anti-windup compensator yielding the $\mathcal{L}_2$ gain $\gamma > 0$ can then be constructed by solving a further LMI [63]. Furthermore, mirroring techniques used in low-order robust controller design, a trace minimisation can be performed to 'remove' the rank constraint as advocated in [84]. Although this procedure is not guaranteed to work, experience in other areas of the control field has shown this to sometimes be successful.

3.3.3 *Mismatch-based anti-windup synthesis:*

The anti-windup problem is often interpreted as one of keeping the behaviour of the system during saturation as close as possible to the behaviour of the system, had saturation not been present. This idea had been around for many years in the anti-windup community but was not formalised until the work of [23, 62, 85], although preliminary work in this spirit can be found in [86, 87]. In this case, it is often useful to define the nominal linear system as

$$\mathcal{P}_{lin} = \begin{cases} \dot{\xi}_{lin} = \mathbb{A}\xi_{lin} + \mathbb{B}_2 w \\ y_{c,lin} = \mathbb{C}_2\xi_{lin} + \mathbb{D}_{22}w \\ z_{lin} = \mathbb{C}_1\xi_{lin} + \mathbb{D}_{12}w \end{cases} \tag{44}$$

This system represents the behaviour of the system when no saturation is encountered and hence does not include the dynamics of the anti-windup compensator. In this representation z_{lin} denotes the performance output when no saturation is present, $y_{c,lin}$ the linear control signal and $\xi_{lin} \in \mathcal{R}^{n+n_c}$ the state when no saturation is present. With this in mind, it is possible (see [77, 81, 85, 88, 89]) to represent the anti-windup compensated system as depicted in Fig. 7.

In Fig. 7, the nominal linear system $\mathcal{P}_{lin}$ is clearly decoupled from the non-linear behaviour of the system

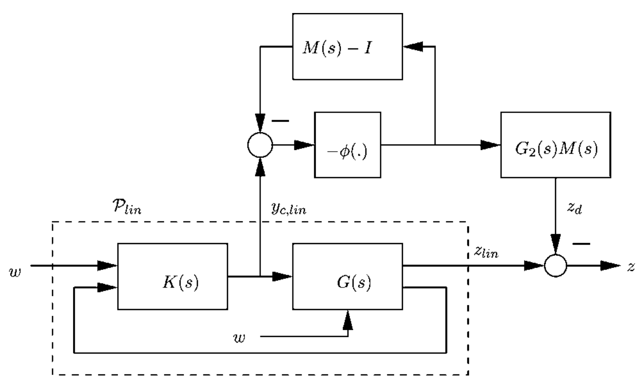

Figure 7 *Structure of mismatch system for anti-windup analysis*

doi: 10.1049/iet-cta:20070435

during saturation. Note that $M(s)$-I and $G_2(s)M(s)$ represent transfer function matrices derived from the dynamics of the linear plant, linear controller and anti-windup compensator and also have attractive 'robust control' interpretations [77, 81, 88]. This figure gives a clear interpretation of the stability and performance problems when trying to keep linear performance deviation minimal: the stability problem is to keep the non-linear loop stable (as, by assumption $\mathbb{A}$ is Hurwitz) and the performance problem is to keep z_d as small as possible.

Although such an expose was not given in [62], that paper effectively defined the anti-windup problem represented by the block diagram in Fig. 7. In particular, the anti-windup problem was framed as one which ensured that:

1. The performance of the 'real' saturated system and the mismatch system are identical unless saturation occurs. i.e. $z - z_\mathrm{lin}(t) \equiv 0 \quad \forall t \geq 0$ if $y_\mathrm{c,lin} \preceq u_0\ \forall t \geq 0$.

2. If $y_\mathrm{c,lin}$ eventually falls below the saturation threshold, which can be captured as ensuring $\phi(y_\mathrm{c,lin}) \in \mathcal{L}_2$, then the real output z will converge to the linear output, z_lin asymptotically.

This is a convenient way in which to look at the anti-windup problem and many authors, [23, 29, 76, 81, 85, 88, 90–92] have adopted this approach for studying the anti-windup problem. One can also strengthen the anti-windup problem to ensuring that the 'gain' from the linear control signal, $y_\mathrm{c,lin}$, to the performance output z_d is less than a certain bound, i.e.

$$\|z_\mathrm{d}\|_2 \leq \gamma \|y_\mathrm{c,lin}\|_2$$

The papers [81, 93, 94] and others have given constructive design procedures for minimising such a cost function. The reader can also consult the related other references proposing interesting work in this context [95–97].

3.4 Optimisation issues

Based on the conditions stated in previous sections in both regional or global contexts, different optimisation strategies can be considered in order to synthesize a suitable anti-windup controller. Hence, the satisfaction of conditions (29) and (30) given in Theorem 2 ensures that the closed-loop system (11) presents bounded trajectories for any admissible disturbance. In this case, it is also ensured that the $\mathcal{L}_2$ gain between the disturbance w and the regulated output z is lower than a constant $\sqrt{\gamma}$. The idea therefore is to use these conditions in order to find the controller considering the following optimisation problems.

- *Maximisation of the disturbance tolerance*. In this case the idea is to maximise the bound on the disturbance, for which we can ensure that the system trajectories remain bounded. This can be accomplished by the following optimisation problem

$$\begin{array}{c} \min \delta \\ \text{subject to relations (29) and (30)} \end{array} \tag{45}$$

Note that, in this case, we are not interested in the value of γ. Indeed, the scalar γ will take a value (as large as necessary) to ensure that relation (29) is verified.

- *Minimisation of the $\mathcal{L}_2$ gain*. Given a bound δ^{-1} on the admissible disturbances, the idea here is to perform a minimisation of the upper bound $\sqrt{\gamma}$ on the $\mathcal{L}_2$ gain, as follows

$$\begin{array}{c} \min \gamma \\ \text{subject to relations (29) and (30)} \end{array} \tag{46}$$

Note that, in the case $w = 0$, the implicit objective of Problem 1 consists in maximising the estimate of the basin of attraction of the closed-loop system. Thus, when the open-loop matrix A is asymptotically stable, if the conditions shown in Section 3.3 are feasible then the region of stability is the whole state space. Otherwise, by using the results given in Section 3.2, the problem of maximising the region of stability consists in maximising the size of $\mathcal{E}(W)$. Different linear optimisation criteria $J(.)$, associated to the size of $\mathcal{E}(W)$, can be considered, like the volume: $J = -\log(\det(W))$, or the size of the minor axis: $J = -\lambda$, with $W \geq \lambda I_n$. A given *shape set* $\Xi_0 \in \Re^n$ and a scaling factor β, where $\Xi_0 = \mathrm{Co}\{v_r \in \Re^n; r = 1, \ldots, n_r\}$ can also be considered and the associated criterion may then be to maximise the scaling factor β such that $\beta\Xi_0 \subset \mathcal{E}(W)$ [5, 98].

4 Extensions

4.1 Rate saturation

Quite frequently limits on the magnitude of control signal which an actuator can handle is less important than limits on the *rate* of the control signal which it can handle. Rate limits are of particular importance in mechanical systems where the inertia in various components of the actuator prevents it from moving very fast, thereby limiting the rate of the control signal which it can pass to the plant.

Modelling of rate saturation non-linearities is not identical within the constrained control literature [99–101] and is even disparate within the anti-windup literature [33, 102, 103]. There are added complications when the rate limit is combined with the magnitude limit which appear to be related to the physics of the actuator (contrast, for example, the electro-hydraulic actuator discussed in the first chapter of [7] with the aircraft actuators used in [48]). However a useful model of the rate limit [68, 100, 102, 103] appears to be obtained by cascading the standard saturation function with

doi: 10.1049/iet-cta:20070435

an integrator and gain and enclosing this within a feedback loop as depicted in Fig. 8 for the scalar case. In this representation, the limits of the saturation function, u_0, now represent the rate-limits of the system and the gain H determines the actuator's linear bandwidth when no rate limits are reached. The state-space equations are thus given by

$$\dot{\hat{x}}_r = \mathrm{sat}_{u_0}(H(y_c - \hat{x}_r)) \qquad (47)$$

$$u = \hat{x}_r \qquad (48)$$

Such a representation of a rate limit is attractive for two reasons: firstly it gives a practical representation of a rate limit, that is both the actuator's linear dynamics and non-linear rate limits are featured; secondly it is evident because the 'non-linear' part of the actuator is simply the standard saturation function, that many magnitude saturation type of techniques can be applied to such systems with little extra difficulty, see [102] for example. One note of caution is that the presence of the integrator (which is not asymptotically stable) causes some extra technical difficulties in obtaining global results should A_p be Hurwitz, but overall the techniques are broadly similar.

A good paper which discusses the merits of different ways to model actuator position and rate limits is [100]. Actuator rate limits have recently attracted a lot of interest because of their role in pilot-induced-oscillations (PIOs) and the subsequent untimely demise of several aircraft because of rate-limited actuators – see [1, 49, 50, 104] for further details. The reader may also consult [105] and [106] in which a different model of rate saturation is proposed. An anti-windup scheme is developed in a similar way to that used when dealing with only magnitude saturation.

4.2 Sensor saturations

The study of systems subject to sensor saturation is less developed, with only a few papers devoted to this topic [107–112]). This frugal treatment in the literature is perhaps because of the less frequent occurrence of sensor saturation, although as it too introduces a non-linearity into largely linear control loops, it is easy to see that it poses similar performance and stability issues. Sensor saturation is normally found in applications where cost prohibits the use of sensors with adequate range, leading to sensor saturation for large reference/disturbance inputs. Alternatively sensor saturation can model the situation where only the *sign* of the output is known. In this case, the sign function can be modelled by a saturation function with a steep gradient.

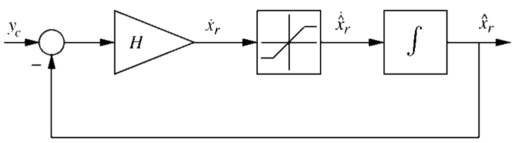

Figure 8 *Simple model of rate limit*

A naive appraisal of the sensor saturation problem suggests that it is similar to the actuator saturation problem with the plant and the controller interchanged. In fact this is not the case [107, 108]; one of the crucial differences between the two problems is the availability of the 'unsaturated' signal. In the actuator saturation case, knowledge of both the output produced by the linear controller and the saturated version of this (i.e. the signals either side of the saturation block) is assumed. In the case of sensor saturation, it is not realistic to assume that the actual plant output is known; only the saturated version of this is known (otherwise there would be no problem!). This is problematic for the anti-windup approach, and hence an observer may be used to overcome this difficulty. If the study of systems subject to sensor saturation is under-developed, the study of anti-windup compensation for this class of systems is less developed still. To the best of the authors' knowledge, the only literature discussing this approach are the papers [40] and [107], that establish conditions for both local stability and global stability with $\mathcal{L}_2$ gain, respectively. These two papers are also related to the paper by Park and Youn [113] in which state-constrained systems are considered and an anti-windup type approach is proposed. Note however that architectures, potentially different from that ones introduced in these papers should be more deeply studied in terms of merits and deficiencies.

4.3 Nested saturations

Following the two previous sections, we can consider that an important class of systems of interest consists of systems presenting nested saturations. In particular, such structures appear when we deal with non-linear actuators and sensors. For instance, it is common in aerospace control systems (e.g. launcher and aircraft control) that actuators are both limited in amplitude and rate, even in dynamics: see, for example, [6, 39, 44, 114]. Few results relative to anti-windup strategies can however be found for such systems. One recalls the problem of rate or dynamics and amplitude limitations representation as mentioned in Section 4.1. Different models are used in the literature [6, 7]. The more common one is directly obtained from systems (47) and (48) as follows

$$\dot{\hat{x}}_r = \mathrm{sat}_{u_0}(H(\mathrm{sat}_{u_1}(y_c) - \hat{x}_r)) \qquad (49)$$

$$u = \hat{x}_r \qquad (50)$$

Indeed, in this context both conditions in local and global case can be derived to design static or dynamic anti-windup compensators. Non-constructive conditions are proposed in [33] to characterise a plant-order anti-windup controller. Constructive conditions to exhibit anti-windup schemes are proposed in [7] for amplitude and rate saturation, in [45] for amplitude and dynamics saturation. Some applications of such studies are given, for example, in [50, 115, 116].

doi: 10.1049/iet-cta:20070435

Another case of nested saturation resides in the presence of both sensor and actuator amplitude limitations, which extend the class of systems discussed in Section 4.2. The saturation of the sensor output induces an incorrect action of the controller, since the actual state or output of the plant is no longer precisely measured. This is the case, for instance, in linear systems controlled by dynamic output feedback controllers in the presence of saturating sensors and actuators [107, 117]. Some application oriented studies can be found for example in [7, 52].

4.4 Time-delay systems

In the last few years, the study of systems presenting time-delays has received special attention in the control systems literature, see for example [118–121]. This interest comes from the fact that time delays appear in many kinds of control systems (e.g. chemical, mechanical and communication systems) and their presence can be source of performance degradation and instability. In this sense, we can find in the literature many works giving conditions for ensuring stability as well as performance and robustness requirements, considering or not the delay dependence. Concerning the delay independent results, the stability is ensured no matter the size of the delay, whereas in the delay dependent results, the size of the delay is directly taken into account and this fact can lead, especially when the time delays are small, to less conservative results.

Considering that many practical systems present both time delays and saturating inputs, from the considerations above, it becomes important to study the stability issues regarding this kind of systems. With this aim, different techniques can be investigated: in particular the characterization of admissible regions of stability is often based on the use of Razumikhin or Lyapunov–Krasovskii functionals. In parallel, another popular technique consists of approximating the delay through a Padé approximation, which implies an increase in the order of the closed-loop system. It may be used in order to prove some robustness properties with respect to the presence of delays. Techniques based on Lyapunov functionals or Padé approximations can be used to analyse the stability and the performance of saturated systems.

In the anti-windup approach context, we can cite [122] and [123]. In these papers, dynamic anti-windup strategies are considered only for systems with input and output delays. It should be highlighted that the results in [122] can be applied only to open-loop stable systems and that in [123], the main focus is the formal definition and characterization of the $\mathcal{L}_2$ gain based anti-windup. Differently from [122, 123], in [124–126], the design of anti-windup gains was studied with the aim of enlarging the region of attraction of the closed-loop system. In [124] a method for computing anti-windup gains for systems presenting only input delays is proposed leading to BMI conditions. In contrast to [122], the proposed techniques can be applied to both stable and unstable open-loop systems.

Some application oriented studies can be found in [53] in the context of open water channels and in [127] in the context of a fighter aircraft, where the delays are replaced by first-order Padé approximations.

4.5 Anti-windup for non-linear systems

The vast majority of the anti-windup literature has concentrated on the development of anti-windup techniques for systems which are largely linear, or at least represent linear approximations of nonlinear systems (in the aerospace case). This has been because even for linear systems, the anti-windup problem has only just begun to be understood in a rigorous technical way and again, only recently, have modern control techniques been harnessed to address the problem. Furthermore, when dealing with saturated linear systems, the control engineer can draw upon the large body of knowledge on, for example, absolute stability theory which is now well-developed.

Of course, systems which are non-linear also have problems with saturation as well, and this has not been lost on the research community. Several anti-windup techniques for non-linear systems have recently been proposed and the intention of many of these is to mimic the anti-windup techniques for linear systems in some way. The literature base is too sparse to warrant a full discussion here but it suffices to say that techniques based on feedback linearisation (non-linear dynamic inversion) [128–130], adaptive control [131–133] and neural network control [134] are beginning to emerge and promise to be exciting and fruitful areas of research.

5 Conclusion

This paper was dedicated to presenting an overview of recent advances in anti-windup techniques which can be used to tackle problems of stability and performance degradation for linear systems with saturated inputs. Noting that the anti-windup techniques which can be found in the literature today have evolved from many sources and may be somewhat disconnected from one another, the objective of the current survey was to show not only the recent developments but also their potential connections. The anti-windup strategy was then explained within the context of its historical emergence and we tried to speculate about the likely future directions of the field. It is important to emphasize that the focus of this paper was on the so-called 'modern' anti-windup techniques which began to emerge during the end of the 20th century and which allow *a priori* guarantees on stability to be made. The survey attempted to provide constructive LMI conditions for the anti-windup compensators design in both global and local contexts. Some interesting extensions and open problems were discussed, such as rate or sensor saturations, nested saturations, the presence of time delays in the state or the input, and anti-windup for non-linear systems. In a certain sense, this paper fills a gap in the anti-windup literature.

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doi: 10.1049/iet-cta:20070435

Our hope is also that this survey will enable practitioners to have a more concise 'guide' to modern anti-windup techniques and that this will encourage future applications.

At this stage, we reveal that promising and exciting avenues of research would be to develop the capability to widen the types of non-linearities which the anti-windup strategy can address. Examples of these include piecewise affine non-linearities, backlash, nested non-linearities of different types, and so on. Such non-linearities pose different technical and philosophical problems to those traditionally present in the anti-windup literature but are sure to keep the field alive with new and adventurous ideas.

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