Driver steering control and a new perspective on car handling qualities

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Abstract: The article is about steering control of cars by drivers, concentrating on following the lateral profile of the roadway, which is presumed visible ahead of the car. It builds on previously published work, in which it was shown how the driver's preview of the roadway can be combined with the linear dynamics of a simple car to yield a problem of discrete-time optimal-linear-control-theory form. In that work, it was shown how an optimal 'driver' of a linear car can convert the path preview sample values, modelled as deriving from a Gaussian white-noise process, into steering wheel displacement commands to cause the car to follow the previewed path with an attractive compromise between precision and ease.

Recognizing that real roadway excitation is not so rich in high frequencies as white-noise, a low-pass filter is added to the system. The white-noise sample values are filtered before being seen by the driver. Numerical results are used to show that the optimal preview control is unaltered by the inclusion of the low-pass filter, whereas the feedback control is affected diminishingly as the preview increases. Then, using the established theoretical basis, new results are generated to show time-invariant optimal preview controls for cars and drivers with different layouts and priorities. Tight and loose controls, representing different balances between tracking accuracy and control effort, are calculated and illustrated through simulation. A new performance criterion with handling qualities implications is set up, involving the minimization of the preview distance required. The sensitivities of this distance to variations in the car design parameters are calculated. The influence of additional rear wheel steering is studied from the viewpoint of the preview distance required and the form of the optimal preview gain sequence. Path-following simulations are used to illustrate relatively high-authority and relatively low-authority control strategies, showing manoeuvring well in advance of a turn under appropriate circumstances.

The results yield new insights into driver steering control behaviour and vehicle design optimization. The article concludes with a discussion of research in progress aimed at a further improved understanding of how drivers control their vehicles.

Keywords: automobile, driving, handling qualities, preview, optimal control

1 INTRODUCTION

Steering a car is a familiar activity. It involves the driver looking ahead at the intended path relative to the car and somehow processing the preview information and the current motion data to yield the steering control inputs needed to make the car follow the path. We take the conventional view that the control input, in the case of the car, is steering wheel angle [1].

The most basic ideas of preview steer angle control can be appreciated by observing a conventional four-wheeled, dolly-steered trailer (Fig. 1).

The dolly wheels are steered through an angle proportional to the single-point path preview lateral position error and it is apparent that such an

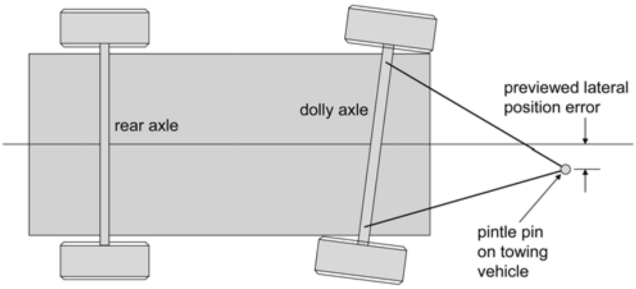

Fig. 1 Conventional four-wheeled trailer with dolly steering – an example of single-point preview control with steering proportional to previewed path lateral position error

arrangement provides at least a moderate tracking capability. It is well known that a short drawbar, corresponding to a high steering gain in control system terms, gives good tracking accuracy at low speeds and oscillatory lateral behaviour at high speeds and vice-versa [**2**, **3**]. Low-speed, modest-accuracy tracking control by preview utilization is not difficult. A large number of studies have been aimed at more general and effective use of the single-point preview path error signal [**4–6**].

With a more general multi-point preview, it is clearly the case that preview information sufficiently far ahead of the car is not relevant to the present control decisions. Indeed, it is a feature of optimal preview control solutions that the use of preview information decays asymptotically to zero for long enough previews [**7**, **8**], giving rise to the notion of 'sufficient' preview for full task performance to be achieved, depending on the task and the plant involved. The extent of the preview necessary for effective car driving has been measured at around 1.5 s at speeds of 9–13 m/s [**9**] and reported as anywhere between 2 and 9 s in other trials. It is also well known and has been shown formally [**10**] that car steering is often initiated well in advance of a change in road curvature.

Drivers can normally differentiate between low-intensity, low-accuracy steering and the opposite. In simple terms, they will employ one or the other, depending on the circumstances. Also, it is clear that tracking a relatively straight path with a full view ahead at low speed is relatively easy and can be done after very little learning, whereas maintaining a precise path under racing conditions demands high skill from much learning, high concentration, and significant effort at the steering wheel. In more demanding circumstances, drivers gain advantage from knowing both the dynamics of their vehicles and the geometry of the track. It is intended to mimic these features in the theoretical world.

The present article is a development of the work reported in reference [**11**]. There, it was shown that the small-perturbation car path-tracking problem can be arranged in standard discrete-time linear-quadratic-optimal-regulator format. The truly optimal control, minimizing a quadratic function describing path errors and control power, is time-varying [**12**]. It depends on the road profile immediately ahead of the car and it would need to be determined on-line. For the infinite time, Gaussian disturbance case, the optimal control is time-invariant and it can be found off-line. This control consists of a car lateral velocity, yaw rate feedback part, and a preview error part. The preview error part of the control reflects the inverse vehicle dynamics and indicates how much preview is necessary as a function of vehicle properties, speed, and objectives. The time-invariant optimal control (referred as TI-optimal subsequently) was determined for several cases of vehicle, speed, and cost function priorities in a 'ground-fixed' framework and transformed to a 'moving-vehicle' framework, shown diagrammatically in Fig. 2. It was argued that the relative simplicity of the steering control action implied by the TI-optimal control, combined with its good tracking performance and its versatility make it a plausible basis for real driving. The driver's understanding of the vehicle dynamics and selection of the balance between accuracy and effort are nicely contained within the theory.

The aims remain much as before, to improve on existing virtual path-following capabilities, in terms of the balance between computational effort and tracking performance, and to advance the simulation of path-based vehicle manoeuvres, several of which are standardized [**13**]. Very high capability will allow the simulation of the limit behaviour of circuit racing and rally cars and appears to offer an efficient way of optimizing such cars, improving on previous methods, which are effective but slow [**14–18**]. In addition, it is hoped to understand better how humans drive, how they learn to drive, how they adapt to unfamiliar vehicle dynamic characteristics,

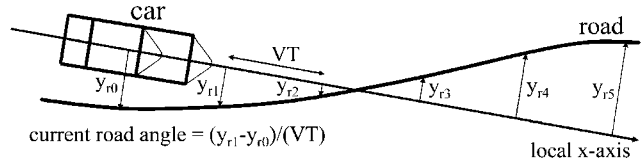

Fig. 2 Structure of the preview part of the time-invariant discrete-time optimal steering control in a moving-vehicle framework, showing the first six road preview sample values. Car states, lateral velocity, and yaw rate, are also fed back to the steer angle

how they gain advantage from knowing the road, and how they interact with their cars, reflecting in a new way on vehicle handling qualities.

In the present context, the focus is on steering control with 'sufficient' preview of the path ahead [**11**]. Responding to unpredicted (not previewed) events, like a wind gust or an animal incursion, is not seriously considered. First, the theory to include a low-pass filter on the road data is developed, so that the path lateral profile may be represented more realistically as a low-pass filtered white-noise process. It is shown that the TI-optimal preview control and the vehicle state feedback control both remain the same irrespective of the filter properties, whereas the filter state feedback gains reduce asymptotically to zero as the preview increases. Then, preview gain sequences and corresponding path-following simulations are used to show parametric influences. The required preview distance is quantified and suggested as an inverse measure of car handling quality. The manner in which car parameters influence the required preview distance is examined through sensitivities. The theory is also used to show the oscillatory premanoeuvre steering commonly observed in rallying. Conclusions are drawn and directions for further work are considered.

2 INCLUSION OF A LOW-PASS ROAD EXCITATION FILTER IN OPTIMAL PREVIEW CONTROL CALCULATIONS

2.1 Setting up the problem with a road filter

There are two significant limitations to the optimality of the time-invariant control law so far relied on [**11**]. One of these is the time-invariance itself, which may prejudice the control taking full advantage of the particularities of the path ahead of the car. The other is the white-noise process disturbance assumption underlying the time-invariant control solution. White-noise is too rich in high frequency components to realistically represent a road lateral profile. Let us now represent the road lateral profile as consisting of sample values from a white-noise process which is low-pass filtered before being input to the system (Fig. 3). The low-pass filter properties can be chosen to define the road spectrum. According to the preview control structure established in reference [**11**], a shift register is used to represent the way in which the road sample values ahead of the car are moved one step nearer to the vehicle as the car moves forwards through one time step. The road sample value at the car, y_{r0} in Fig. 2, leaves the problem and a new road sample, y_{ri} (Fig. 3), enters as the disturbance input.

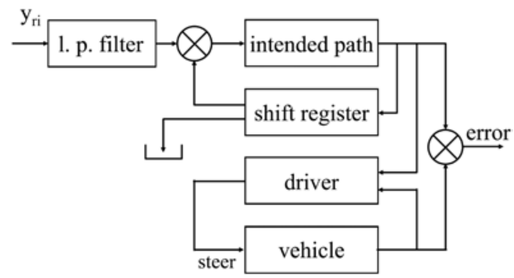

Fig. 3 Optimal preview path-following problem structure with low-pass filtered road excitation

The states of the shift register are included as states of the complete dynamic system, which has low-pass filter, shift register, and vehicle dynamics parts. Mathematically, the system is of standard linear discrete-time form and it has a single input, y_{ri}.

In state space discrete-time form, the filter equations, of order n_f, are

$$\begin{aligned} x_f(k+1) &= \mathbf{A}_f x_f(k) + \mathbf{B}_f y_{ri}(k) \\ y_f(k) &= \mathbf{C}_f x_f(k) + \mathbf{D}_f y_{ri}(k) \end{aligned} \tag{1}$$

The vehicle, of order n_c, is represented by

$$\begin{aligned} x_c(k+1) &= \mathbf{A}_c x_c(k) + \mathbf{B}_c \delta_{sw}(k) \\ y_c(k+1) &= \mathbf{C}_c x_c(k) + \mathbf{D}_c \delta_{sw}(k) \end{aligned} \tag{2}$$

and the road sample value shift register process, of order $(q+1)$, by

$$\begin{aligned} y_r(k+1) &= \mathbf{A}_r y_r(k) + \mathbf{B}_r y_f(k) \\ &= \mathbf{A}_r y_r(k) + \mathbf{B}_r \{\mathbf{C}_f x_f(k) + \mathbf{D}_f y_{ri}(k)\}, \end{aligned}$$

from equation (1),
where A_r is of the form

$$\begin{bmatrix} 0 & 1 & 0 & \dots & \dots 0 \\ 0 & 0 & 1 & \dots & \dots 0 \\ 0 & 0 & 0 & 1 & \dots 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots 0 \end{bmatrix} \quad \text{and}$$

B_r is of the form

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 1 \end{bmatrix} \tag{3}$$

Combining filter, car, and road equations (1) to (3), the whole problem can be stated as

$$\begin{bmatrix} x_f(k+1) \\ x_c(k+1) \\ y_r(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_f & 0 & 0 \\ 0 & \mathbf{A}_d & 0 \\ \mathbf{B}_r\mathbf{C}_f & 0 & \mathbf{A}_r \end{bmatrix} \begin{bmatrix} x_f(k) \\ x_c(k) \\ y_r(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{B}_d \\ 0 \end{bmatrix} \delta_{sw}(k) + \begin{bmatrix} \mathbf{B}_f \\ 0 \\ \mathbf{B}_r\mathbf{D}_f \end{bmatrix} y_{ri}(k) \tag{4}$$

$\mathbf{A_f}$ is $(n_f{}^*n_f)$; $\mathbf{A_d}$ is $(n_c{}^*n_c)$; $\mathbf{B_rC_f}$ is $(q+1)^*n_f$; $\mathbf{A_r}$ is $(q+1)^*(q+1)$; $\mathbf{B_d}$ is $(n_c{}^*1)$; $\mathbf{B_f}$ is $(n_f{}^*1)$; $\mathbf{B_rD_f}$ is $(q+1)^*1$. q is the number of sample values of the roadway ahead of the car within the driver's view. The cost function to be minimized by the optimal control represents a weighted sum of squares of tracking error and path tangent error, with a fixed weighting on control power or squared steering wheel angle, as in reference [**11**].

The complete problem takes the standard form

$$z(k+1) = \mathbf{A}z(k) + \mathbf{B}u(k) + \mathbf{E}y_{ri}(k)$$
$$y(k) = \mathbf{C}z(k)$$

If y_{ri} is a sample from a random sequence, the TI-optimal control, given that the pair ($\mathbf{A}$, $\mathbf{B}$) is stabilizable and the pair ($\mathbf{A}$, $\mathbf{C}$) is detectable [**12**], is

$$u^*(k) = -Kz(k),$$
$$\text{where } K = \{R + \mathbf{B}^T\mathbf{PB}\}^{-1}\mathbf{B}^T\mathbf{PA} \tag{5}$$

and $\mathbf{P}$ satisfies the matrix-difference-Riccati equation

$$\mathbf{P} = \mathbf{A}^T\mathbf{PA} - \mathbf{A}^T\mathbf{PB}\{R + \mathbf{B}^T\mathbf{PB}\}^{-1}\mathbf{B}^T\mathbf{PA} + \mathbf{C}^T\mathbf{QC} \tag{6}$$

in which $\mathbf{Q}$ is the weighting matrix diag$[q_1, q_2]$, corresponding to the two performance terms contributing to the cost function, and $R = 1$, corresponding to the single control input, steering wheel angle.

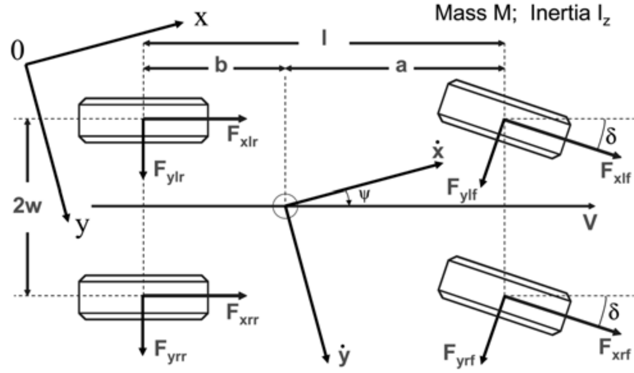

Fig. 4 Yaw-sideslip car model in ground-fixed, 0*xyz*, framework

samples to find the preview gains. That method has computational speed and insight advantages. It suggests a relationship between the closed-loop system eigen-properties and the nature of the preview gain sequence. However, it cannot be applied straightforwardly, when the low-pass road filter is in place, because of the coupling term $\mathbf{B_rC_f}$ in the $\mathbf{A}$ matrix, see equation (4). Rather, a direct solution is obtained by simply specifying the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{Q}$, and $\mathbf{R}$ which appear in equation (6) and employing the MATLAB function DLQRY.

In our specific problem, let us consider using a fourth-order Butterworth low-pass filter, with selectable passband and the car model and parameter set from reference [**11**], illustrated by Fig. 4.

In the state space equations describing the low-pass filter, $\mathbf{A_f}$ is (4*4), with element values which depend on the cut-off circular frequency selected, Ω_f, $\mathbf{B_f}$ is $[\Omega_f\ 0\ 0\ 0]$, $\mathbf{C_f}$ is $[0\ 0\ 0\ 1]$, and $\mathbf{D_f}$ is [0]. The car equations of motion are written in a ground-fixed (0*xyz*) axis system, so that the problem form is aligned with the discrete-time linear-quadratic-regulator theory available. They come directly from reference [**11**]. With the order of variables: $x_{c1} = y$; $x_{c2} = \dot{y}$; $x_{c3} = \psi$; $x_{c4} = \dot{\psi}$; and writing the equations in standard state space form, $\dot{x}_c = \mathbf{A}_c x_c + \mathbf{B}_c \delta_{sw}$; then

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -(\mathbf{C}_{af}+\mathbf{C}_{ar})/MV & (\mathbf{C}_{af}+\mathbf{C}_{ar})/M & (b\mathbf{C}_{ar}-a\mathbf{C}_{af})/MV \\ 0 & 0 & 0 & 1 \\ 0 & (b\mathbf{C}_{ar}-a\mathbf{C}_{af})/I_zV & (a\mathbf{C}_{af}-b\mathbf{C}_{ar})/I_z & -(a^2\mathbf{C}_{af}+b^2\mathbf{C}_{ar})/I_zV \end{bmatrix} \quad \text{and} \quad \mathbf{B}_c = \begin{bmatrix} 0 \\ C_{af}/MG \\ 0 \\ a\mathbf{C}_{af}/I_zG \end{bmatrix}$$

In prior work, in which the road input was fed directly to the system [**11**, **19**, **20**], the optimal control was found by partitioning the problem into non-preview and preview parts. First, the non-preview, standard LQR optimal control was found. Then, the state-transition-matrix of the closed-loop system was applied recursively to the preview error

Here, the road–wheel steer angle is taken to be proportional to the steering wheel angle; $\delta = \delta_{sw}/G$, G being a simple gear ratio. Transformation of the equations of motion into discrete-time form is a standard process [**6**] that can be done automatically in MATLAB [**9**]. The equation form is then: $z(k+1) = \mathbf{A}z(k) + \mathbf{B}\delta_{sw}(k) + \mathbf{E}y_{ri}(k)$, in which k is the index of time; that is, the elapsed time from the chosen zero to the present time is kT, T being the sampling interval. Baseline numerical values of the car parameters are: $M = 1050$ kg; $I_z = 1500$ kg m^2; $a = 0.92$ m; $b = 1.38$ m; $C_f = 120\,000$ N/rad; $C_r = 80\,000$ N/rad; $G = 17$. A discrete-time step of $T = 0.02$ s is used throughout. The matrix **C**, helping to define the cost function, is given by

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-1}{VT} & \frac{1}{VT} & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{7}$$

2.2 Some optimal control solutions

The MATLAB software which allows the generation of optimal controls requires the specification of the filter type and order, of the car and its parameter values, of the car speed, the number of preview points assumed, the cost function weights on path, and attitude angle errors. Then it solves for the optimal gains, the first four (for a fourth-order filter) defining the feedback of filter states to steer angle, the next four (for the fourth-order car) defining the feedback of car states to the steer angle, the remainder defining the feedback of preview errors to the steer angle. It will be convenient to call these sequences $\boldsymbol{K_f}$, $\boldsymbol{K_c}$, and $\boldsymbol{K_p}$.

For the baseline car at 20 m/s and 2 Hz low-pass filter, we calculate the TI-optimal controls with variations over a wide range of the road preview allowed. Each of the $\boldsymbol{K_f}$ gains is plotted in Fig. 5 as a function of the number of preview points. The important feature of these results is that all the gains go to zero as the preview increases. In the region for which the preview is sufficient for full performance, a notion to be elaborated later, the filter states are not needed at all for control. The feature is independent of the car or filter parameters. Further, the gains $\mathbf{K_c}$ are the same for all the cases treated, while the gain sequence $\mathbf{K_p}$ is simply extended, without change to existing values, by extending the preview. The pattern of these results, which is fundamental and which will be published in due course, remains the same, whatever car or filter parameters, filter order, or cost function weightings are chosen.

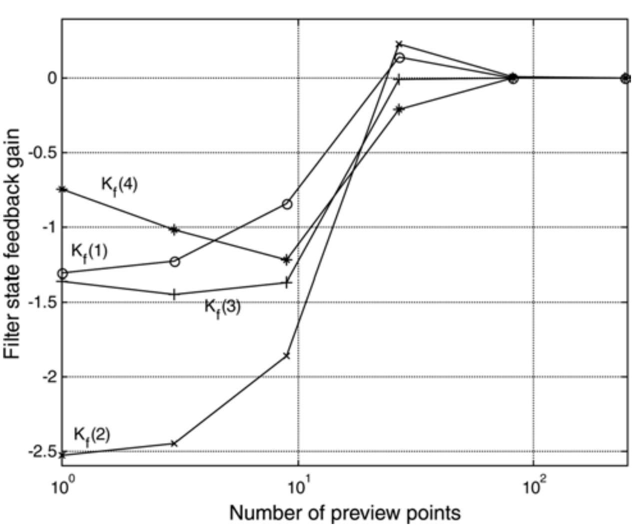

Fig. 5 Optimal filter state-feedback gains for standard car at 20 m/s and 2 Hz low-pass filter cut-off frequency with $q_1 = 100$, $q_2 = 0$, and variations in the number of preview points allowed

3 PARAMETRIC INFLUENCES ON CONTROL AND TRACKING PERFORMANCE

Optimal feedback and preview gain sets are functions of the car parameters, the car speed, and the weights applied to path-following errors (q_1) and to attitude angle errors (q_2). In fact, to avoid a proliferation of results, the attitude angle component will be excluded here ($q_2 = 0$), the main purpose being path-tracking accuracy. Variations in the relative values of q_1, q_2, and R were studied in reference [**11**], where it transpired that the optimal control can be influenced strongly by the performance objectives set for the system. Without rear steering, tracking accuracy, and precise alignment of the car with the road are in conflict, as the rear axle forces necessary for good tracking can only be generated by side-slipping of the rear axle. Thus, it is considered realistic to put the performance accent on good tracking and to associate no cost with attitude errors.

First, the properties of the baseline car are illustrated, Figs 6, 7, and 8. In reference [**11**], the transformation of the optimal control from the inertial coordinates, in which it is calculated, into the local frame of the driver is explained and, in that transformation, the car state-feedback gains related to lateral position and attitude angle are eliminated. Consequently, in Fig. 6, only the feedback gains related to lateral velocity and yaw rate, as functions of speed and for wide variations in q_1, are shown.

The well-known diminishing value of additional preview information as the preview time increases is clear in Figs 7 and 8. Also, in Fig. 7, it can be seen that the oscillatory pattern in the preview gains increases with speed, corresponding to the increasingly oscillatory vehicle dynamics as speed increases. We can distinguish 'tight control' for high q_1 and 'loose control' for low q_1, with relatively small gains and long preview usage being associated with the latter. It is implied that the TI-optimal driver can trade-off path-tracking accuracy and steering

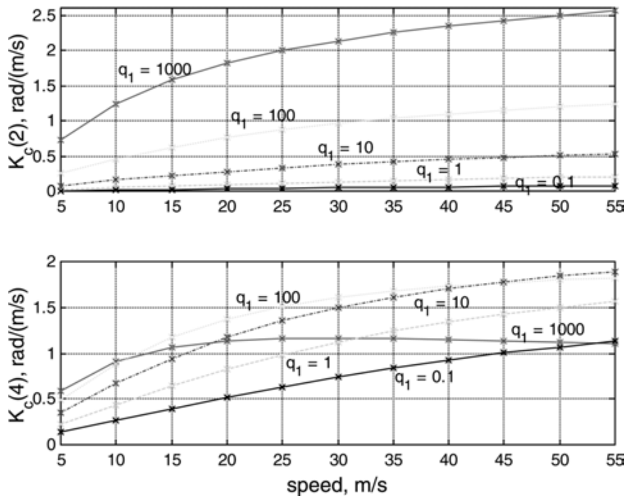

Fig. 6 TI-optimal lateral velocity (upper) and yaw rate (lower) feedback gains for the baseline car as functions of speed

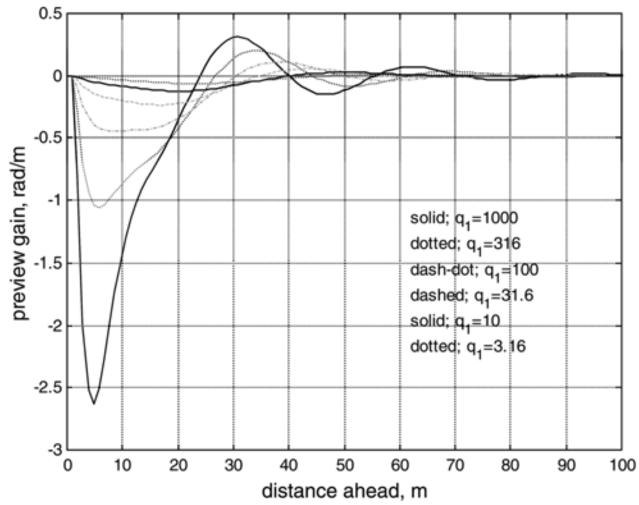

Fig. 8 TI-optimal preview gains for the baseline car as functions of the q_1 weighting for speed 48 m/s. Although shown by a continuous line, the gains are discrete values at 0.96 m intervals for this speed

effort, as real drivers undoubtedly do. Loose control appears to involve somewhat longer preview distances than does tight control, which issue will be re-visited.

Let us now compare, in our context, two contrasting cars; one, a large saloon; the other, a sports car. Parameter values from reference [**21**], given in Table 1, are used to describe the vehicles. Corresponding preview gain sequences are shown in Fig. 9. There is a clear implication that the saloon car requires the driver to use much extended preview relative to the sports car to achieve full steering control. The possibility arises that this is an important constituent of a vehicle's behaviour that needs to be understood better.

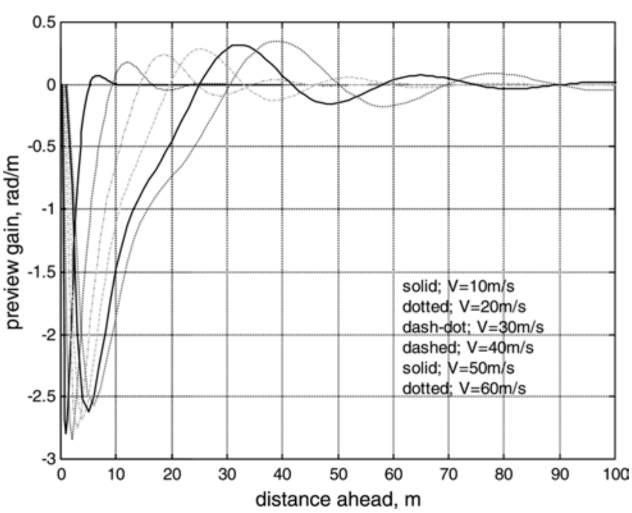

Fig. 7 TI-optimal preview gains for the baseline car as functions of speed for $q_1 = 1000$. Gains are discrete values calculated at 0.02 s intervals and joined together with a continuous line in each case

4 A NEW VEHICLE HANDLING QUALITIES CRITERION

It is now postulated that the preview distance required by a car, for a given speed and balance between tracking accuracy and control power, is a significant handling quality. We imagine that, the responsiveness to steering control which belongs to the sports car is reflected in the relatively short preview distance and that we can use the preview distance to quantify this property. As the criterion for the length of the preview required, let us use the distance ahead of the car mass centre for which 98 per cent of the area under the full preview gain curve is contained. This is $(i \cdot V \cdot T)$ for the smallest i for which $\{\sum \mathrm{abs}(\mathbf{K_p}(1{:}i)) > 0.98^* \sum(\mathrm{abs}(\mathbf{K_p}(:))\}$. Applying the criterion to the baseline car at arbitrarily chosen speeds of 16 and 48 m/s, the influence of control tightness on preview distance can be seen, in Fig. 10. Looser control from smaller q_1 implies a need for more preview for full performance,

Table 1 Parameter values for large saloon (upper) and sports car (lower)

M (kg)	I_z (kg m^2)	a (m)	b (m)	C_f (N/rad)	C_r (N/rad)	G
2045	5428	1.488	1.712	77 847	76 512	21
1008	1031	1.234	1.022	117 438	144 929	15

Source: Adapted from reference [**21**].

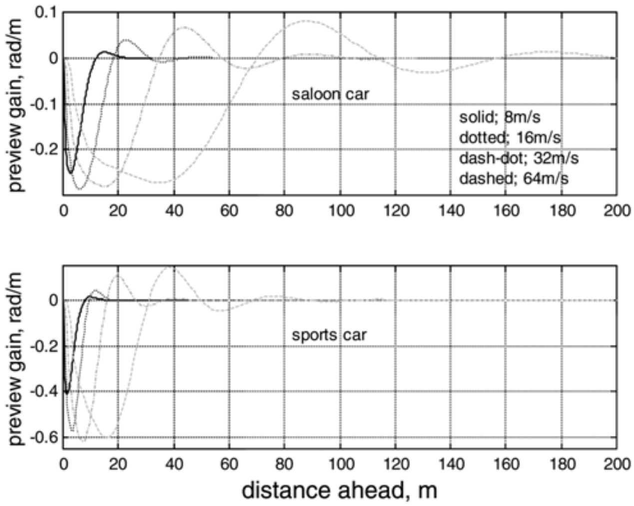

Fig. 9 TI-optimal preview gain sequences for a large saloon and a sports car at 8, 16, 32, and 64 m/s, with $q_1 = 100$ in each case

but the effect is quite weak over the interesting range $0.1 < q_1 < 1000$.

In view of the remarkable contrast between the saloon car and the sports car in their need for preview distance, we now enquire into the parametric influences. This is done by performing the preview distance calculations repeatedly, first for the baseline car and then for variants of it, at each of the four speeds, 8, 16, 32, and 64 m/s. Each variant involves a 10 per cent increase from the baseline value of one parameter. The percentage change in the preview distance due to the 10 per cent parameter change is calculated, with results shown in Fig. 11.

For good tracking with short preview, the yaw inertia and the mass should be small, whereas the wheelbase and the tyre cornering stiffnesses should be large. Owing to the greater influence of the separation of the mass centre from the rear axle with reference to that from the front, moving the mass centre forwards is also helpful. Curiously, the greatest influence is generally exerted at moderate speeds.

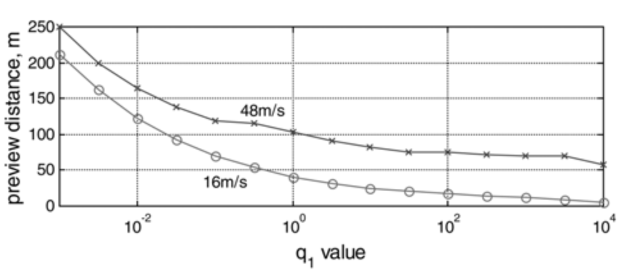

Fig. 10 The influence of the q_1 (path-tracking accuracy) weighting on the preview distance required for full control of the baseline car at 16 and 48 m/s

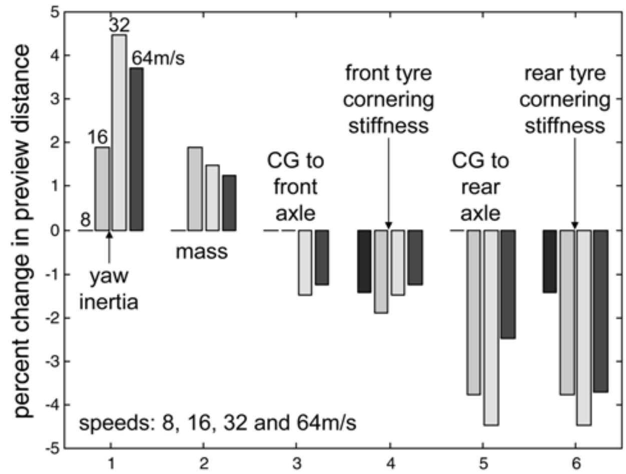

Fig. 11 Percentage changes in preview distance for the baseline car with 10 per cent increase in each of its parameter values in turn, whereas all other parameters retain their baseline values, at each of four speeds

The influence of rear wheel steering in proportion to that at the front on the required preview distance can be determined similarly. The equations of motion for the car need only very minor changes to cover the case [**22**]. The ratio of rear road–wheel to front road–wheel steer angle is fixed in turn at −0.2, 0 (the standard car), 0.2, 0.4, 0.6, and 0.8 and the preview distance found for speeds 8, 16, 32, and 64 m/s (Fig. 12). At the lower speeds, the rear steering has only modest consequences. As the speed increases, the required preview first goes up and then goes down again, quite markedly for the very high ratio of 0.8. The preview control gain sequence also simplifies for this case (Fig. 13). The case is

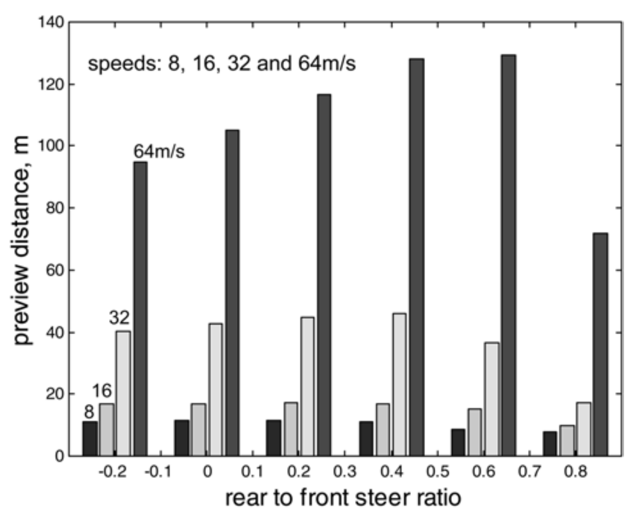

Fig. 12 Preview distances as a function of rear to front steer angle ratio for each of four speeds

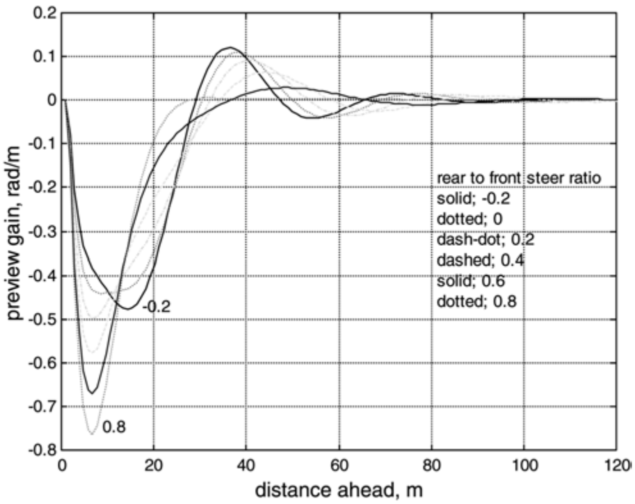

Fig. 13 Preview gain sequences for baseline car at 48 m/s with rear steering of various ratios ($q_1 = 100$; $q_2 = 0$). Although shown by a continuous lines, the gains are discrete values at 0.96 m intervals for this speed

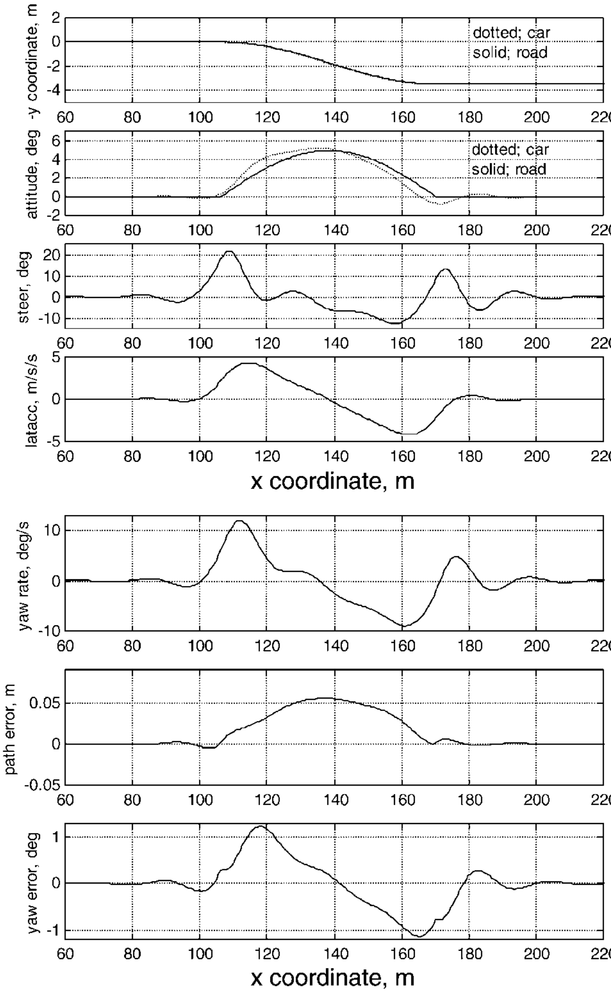

Fig. 14 Lane change simulation with baseline car at 32 m/s, with tight control from $q_1 = 1000$ and 150 preview points. 'Steer' refers to the steering wheel angle

interesting, with variable rear axle steering apparently providing an opportunity to test the idea that the extent of the preview needed is a significant factor in the determination of handling qualities. Experiments involving real drivers are probably necessary to determine the implications of these results.

5 PATH-FOLLOWING SIMULATIONS

The operation of the TI-optimal steering controllers in path-following simulations is demonstrated next. Simulations are carried out in the local framework of the driver as explained in reference [**11**]. The small angle assumption of the linear theory then requires that the road and vehicle angles are small within the envelope that contains the car and the preview data. The car can track a full circle in this framework, if desired, as long as this is true.

First, the influence of the control tightness on the tracking accuracy and the steering utilization in a lane change manoeuvre is shown in Figs 14 and 15. The loose control, deriving from setting q_1 to 1, leads to a corner cutting strategy, with smaller and smoother control inputs than that with tighter control.

Secondly, a rally car is crudely represented by drastically reducing the tyre cornering stiffnesses of the baseline car (deriving from running on a loose surface mainly). This car tracks a sharp change of direction in the roadway with a control that is necessarily quite loose ($q_1 = 1$) to avoid the occurrence of unrealistically large lateral accelerations. Steering well in advance of the direction change is demonstrated and the first steering observed is in the 'wrong' sense (Fig. 16). The oscillatory nature of the vehicle dynamics associated with fairly high speed and low cornering stiffnesses leads to an oscillatory preview gain sequence, which, in turn, leads to an oscillatory approach to a sharp corner, as commonly seen in practice. Single-point preview control schemes, without memory elements, appear incapable of reproducing this known behaviour.

6 CONCLUSIONS

The article has been built on the framework set out in reference [**11**], in which discrete-time linear-quadratic-regulator theory was applied to planned car steering control. It has been newly shown that the time-invariant optimal steering control for full preview is independent of the frequency content of

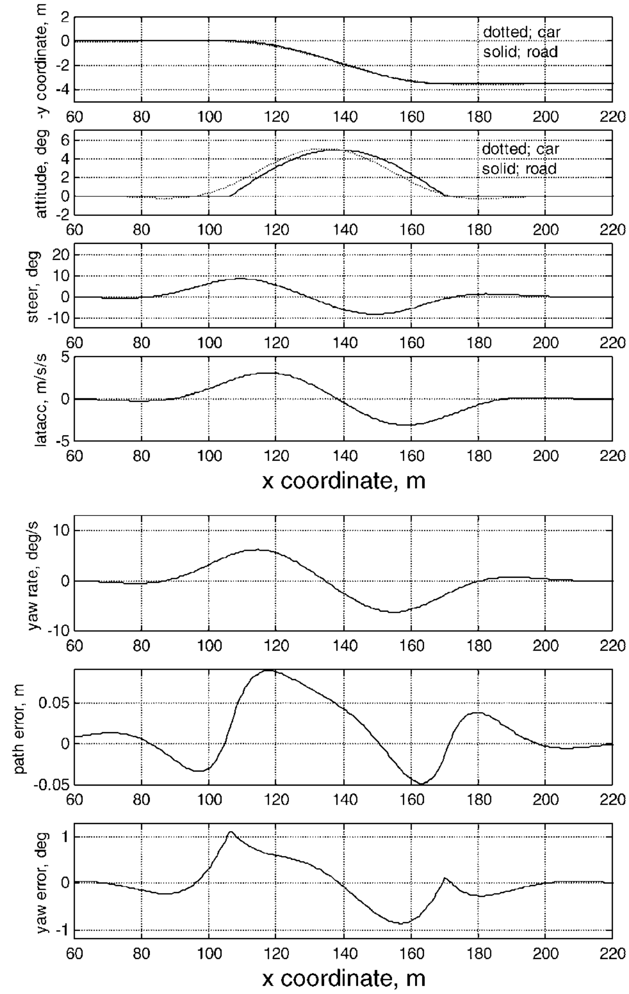

Fig. 15 Lane change simulation with baseline car at 32 m/s, with loose control from $q_1 = 1$ and 150 preview points. 'Steer' refers to the steering wheel angle

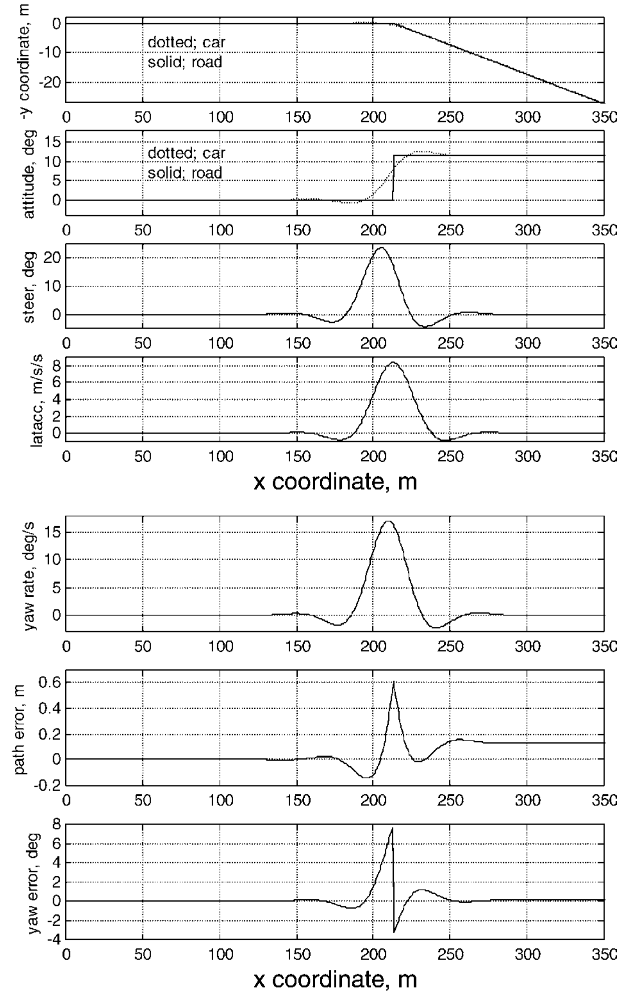

Fig. 16 Direction change simulation with rally car (20 per cent normal tyre cornering stiffnesses) at 32 m/s, with loose control from $q_1 = 1$ and 350 preview points. 'Steer' refers to the steering wheel angle

the roadway excitation. For less than full preview, feedback of the low-pass road filter states is needed for TI-optimality but the car state and preview gains are unaffected by the presence or nature of the filter. Thus, the white-noise disturbance assumption originally associated with this control does not restrict the optimality, when full preview is available. Nevertheless, it may be expected that a modified time-invariant control or a time varying control will be able to take some advantage from the particularities of the path to be tracked. It should be capable of more accurate tracking, in general, than the time-invariant version.

Tight and loose controls have been illustrated, by giving different priorities to path errors and control effort.

The results reinforce the idea that the TI-optimal control represents an attractive compromise between path-following accuracy and driver workload. The theory highlights the existence of full preview information and accentuates the value of the uncomplicated use of multipoint-preview data, in contrast to popular and highly developed single-point-preview based theories. It accounts properly for the need for a certain amount of road preview for good and effortless tracking performance and for the degradation which is observed when preview is restricted. Also, it accounts for the commonly observed oscillatory steering control used by rally drivers, when approaching a monotonic turn. The extent of the preview information predicted as necessary for full car control is very well aligned with reported experimental findings. These features are seen as providing much more convincing 'validation' than that often claimed from path-following vehicle or simulator experiments, as it is clear that if a model driver and a real driver track the same path with reasonable precision, the steering control inputs used must match each other well. The vehicle dynamics part of the problem is not really an issue.

Remarkable differences between the full preview requirements of a large saloon car and a sports car have been demonstrated and this required preview distance has been suggested as a car handling qualities criterion. A sensitivity study has shown the parametric influences on the preview needed, the vehicle design parameters being more significant than the weights chosen for the optimization. The preview distance has been computed for a simple car with rear wheel steering geared to that at the front and it has been shown that the influence of the rear steering is modest at low speeds and much more substantial at high speeds. As the rear to front steering ratio increases from a negative value, the preview distance first increases and then decreases again. For an extreme ratio of 0.8, the preview distance becomes relatively short and the preview gain sequence becomes relatively simple. It is suggested that this needs testing experimentally.

Further work is in progress to incorporate a low-pass filter into the driver structure, representing control bandwidth limitations; to apply the methodology to more complex and realistic cars and to other kinds of vehicles; to understand the influence of limiting the path preview available; to examine the performance of the linear controller in conjunction with non-linear cars, having saturating tyre lateral forces; to improve on the linear controller for dealing with tyre force saturation; to apply the technique to speed control in addition to steering control; and to examine efficient implementation issues for vehicle path-based simulation and optimization studies.

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APPENDIX

Notation

a, b, l, w	car dimensions (Fig. 4)
k	time stepping parameter
n_f, n_c	filter and car model orders, respectively
q	number of preview samples
q_1, q_2	cost function weights relating to tracking and attitude angle errors, respectively
u^*	optimal control
x_c, y_c	car state and output vectors
x_f, y_f	filter state and output vectors
y	generalized output vector, car lateral displacement
y_r	road state vector with components $y_{r0}, y_{r1}, y_{r2}\ldots$ (Fig. 2)
y_{ri}	single input from road to low-pass filter
z, u	generalized state vector and control
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$	generalized state space matrices
$\mathbf{A_c}, \mathbf{B_c}, \mathbf{C_c}, \mathbf{D_c}$	car state space matrices
$\mathbf{A_f}, \mathbf{B_f}, \mathbf{C_f}, \mathbf{D_f}$	filter state space matrices
$\mathbf{A_r}, \mathbf{B_r}$	road model state space matrices
C_{af}, C_{ar}	car front and rear axle cornering stiffnesses
$\boldsymbol{E}$	generalized input distribution vector
$F_{xfr}, F_{xfl}, F_{xrr}, F_{xrl}, F_{yfr}, F_{yfl}, F_{yrr}, F_{yrl}$	tyre longitudinal and lateral force components (Fig. 4)
G	steering gear ratio
$\boldsymbol{K}$	optimal gain vector with components $\boldsymbol{K_f}$, $\boldsymbol{K_c}$, and $\boldsymbol{K_p}$
M, I_z	car mass and yaw inertia
$\mathbf{Q}$	weighting matrix for performance indicators
$\mathbf{P}$	Riccati matrix
R	weighting parameter for control power (square of steer angle), set at 1
T	discrete-time increment, set at 0.02 s (Fig. 2)
V	car velocity (Figs 2 and 4)
δ	front road wheel steer angle (Fig. 4)
δ_{sw}	steering wheel angle
ψ	car attitude angle (Fig. 4)
Ω_f	filter roll-off radian frequency