Gaussian_Process_Code

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2/3/2020

##Part 1: Standard Gaussian Process

1-1: Fitting

```
library(rstan)
```

```
## Loading required package: StanHeaders
```

```
## Loading required package: ggplot2
```

```
## rstan (Version 2.19.3, GitRev: 2e1f913d3ca3)
```

```
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
```

```
source("gp.utility.R")

# Fitting GP model
stan_dat <- read_rdump('Financial_Data_Put_American.R')
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )
```

```
## See spec(...) for full column specifications.
```

```
## Warning: 98350 parsing failures.
##    row        col expected actual
```

```
## 142894 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 12/20/2019 a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## ...... .......... ........ ...... ......................................................................
## See problems(...) for more details.

## Loading required package: limSolve

##
## Attaching package: 'limSolve'

## The following object is masked from 'package:ggplot2':
##
##     resolution

## Loading required package: futile.logger

## Welcome to ragtop.  Logging can be enabled with commands such as
##   futile.logger::flog.threshold(futile.logger::INFO, name='ragtop.calibration')

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

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```

```r
fit_gp_SGP_American <- stan(file="gp-fit-6dimension_withBS.stan", data=stan_dat,
               iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.

## Trying to compile a simple C file

## Running /usr/lib64/R/bin/R CMD SHLIB foo.c
## gcc -m64 -I"/usr/include/R" -DNDEBUG   -I"/usr/lib64/R/library/Rcpp/include/"  -I"/usr/lib64/R/libra
## In file included from /usr/lib64/R/library/RcppEigen/include/Eigen/Dense:1,
##                  from /usr/lib64/R/library/StanHeaders/include/stan/math/prim/mat/fun/Eigen.hpp:13,
##                  from <command-line>:
## /usr/lib64/R/library/RcppEigen/include/Eigen/Core:82:12: fatal error: new: No such file or directory
##    #include <new>
##             ^~~~~
## compilation terminated.
## make: *** [/usr/lib64/R/etc/Makeconf:167: foo.o] Error 1
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.020266 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 202.66 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
```

```
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 34.1436 seconds (Warm-up)
## Chain 1:                44.7943 seconds (Sampling)
## Chain 1:                78.9379 seconds (Total)
## Chain 1:
```

```
## Warning: The largest R-hat is 1.25, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat
```

```
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess
```

```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quanti
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
print(fit_gp_SGP_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##               mean   se_mean        sd    2.5%      25%      50%       75%
## theta[1]  50104.27   8844.34  35877.26   40.75 25192.06 40416.64  68015.44
## theta[2]    227.02      9.76     89.98  102.63   153.98   210.65    279.24
## theta[3]     17.31      0.63      4.15   10.24    14.60    16.60     20.36
## theta[4]      0.02      0.00      0.01    0.01     0.02     0.02      0.02
## theta[5] 405784.93 260969.37 801752.67    0.11    43.23  4141.26 534703.72
## theta[6]     11.26      1.67     10.00    1.24     5.08     8.20     14.58
## sigma2        0.00      0.00      0.00    0.00     0.00     0.00      0.00
## gamma2    12287.88   1169.72   8136.02 4030.35  7739.18  9749.47  15591.34
##               97.5% n_eff Rhat
## theta[1]  129521.00    16 1.19
```

```
## theta[2]     407.86    85 1.01
## theta[3]      23.84    43 0.98
## theta[4]       0.04    34 1.01
## theta[5] 2198636.27     9 1.08
## theta[6]      35.63    36 0.98
## sigma2         0.00    50 1.05
## gamma2     33164.20    48 0.98
##
## Samples were drawn using NUTS(diag_e) at Thu Mar 26 16:10:07 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_SGP_American <- extract(fit_gp_SGP_American,permuted=FALSE)
```

```
# Predicting from GP model
post_mean_theta_1_SGP <- mean(sum_gp_SGP_American[,1,1]) #theta
post_mean_theta_2_SGP <- mean(sum_gp_SGP_American[,1,2]) #theta
post_mean_theta_3_SGP <- mean(sum_gp_SGP_American[,1,3]) #theta
post_mean_theta_4_SGP <- mean(sum_gp_SGP_American[,1,4]) #theta
post_mean_theta_5_SGP <- mean(sum_gp_SGP_American[,1,5]) #theta
post_mean_theta_6_SGP <- mean(sum_gp_SGP_American[,1,6]) #theta
post_mean_sigma2_SGP <- mean(sum_gp_SGP_American[,1,7]) #sigma2
post_mean_gamma2_SGP <- mean(sum_gp_SGP_American[,1,8]) #gamma2
post_mean_mu_SGP <- stan_dat$blackscholes


# x2 <- as.numeric(unlist(spx_spy_2019_06_30_put_2017_06_500rows_test['strike_price']))
# x2<- cbind(spy_2013_01_01_2013_01_31_put$strike_price[201:300],spy_2013_01_01_2013_01_31_put$impl_vol
# x2 <- seq(from=-2,to=2,by=0.01)

# x2 <- cbind(seq(from=0,to=1,by=0.01),seq(from=0,to=1,by=0.01))

# test_start <- 323 #06/10
# test_end <- 559 #06/14

test_start <- 560 #06/17
test_end <- 852 #06/20

x.grid_1 <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2 <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_3 <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_4 <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])
x.grid_5 <- as.numeric(stan_dat$total_puts_American$dividend[test_start:test_end])
x.grid_6 <- as.numeric(stan_dat$total_puts_American$interest_rate[test_start:test_end])
x2 <- cbind(x.grid_1,x.grid_2,x.grid_3,x.grid_4,x.grid_5,x.grid_6)
```

```
library('qrmtools')
library('ragtop')
blackscholes_2 <- rep(NA,length(x2[,1]))
for (row in 1:nrow(data.frame(x2))){
  blackscholes_2[row] <- as.numeric(blackscholes(-1,S0=x.grid_1[row],K=x.grid_2[row],r=x.grid_6[row],t=
  # blackscholes_2[row] <- Black_Scholes(0,x.grid_1[row],x.grid_6[row],x.grid_3[row],x.grid_2[row],x.gr
}
```

1-2: Predictions

```
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_SGP_American <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_theta_3_SGP
# post_data

pred_gp_SGP <- stan(file="Predictive GP_6dimension_withBS.stan", data=post_data_SGP_American,iter=200, w
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
```

```
## Trying to compile a simple C file
```

```
## Running /usr/lib64/R/bin/R CMD SHLIB foo.c
## gcc -m64 -I"/usr/include/R" -DNDEBUG   -I"/usr/lib64/R/library/Rcpp/include/"  -I"/usr/lib64/R/libra
## In file included from /usr/lib64/R/library/RcppEigen/include/Eigen/Dense:1,
##                  from /usr/lib64/R/library/StanHeaders/include/stan/math/prim/mat/fun/Eigen.hpp:13,
##                  from <command-line>:
## /usr/lib64/R/library/RcppEigen/include/Eigen/Core:82:12: fatal error: new: No such file or directory
##    #include <new>
##             ^~~~
## compilation terminated.
## make: *** [/usr/lib64/R/etc/Makeconf:167: foo.o] Error 1
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                9.49489 seconds (Sampling)
## Chain 1:                9.49489 seconds (Total)
## Chain 1:
```

##Part2: Bdrycov Gaussian Process

2-1: Fitting

```
# Fitting GP model for Bdrycov
stan_dat <- read_rdump('Financial_Data_Put_American.R')
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )
```

```
## See spec(...) for full column specifications.
```

```
## Warning: 98350 parsing failures.
##    row        col expected actual
```

```
## 142894 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 12/20/2019 a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## ...... .......... ........ ...... ........................................................................
## See problems(...) for more details.
```

```
## Warning in blackscholes[row] <- as.numeric(blackscholes(-1, S0 = x_1[row], :
## number of items to replace is not a multiple of replacement length

## Warning in blackscholes[row] <- as.numeric(blackscholes(-1, S0 = x_1[row], :
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## Warning in blackscholes[row] <- as.numeric(blackscholes(-1, S0 = x_1[row], :
## number of items to replace is not a multiple of replacement length

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```
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## Warning in blackscholes[row] <- as.numeric(blackscholes(-1, S0 = x_1[row], :
## number of items to replace is not a multiple of replacement length
```

```
fit_gp_Bdrycov_American <- stan(file="gp-fit-6dimension_withBS_Bdrycov.stan", data=stan_dat,
               iter=100, chains=1);
```

```
## Trying to compile a simple C file

## Running /usr/lib64/R/bin/R CMD SHLIB foo.c
## gcc -m64 -I"/usr/include/R" -DNDEBUG   -I"/usr/lib64/R/library/Rcpp/include/"  -I"/usr/lib64/R/libra
## In file included from /usr/lib64/R/library/RcppEigen/include/Eigen/Dense:1,
##                  from /usr/lib64/R/library/StanHeaders/include/stan/math/prim/mat/fun/Eigen.hpp:13,
##                  from <command-line>:
## /usr/lib64/R/library/RcppEigen/include/Eigen/Core:82:12: fatal error: new: No such file or directory
##    #include <new>
##             ^~~~~
## compilation terminated.
## make: *** [/usr/lib64/R/etc/Makeconf:167: foo.o] Error 1
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.126055 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 1260.55 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
```

```
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 722.367 seconds (Warm-up)
## Chain 1:                769.165 seconds (Sampling)
## Chain 1:                1491.53 seconds (Total)
## Chain 1:
```

```
## Warning: The largest R-hat is 1.18, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat
```

```
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess
```

```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quanti
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
print(fit_gp_Bdrycov_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS_Bdrycov.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##                  mean      se_mean           sd 2.5%   25%        50%
## theta[1] 3.248405e+05 2.196982e+05 1.452299e+06 0.11  1.50      13.22
## theta[2] 1.010569e+17 9.679251e+16 5.321590e+17 0.07  7.14 1470898.35
## theta[3] 8.184325e+08 6.289533e+08 4.406763e+09 0.13  2.87     172.33
## theta[4] 1.435375e+14 1.155000e+14 8.589929e+14 0.08  7.52     305.79
## theta[5] 1.248538e+15 1.133214e+15 5.547606e+15 0.09  2.02      67.24
## theta[6] 1.675240e+18 1.603950e+18 1.184573e+19 0.21 19.96   10703.10
## sigma2   5.845000e+01 7.940000e+00 5.033000e+01 0.03  3.10      88.29
## gamma2   4.751000e+01 7.890000e+00 4.868000e+01 0.06  0.91      15.41
##                  75%        97.5% n_eff Rhat
## theta[1] 8.697690e+03 2.128426e+06    44 1.03
## theta[2] 1.793884e+10 1.107864e+18    30 1.01
## theta[3] 3.983270e+03 5.353208e+09    49 0.99
## theta[4] 2.277726e+05 8.808142e+14    55 0.99
## theta[5] 1.243348e+05 1.516744e+16    24 1.03
## theta[6] 3.213058e+08 3.819663e+13    55 1.00
## sigma2   1.025800e+02 1.204000e+02    40 0.99
## gamma2   9.717000e+01 1.175300e+02    38 0.98
##
## Samples were drawn using NUTS(diag_e) at Thu Mar 26 16:39:06 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_Bdrycov_American <- extract(fit_gp_Bdrycov_American,permuted=FALSE)
# saveRDS(fit_gp,file ="fit_gp_vol50_within50spot_7to19days")
```

```
# Predicting from GP model - 2 dimensional case
post_mean_theta_1_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,1]) #theta
post_mean_theta_2_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,2]) #theta
post_mean_theta_3_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,3]) #theta
post_mean_theta_4_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,4]) #theta
post_mean_theta_5_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,5]) #theta
post_mean_theta_6_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,6]) #theta
post_mean_sigma2_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,7]) #sigma2
post_mean_gamma2_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,8]) #gamma2
post_mean_mu_Bdrycov <- stan_dat$blackscholes


# x2 <- as.numeric(unlist(spx_spy_2019_06_30_put_2017_06_500rows_test['strike_price']))
# x2<- cbind(spy_2013_01_01_2013_01_31_put$strike_price[201:300],spy_2013_01_01_2013_01_31_put$impl_vol
# x2 <- seq(from=-2,to=2,by=0.01)

# x2 <- cbind(seq(from=0,to=1,by=0.01),seq(from=0,to=1,by=0.01))
```

2-2: Predictions

```
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_Bdrycov_American <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean
# post_data

pred_gp_Bdrycov <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_Bdrycov_Amer
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
```

```
## Trying to compile a simple C file
```

```
## Running /usr/lib64/R/bin/R CMD SHLIB foo.c
## gcc -m64 -I"/usr/include/R" -DNDEBUG   -I"/usr/lib64/R/library/Rcpp/include/"  -I"/usr/lib64/R/libra
## In file included from /usr/lib64/R/library/RcppEigen/include/Eigen/Dense:1,
##                  from /usr/lib64/R/library/StanHeaders/include/stan/math/prim/mat/fun/Eigen.hpp:13,
##                  from <command-line>:
## /usr/lib64/R/library/RcppEigen/include/Eigen/Core:82:12: fatal error: new: No such file or directory
##    #include <new>
##             ^~~~
## compilation terminated.
## make: *** [/usr/lib64/R/etc/Makeconf:167: foo.o] Error 1
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                25.2826 seconds (Sampling)
## Chain 1:                25.2826 seconds (Total)
## Chain 1:
```

##Part 3 Predictions Versus Truth

3-1: Computing Means Standard GP

```
#Computing Mean
y_predict_values_SGP <- extract(pred_gp_SGP,permuted=FALSE)
y_mean_values_SGP <- c(colMeans(y_predict_values_SGP))
y_mean_values_SGP <- y_mean_values_SGP[1:(length(y_mean_values_SGP)-1)]

#Computing Standard Deviation
pred_gp_summary_SGP <- summary(pred_gp_SGP, sd=c("sd"))$summary
pred_gp_sd_SGP <- pred_gp_summary_SGP[, c("sd")]
y_sd_values_SGP <- pred_gp_sd_SGP[1:(length(pred_gp_sd_SGP)-1)]
```

3-2: Computing Means Bdrycov

```
#Computing Mean
y_predict_values_Bdrycov <- extract(pred_gp_Bdrycov,permuted=FALSE)
y_mean_values_Bdrycov <- c(colMeans(y_predict_values_Bdrycov))
y_mean_values_Bdrycov <- y_mean_values_Bdrycov[1:(length(y_mean_values_Bdrycov)-1)]

#Computing Standard Deviation
pred_gp_summary_Bdrycov <- summary(pred_gp_Bdrycov, sd=c("sd"))$summary
pred_gp_sd_Bdrycov <- pred_gp_summary_Bdrycov[, c("sd")]
y_sd_values_Bdrycov <- pred_gp_sd_Bdrycov[1:(length(pred_gp_sd_Bdrycov)-1)]
```

3-3: Plotting Predicted Values against Truth

```
par(mfrow=c(1,3))
#Plotting Standard GP
plot(log(y_mean_values_SGP),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
```

```
## Warning in log(y_mean_values_SGP): NaNs produced
```

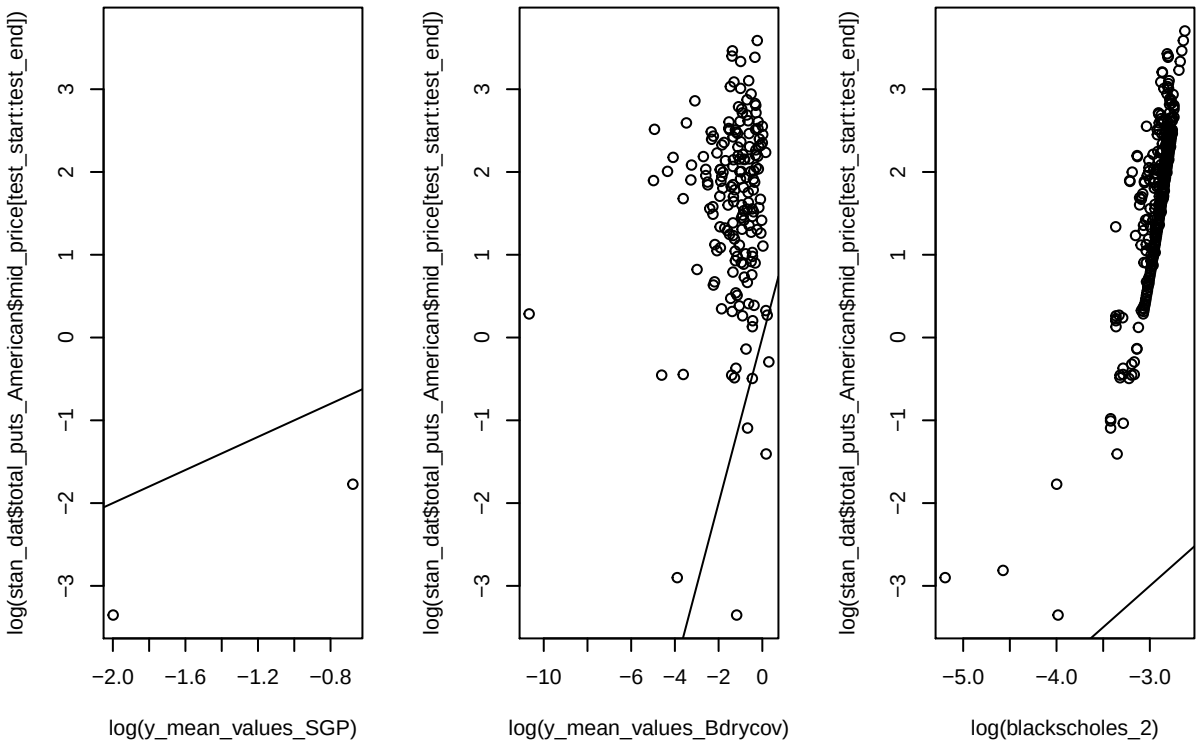
```
abline(0,1)

#Plotting BDrycov
plot(log(y_mean_values_Bdrycov),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
```

```
## Warning in log(y_mean_values_Bdrycov): NaNs produced
```

```
abline(0,1)

#Plotting Blackscholes
plot(log(blackscholes_2),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
abline(0,1)
```




```
#MSE
library('MLmetrics')
```

```
##
## Attaching package: 'MLmetrics'
```

```
## The following object is masked from 'package:base':
##
##     Recall
```

```
MSE(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 205.4539
```

```
MSE(y_mean_values_Bdrycov,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 100.1495
```

```
MSE(blackscholes_2,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 100.3091
```

##Part 4 Visualizations

4-1: Contour Plots of Forward Price & Strike Price

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility[test_start:test_end]))
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end])),nr
```

```r
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end]))
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end])),r

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

blackscholes_2_cont <- rep(NA,length(x2_cont[,1]))
for (row in 1:nrow(data.frame(x2_cont))){
  blackscholes_2_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont[row,1],K=x2_cont[row,2],r=x2_cont[r
}

post_data_cont <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean_theta_3_Bd
# post_data

pred_gp_cont <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_cont,iter=200,
```

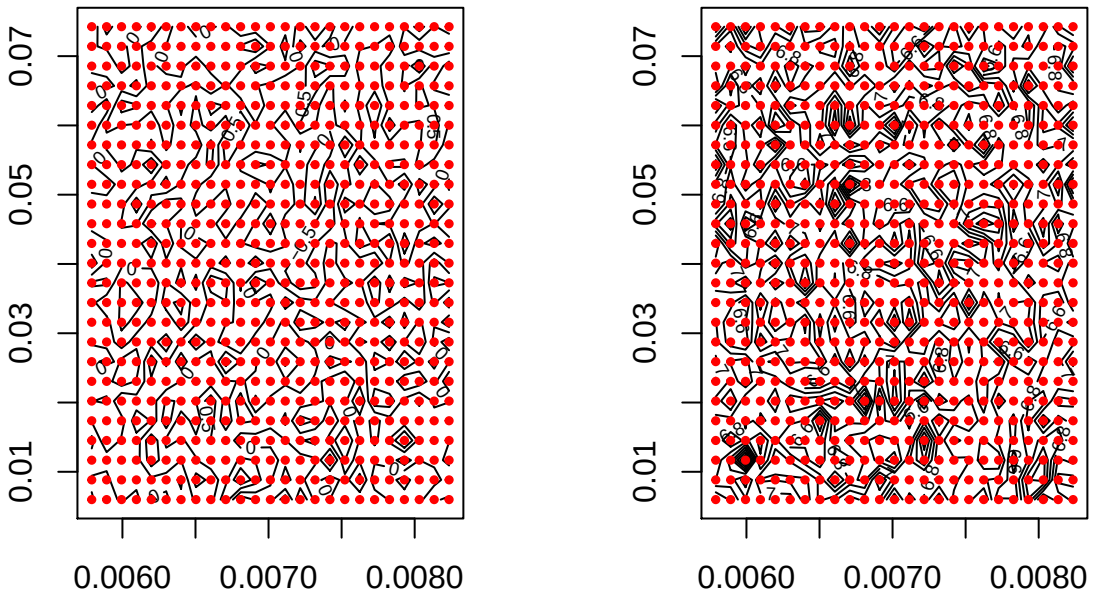
```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                69.8157 seconds (Sampling)
## Chain 1:                69.8157 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont <- extract(pred_gp_cont,permuted=FALSE)
y_mean_values_cont <- c(colMeans(y_predict_values_cont))
y_mean_values_cont <- y_mean_values_cont[1:(length(y_mean_values_cont)-1)]

#Computing Standard Deviation
pred_gp_summary_cont <- summary(pred_gp_cont, sd=c("sd"))$summary
pred_gp_sd_cont <- pred_gp_summary_cont[, c("sd")]
y_sd_values_cont <- pred_gp_sd_cont[1:(length(pred_gp_sd_cont)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
# x1_grid_cont <- seq(from=min(x.grid_1_cont), to=max(x.grid_1_cont), length.out=length(x.grid_1_cont))
# x2_grid_cont <- seq(from=min(x.grid_2_cont), to=max(x.grid_2_cont), length.out=length(x.grid_2_cont))

contour(dim1, dim2, matrix(y_mean_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



4-2: Contour Plots of Implied VOlatility & Time to Expiration

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end])),
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end])),n
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end]))
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end])),

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

blackscholes_2_cont <- rep(NA,length(x2_cont[,1]))
for (row in 1:nrow(data.frame(x2_cont))){
  blackscholes_2_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont[row,3],K=x2_cont[row,4],r=x2_cont[r
}

post_data_cont <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean_theta_3_Bd
# post_data

pred_gp_cont <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_cont,iter=200,

## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
```

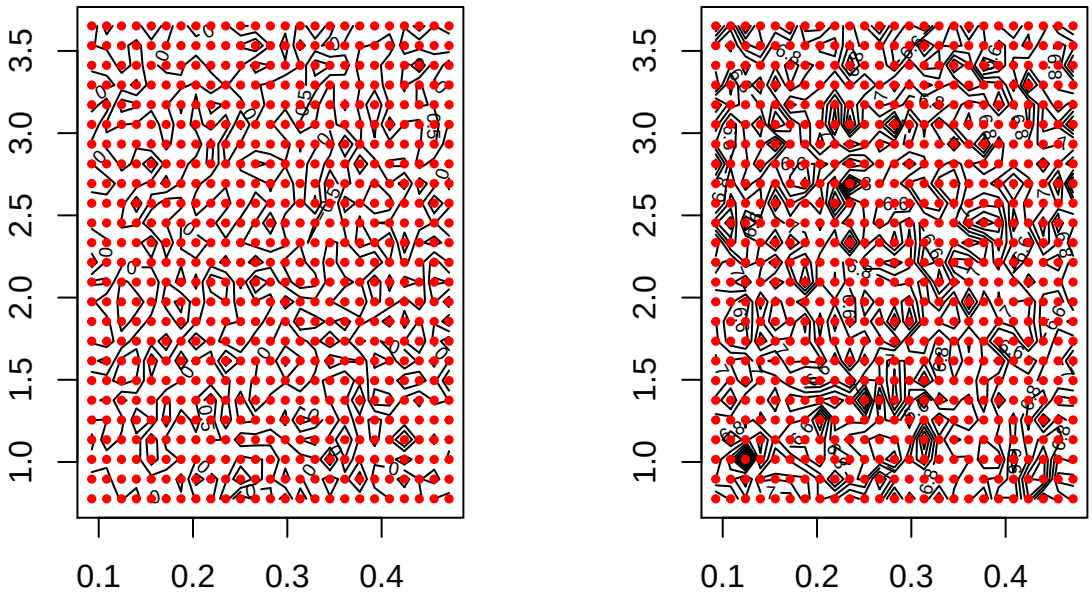
```
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                69.5378 seconds (Sampling)
## Chain 1:                69.5378 seconds (Total)
## Chain 1:
```

```
#Computing Mean
y_predict_values_cont <- extract(pred_gp_cont,permuted=FALSE)
y_mean_values_cont <- c(colMeans(y_predict_values_cont))
y_mean_values_cont <- y_mean_values_cont[1:(length(y_mean_values_cont)-1)]

#Computing Standard Deviation
pred_gp_summary_cont <- summary(pred_gp_cont, sd=c("sd"))$summary
pred_gp_sd_cont <- pred_gp_summary_cont[, c("sd")]
y_sd_values_cont <- pred_gp_sd_cont[1:(length(pred_gp_sd_cont)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
# x1_grid_cont <- seq(from=min(x.grid_1_cont), to=max(x.grid_1_cont), length.out=length(x.grid_1_cont))
# x2_grid_cont <- seq(from=min(x.grid_2_cont), to=max(x.grid_2_cont), length.out=length(x.grid_2_cont))

contour(dim1, dim2, matrix(y_mean_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



4-3: Contour Plots of Interest Rate & Time to Expiration

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$interest_rate[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end])),
```

```r
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end])),n
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end]))

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

blackscholes_2_cont <- rep(NA,length(x2_cont[,1]))
for (row in 1:nrow(data.frame(x2_cont))){
  blackscholes_2_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont[row,3],K=x2_cont[row,4],r=x2_cont[r
}

post_data_cont <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean_theta_3_Bd
# post_data

pred_gp_cont <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_cont,iter=200,
```

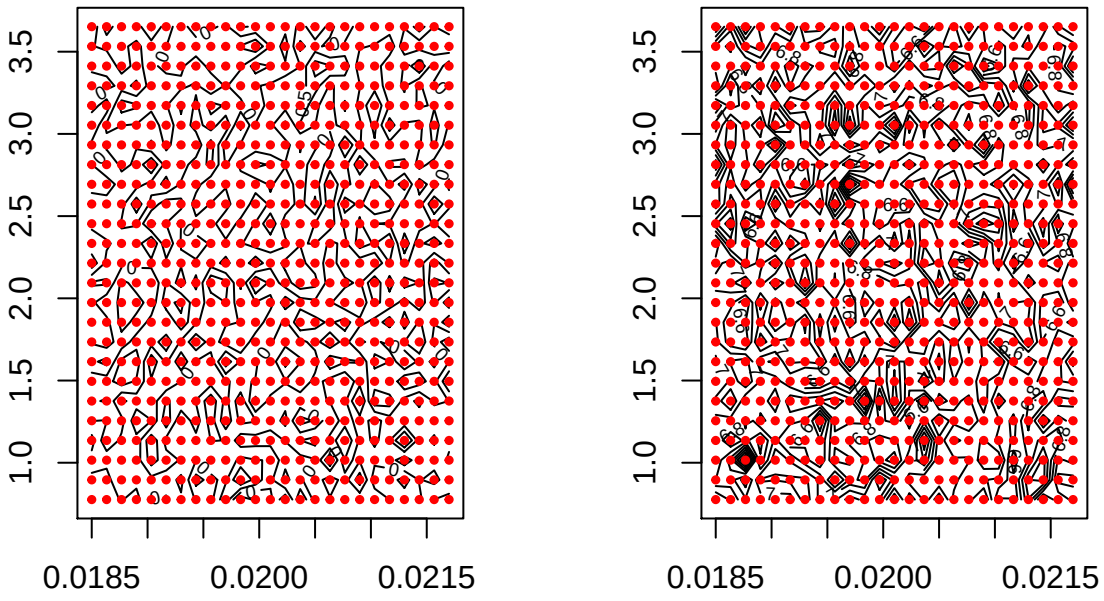
```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                70.8091 seconds (Sampling)
## Chain 1:                70.8091 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont <- extract(pred_gp_cont,permuted=FALSE)
y_mean_values_cont <- c(colMeans(y_predict_values_cont))
y_mean_values_cont <- y_mean_values_cont[1:(length(y_mean_values_cont)-1)]

#Computing Standard Deviation
pred_gp_summary_cont <- summary(pred_gp_cont, sd=c("sd"))$summary
pred_gp_sd_cont <- pred_gp_summary_cont[, c("sd")]
y_sd_values_cont <- pred_gp_sd_cont[1:(length(pred_gp_sd_cont)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
# x1_grid_cont <- seq(from=min(x.grid_1_cont), to=max(x.grid_1_cont), length.out=length(x.grid_1_cont))
# x2_grid_cont <- seq(from=min(x.grid_2_cont), to=max(x.grid_2_cont), length.out=length(x.grid_2_cont))

contour(dim1, dim2, matrix(y_mean_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



##Part 5: Improving the model by incorporating discrepancy

5-1: Computing Predicted European Option Prices

```
library(rstan)
source("gp.utility.R")

# Fitting GP model
stan_dat_European <- read_rdump('Financial_Data_Put_European.R')
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )
```

```
## See spec(...) for full column specifications.
```

```
## Warning: 98350 parsing failures.
##    row        col expected actual
## 142894 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 12/20/2019 a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## ...... .......... ........ ...... ..................................................................
## See problems(...) for more details.
```

```
## Warning in blackscholes[row] <- as.numeric(blackscholes(-1, S0 = x_1[row], :
## number of items to replace is not a multiple of replacement length

## Warning in blackscholes[row] <- as.numeric(blackscholes(-1, S0 = x_1[row], :
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```

```
fit_gp_SGP_European <- stan(file="gp-fit-6dimension_withBS.stan", data=stan_dat_European,
              iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.016684 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 166.84 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 16.6719 seconds (Warm-up)
## Chain 1:                27.9789 seconds (Sampling)
## Chain 1:                44.6508 seconds (Total)
## Chain 1:
```

```
## Warning: The largest R-hat is 1.67, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat
```

```
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess
```

```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
print(fit_gp_SGP_European, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##               mean  se_mean        sd      2.5%       25%       50%       75%
## theta[1]      3.09     0.29      1.84      1.15      1.79      2.62      3.84
## theta[2]      1.88     0.13      0.64      1.03      1.49      1.65      2.29
## theta[3]     17.67     0.37      2.59     12.94     16.26     17.93     18.88
## theta[4]      0.12     0.00      0.02      0.09      0.11      0.12      0.13
## theta[5]    507.67   373.20   1902.90      0.07      0.23      0.51     16.06
## theta[6]     25.29     2.18     14.49      6.06     13.63     22.15     33.64
## sigma2        0.00     0.00      0.00      0.00      0.00      0.00      0.00
## gamma2   831203.51 73980.94 373231.18 321447.70 602426.75 792242.88 971858.93
##               97.5% n_eff Rhat
## theta[1]       7.30    40 0.98
## theta[2]       3.42    23 1.01
## theta[3]      22.99    49 1.02
## theta[4]       0.16    43 0.98
## theta[5]    7687.69    26 1.05
## theta[6]      56.66    44 0.99
## sigma2         0.00    85 1.00
## gamma2   1721184.73    25 0.98
##
## Samples were drawn using NUTS(diag_e) at Thu Mar 26 16:45:54 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_SGP_European <- extract(fit_gp_SGP_European,permuted=FALSE)
```

```
# Predicting from GP model
post_mean_theta_1_SGP <- mean(sum_gp_SGP_European[,1,1]) #theta
post_mean_theta_2_SGP <- mean(sum_gp_SGP_European[,1,2]) #theta
post_mean_theta_3_SGP <- mean(sum_gp_SGP_European[,1,3]) #theta
post_mean_theta_4_SGP <- mean(sum_gp_SGP_European[,1,4]) #theta
post_mean_theta_5_SGP <- mean(sum_gp_SGP_European[,1,5]) #theta
post_mean_theta_6_SGP <- mean(sum_gp_SGP_European[,1,6]) #theta
post_mean_sigma2_SGP <- mean(sum_gp_SGP_European[,1,7]) #sigma2
post_mean_gamma2_SGP <- mean(sum_gp_SGP_European[,1,8]) #gamma2
post_mean_mu_SGP <- stan_dat_European$blackscholes


# x2 <- as.numeric(unlist(spx_spy_2019_06_30_put_2017_06_500rows_test['strike_price']))
# x2<- cbind(spy_2013_01_01_2013_01_31_put$strike_price[201:300],spy_2013_01_01_2013_01_31_put$impl_vol
# x2 <- seq(from=-2,to=2,by=0.01)

# x2 <- cbind(seq(from=0,to=1,by=0.01),seq(from=0,to=1,by=0.01))

# test_start <- 323 #06/10
# test_end <- 559 #06/14

test_start <- 560 #06/17
```

```
test_end <- 852 #06/20

x.grid_1 <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2 <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_3 <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_4 <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])
x.grid_5 <- as.numeric(stan_dat$total_puts_American$dividend[test_start:test_end])
x.grid_6 <- as.numeric(stan_dat$total_puts_American$interest_rate[test_start:test_end])
x2 <- cbind(x.grid_1,x.grid_2,x.grid_3,x.grid_4,x.grid_5,x.grid_6)
```

```
library('qrmtools')
library('ragtop')
blackscholes_2 <- rep(NA,length(x2[,1]))
for (row in 1:nrow(data.frame(x2))){
  blackscholes_2[row] <- as.numeric(blackscholes(-1,S0=x.grid_1[row],K=x.grid_2[row],r=x.grid_6[row],t=
  # blackscholes_2[row] <- Black_Scholes(0,x.grid_1[row],x.grid_6[row],x.grid_3[row],x.grid_2[row],x.gr
}
```

```
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_Bdrycov_American_disc <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_t
# post_data

pred_gp_Bdrycov_disc <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_Bdryco
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                24.8167 seconds (Sampling)
## Chain 1:                24.8167 seconds (Total)
## Chain 1:
```

```
#Computing Mean
y_predict_values_Bdrycov_disc <- extract(pred_gp_Bdrycov_disc,permuted=FALSE)
y_mean_values_Bdrycov_disc <- c(colMeans(y_predict_values_Bdrycov_disc))
y_mean_values_Bdrycov_disc <- y_mean_values_Bdrycov_disc[1:(length(y_mean_values_Bdrycov_disc)-1)]

#Computing Standard Deviation
pred_gp_summary_Bdrycov_disc <- summary(pred_gp_Bdrycov_disc, sd=c("sd"))$summary
pred_gp_sd_Bdrycov_disc <- pred_gp_summary_Bdrycov_disc[, c("sd")]
y_sd_values_Bdrycov_disc <- pred_gp_sd_Bdrycov_disc[1:(length(pred_gp_sd_Bdrycov_disc)-1)]
```

3-3: Plotting Predicted Values against Truth

```
par(mfrow=c(1,4))
#Plotting Standard GP
plot(log(y_mean_values_SGP),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
```

```
## Warning in log(y_mean_values_SGP): NaNs produced
```

```
abline(0,1)

#Plotting BDrycov
plot(log(y_mean_values_Bdrycov),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
```

```
## Warning in log(y_mean_values_Bdrycov): NaNs produced
```

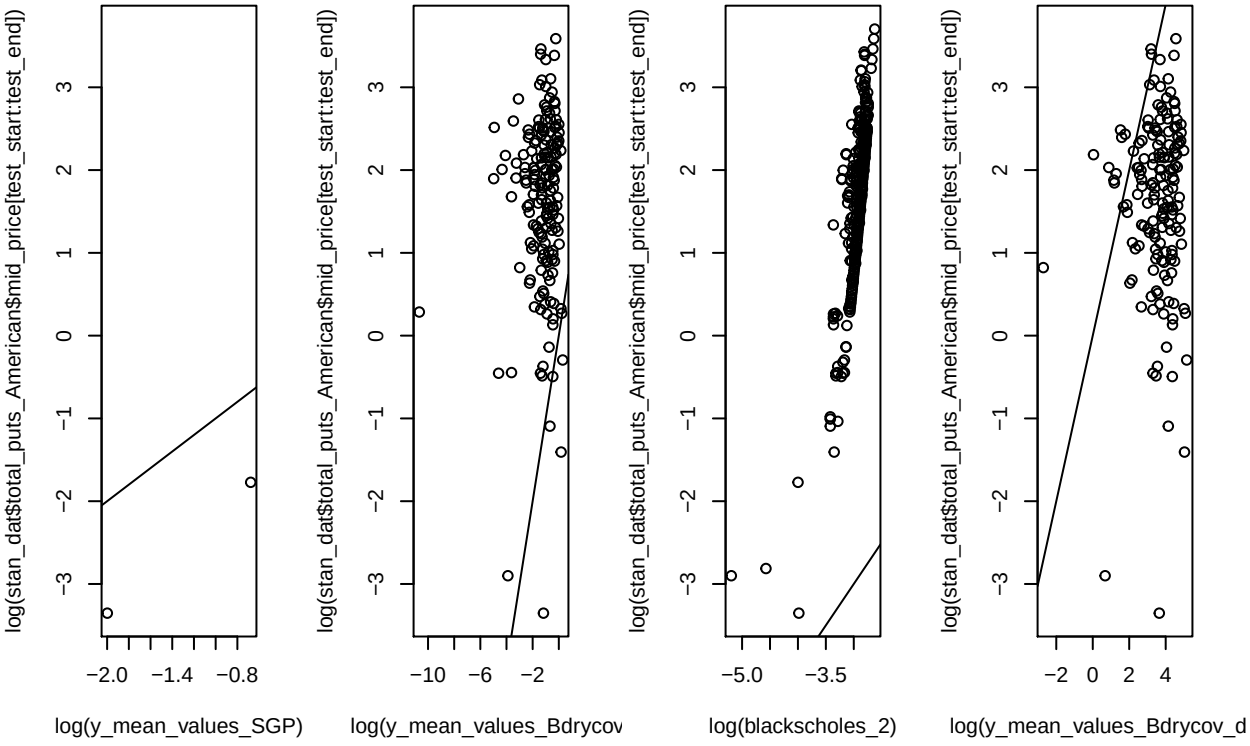
```
abline(0,1)

#Plotting Blackscholes
plot(log(blackscholes_2),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
abline(0,1)

#Plotting Blackscholes
plot(log(y_mean_values_Bdrycov_disc),log(stan_dat$total_puts_American$mid_price[test_start:test_end]))
```

```
## Warning in log(y_mean_values_Bdrycov_disc): NaNs produced
```

```
abline(0,1)
```




```
#MSE
library('MLmetrics')
MSE(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 205.4539
```

```
MSE(y_mean_values_Bdrycov,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 100.1495
```

```
MSE(blackscholes_2,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 100.3091
```

```r
MSE(y_mean_values_Bdrycov_disc,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 4205.734
```

MY EXPERIMENT===

```r
library(rstan)
source("gp.utility.R")

# Fitting GP model
stan_dat_European_American<- read_rdump('Financial_Data_Put_European_American.R')
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )

## See spec(...) for full column specifications.

## Warning: 98350 parsing failures.
##    row        col expected actual
## 142894 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142894 12/20/2019 a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 6/21/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## 142895 9/20/2019  a double  FALSE '~/projects/Independent_Study/spy_spx_(2019.06.01~2019.06.30)_Puts
## ...... .......... ........ ...... ..................................................................
## See problems(...) for more details.

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```
fit_gp_SGP_European_American <- stan(file="gp-fit-6dimension_withBS.stan", data=stan_dat_European_Ameri
               iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.090324 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 903.24 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 167.294 seconds (Warm-up)
## Chain 1:                146.327 seconds (Sampling)
## Chain 1:                313.62 seconds (Total)
## Chain 1:
```

```
## Warning: The largest R-hat is 1.09, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat
```

```
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess
```

```
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quanti
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
print(fit_gp_SGP_European_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##               mean  se_mean        sd      2.5%       25%       50%       75%
```

```
## theta[1]      2.54     0.35      1.76      0.83      1.67      2.00      2.71
## theta[2]      3.63     0.24      1.22      2.17      2.60      3.37      4.44
## theta[3]     13.55     0.16      1.20     11.80     12.56     13.61     14.00
## theta[4]      0.20     0.00      0.02      0.16      0.18      0.19      0.21
## theta[5]   1884.90   355.67   1714.03     23.66    702.13   1432.94   2404.81
## theta[6]      5.64     0.45      2.49      1.34      3.70      5.95      6.79
## sigma2        0.00     0.00      0.00      0.00      0.00      0.00      0.00
## gamma2   563096.03 61134.86 226781.11 263875.71 368746.68 498783.76 746708.26
##              97.5% n_eff Rhat
## theta[1]      8.29    25 1.07
## theta[2]      6.15    25 0.99
## theta[3]     16.00    59 0.99
## theta[4]      0.23    30 1.04
## theta[5]   5216.95    23 1.01
## theta[6]     10.54    31 0.98
## sigma2        0.00    52 1.00
## gamma2   969048.96    14 1.06
##
## Samples were drawn using NUTS(diag_e) at Thu Mar 26 16:51:39 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_SGP_European_American <- extract(fit_gp_SGP_European_American,permuted=FALSE)
```

```
# Predicting from GP model
post_mean_theta_1_SGP <- mean(sum_gp_SGP_European_American[,1,1]) #theta
post_mean_theta_2_SGP <- mean(sum_gp_SGP_European_American[,1,2]) #theta
post_mean_theta_3_SGP <- mean(sum_gp_SGP_European_American[,1,3]) #theta
post_mean_theta_4_SGP <- mean(sum_gp_SGP_European_American[,1,4]) #theta
post_mean_theta_5_SGP <- mean(sum_gp_SGP_European_American[,1,5]) #theta
post_mean_theta_6_SGP <- mean(sum_gp_SGP_European_American[,1,6]) #theta
post_mean_sigma2_SGP <- mean(sum_gp_SGP_European_American[,1,7]) #sigma2
post_mean_gamma2_SGP <- mean(sum_gp_SGP_European_American[,1,8]) #gamma2
post_mean_mu_SGP <- stan_dat_European_American$blackscholes


# x2 <- as.numeric(unlist(spx_spy_2019_06_30_put_2017_06_500rows_test['strike_price']))
# x2<- cbind(spy_2013_01_01_2013_01_31_put$strike_price[201:300],spy_2013_01_01_2013_01_31_put$impl_vol
# x2 <- seq(from=-2,to=2,by=0.01)

# x2 <- cbind(seq(from=0,to=1,by=0.01),seq(from=0,to=1,by=0.01))

# test_start <- 323 #06/10
# test_end <- 559 #06/14

test_start <- 560 #06/17
test_end <- 852 #06/20

x.grid_1 <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2 <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_3 <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_4 <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])
x.grid_5 <- as.numeric(stan_dat$total_puts_American$dividend[test_start:test_end])
x.grid_6 <- as.numeric(stan_dat$total_puts_American$interest_rate[test_start:test_end])
```

```
x2 <- cbind(x.grid_1,x.grid_2,x.grid_3,x.grid_4,x.grid_5,x.grid_6)
```

```
library('qrmtools')
library('ragtop')
blackscholes_2 <- rep(NA,length(x2[,1]))
for (row in 1:nrow(data.frame(x2))){
  blackscholes_2[row] <- as.numeric(blackscholes(-1,S0=x.grid_1[row],K=x.grid_2[row],r=x.grid_6[row],t=
  # blackscholes_2[row] <- Black_Scholes(0,x.grid_1[row],x.grid_6[row],x.grid_3[row],x.grid_2[row],x.gr
}
```

```
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_Bdrycov_European_American_disc <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,pos
# post_data

pred_gp_Bdrycov_disc <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_Bdrycov
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                52.4274 seconds (Sampling)
## Chain 1:                52.4274 seconds (Total)
## Chain 1:
```

```
#Computing Mean
y_predict_values_Bdrycov_disc <- extract(pred_gp_Bdrycov_disc,permuted=FALSE)
y_mean_values_Bdrycov_disc <- c(colMeans(y_predict_values_Bdrycov_disc))
y_mean_values_Bdrycov_disc <- y_mean_values_Bdrycov_disc[1:(length(y_mean_values_Bdrycov_disc)-1)]

#Computing Standard Deviation
pred_gp_summary_Bdrycov_disc <- summary(pred_gp_Bdrycov_disc, sd=c("sd"))$summary
pred_gp_sd_Bdrycov_disc <- pred_gp_summary_Bdrycov_disc[, c("sd")]
y_sd_values_Bdrycov_disc <- pred_gp_sd_Bdrycov_disc[1:(length(pred_gp_sd_Bdrycov_disc)-1)]
```

3-3: Plotting Predicted Values against Truth

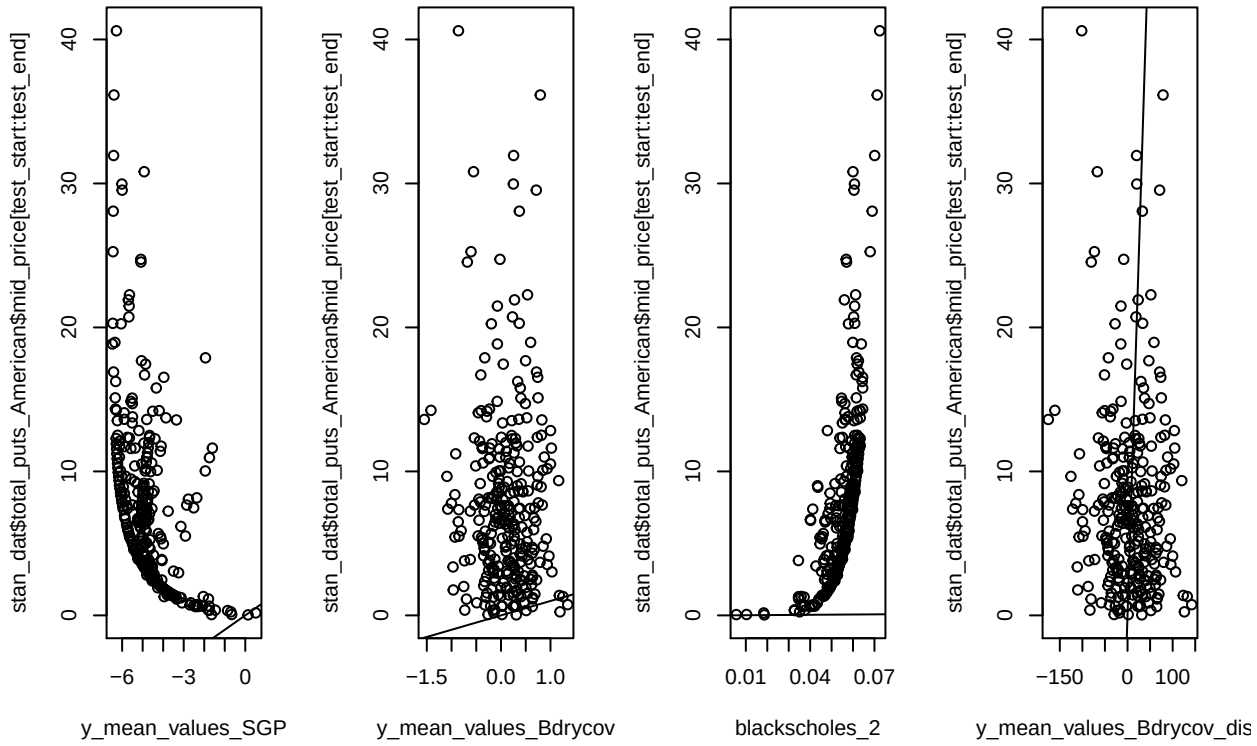
```
par(mfrow=c(1,4))
#Plotting Standard GP
plot(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
abline(0,1)

#Plotting BDrycov
plot(y_mean_values_Bdrycov,stan_dat$total_puts_American$mid_price[test_start:test_end])
abline(0,1)

#Plotting Blackscholes
plot(blackscholes_2,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```r
abline(0,1)

#Plotting Blackscholes
plot(y_mean_values_Bdrycov_disc,stan_dat$total_puts_American$mid_price[test_start:test_end])
abline(0,1)
```




```r
#MSE
library('MLmetrics')
MSE(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 205.4539
```

```r
MSE(y_mean_values_Bdrycov,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 100.1495
```

```r
MSE(blackscholes_2,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 100.3091
```

```r
MSE(y_mean_values_Bdrycov_disc,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 2873.863
```