Gaussian_Process_Code

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##Part 1: Standard Gaussian Process

1-1: Fitting

```
library(rstan)
```

```
## Loading required package: StanHeaders

## Loading required package: ggplot2

## rstan (Version 2.19.2, GitRev: 2e1f913d3ca3)

## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)

## For improved execution time, we recommend calling
## Sys.setenv(LOCAL_CPPFLAGS = '-march=native')
## although this causes Stan to throw an error on a few processors.
```

```
source("gp.utility.R")

# Fitting GP model
stan_dat <- read_rdump('Financial_Data_Put_American.R')
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

## Loading required package: limSolve

##
## Attaching package: 'limSolve'

## The following object is masked from 'package:ggplot2':
##
##     resolution
```

```
## Loading required package: futile.logger

## Welcome to ragtop.  Logging can be enabled with commands such as
##   futile.logger::flog.threshold(futile.logger::INFO, name='ragtop.calibration')

## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )

## See spec(...) for full column specifications.
```

```
fit_gp_SGP_American <- stan(file="gp-fit-6dimension_SGP.stan", data=stan_dat,
               iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_SGP' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.097 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 970 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
```

```
## Chain 1:
## Chain 1:  Elapsed Time: 297.406 seconds (Warm-up)
## Chain 1:                298.888 seconds (Sampling)
## Chain 1:                596.294 seconds (Total)
## Chain 1:
```

```
print(fit_gp_SGP_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_SGP.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##             mean se_mean      sd   2.5%     25%     50%     75%   97.5%
## theta[1]    0.05    0.00    0.03   0.01    0.03    0.04    0.05    0.12
## theta[2]    1.33    0.14    0.77   0.56    0.82    1.03    1.79    3.20
## theta[3]    3.11    0.10    0.81   1.82    2.59    2.99    3.53    4.90
## theta[4]    0.27    0.02    0.13   0.09    0.18    0.25    0.33    0.54
## theta[5]    0.05    0.00    0.03   0.02    0.03    0.04    0.06    0.12
## theta[6]    0.02    0.00    0.01   0.01    0.01    0.01    0.02    0.03
## sigma2      0.00    0.00    0.00   0.00    0.00    0.00    0.00    0.00
## gamma2   2548.58  265.04 1525.68 651.55 1544.73 2054.95 3234.42 5866.57
##          n_eff Rhat
## theta[1]    85 0.98
## theta[2]    29 0.98
## theta[3]    63 1.02
## theta[4]    35 0.98
## theta[5]    81 1.01
## theta[6]    50 1.03
## sigma2      49 1.03
## gamma2      33 1.00
##
## Samples were drawn using NUTS(diag_e) at Sun Apr 05 02:46:28 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_SGP_American <- extract(fit_gp_SGP_American,permuted=FALSE)
```

```
# Predicting from GP model
post_mean_theta_1_SGP <- mean(sum_gp_SGP_American[,1,1]) #theta
post_mean_theta_2_SGP <- mean(sum_gp_SGP_American[,1,2]) #theta
post_mean_theta_3_SGP <- mean(sum_gp_SGP_American[,1,3]) #theta
post_mean_theta_4_SGP <- mean(sum_gp_SGP_American[,1,4]) #theta
post_mean_theta_5_SGP <- mean(sum_gp_SGP_American[,1,5]) #theta
post_mean_theta_6_SGP <- mean(sum_gp_SGP_American[,1,6]) #theta
post_mean_sigma2_SGP <- mean(sum_gp_SGP_American[,1,7]) #sigma2
post_mean_gamma2_SGP <- mean(sum_gp_SGP_American[,1,8]) #gamma2
post_mean_mu_SGP <- mean(sum_gp_SGP_American[,1,9])
```

```
test_start <- 323 #06/10 Puts
test_end <- 559 #06/14 Puts
```

```
# test_start <- 560 #06/17 Puts
# test_end <- 852 #06/20 Puts

# test_start <- 609 #06/10 Calls
# test_end <- 999 #06/14 Calls

# test_start <- 433 #06/17 Calls
# test_end <- 700 #06/20 Calls

x_1 <- as.numeric(stan_dat$total_puts_American$forward_price_scaled[test_start:test_end])
x_2 <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])
x_3 <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])
x_4 <- as.numeric(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_end])
x_5 <- as.numeric(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_end])
x_6 <- as.numeric(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_end])
x2 <- cbind(x_1,x_2,x_3,x_4,x_5,x_6)
```

```
x2_bs <- cbind(as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end]),as.numeric(s


library('qrmtools')
```

```
## Registered S3 method overwritten by 'xts':
##   method     from
##   as.zoo.xts zoo
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library('ragtop')
blackscholes_test <- rep(NA,length(x2_bs[,1]))
for (row in 1:nrow(data.frame(x2_bs))){
  blackscholes_test[row] <- as.numeric(blackscholes(-1,S0=as.numeric(stan_dat$total_puts_American$forwa
}
```

1-2: Predictions

```
post_data_SGP_American <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_theta_3_SG
# post_data

pred_gp_SGP <- stan(file="Predictive GP_6dimension_SGP.stan", data=post_data_SGP_American,iter=200, war
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
```

```
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                27.983 seconds (Sampling)
## Chain 1:                27.983 seconds (Total)
## Chain 1:
```

##Part2: BS Integrated SGP

2-1: Fitting

```
# Fitting GP model for Bdrycov
fit_gp_bs_American <- stan(file="gp-fit-6dimension_withBS_SGP.stan", data=stan_dat,
               iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.101 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 1010 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 87.806 seconds (Warm-up)
## Chain 1:                102.405 seconds (Sampling)
## Chain 1:                190.211 seconds (Total)
## Chain 1:
```

```r
print(fit_gp_bs_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS_SGP.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##           mean se_mean    sd 2.5%   25%   50%   75% 97.5% n_eff Rhat
## theta[1]  0.07    0.01  0.04 0.02  0.04  0.05  0.09  0.15    33 1.04
## theta[2]  2.01    0.16  0.86 0.64  1.51  1.83  2.58  3.80    27 0.99
## theta[3]  4.80    0.27  1.91 2.10  3.59  4.23  5.92  8.70    51 0.99
## theta[4]  0.47    0.03  0.15 0.24  0.36  0.45  0.56  0.79    31 0.98
## theta[5]  0.07    0.01  0.04 0.02  0.04  0.05  0.08  0.16    41 0.98
## theta[6]  0.05    0.00  0.02 0.02  0.04  0.05  0.06  0.10    28 0.99
## sigma2    0.00    0.00  0.00 0.00  0.00  0.00  0.00  0.00    36 1.03
## gamma2   30.68    5.30 23.55 8.51 14.32 23.01 34.26 94.96    20 0.98
##
## Samples were drawn using NUTS(diag_e) at Sun Apr 05 02:53:05 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```r
sum_gp_bs_American <- extract(fit_gp_bs_American,permuted=FALSE)
```

```r
# Predicting from GP model - 2 dimensional case
post_mean_theta_1_bs <- mean(sum_gp_bs_American[,1,1]) #theta
post_mean_theta_2_bs <- mean(sum_gp_bs_American[,1,2]) #theta
post_mean_theta_3_bs <- mean(sum_gp_bs_American[,1,3]) #theta
post_mean_theta_4_bs <- mean(sum_gp_bs_American[,1,4]) #theta
post_mean_theta_5_bs <- mean(sum_gp_bs_American[,1,5]) #theta
post_mean_theta_6_bs <- mean(sum_gp_bs_American[,1,6]) #theta
post_mean_sigma2_bs <- mean(sum_gp_bs_American[,1,7]) #sigma2
post_mean_gamma2_bs <- mean(sum_gp_bs_American[,1,8]) #gamma2
post_mean_mu_bs <- stan_dat$blackscholes
```

2-2: Predictions

```r
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_bs_American <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,pos
# post_data

pred_gp_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_bs_American,iter=200
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
```

```
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                27.922 seconds (Sampling)
## Chain 1:                27.922 seconds (Total)
## Chain 1:
```

##Part 3 Predictions Versus Truth

3-1: Computing Means Standard GP

```
#Computing Mean
y_predict_values_SGP <- extract(pred_gp_SGP,permuted=FALSE)
y_mean_values_SGP <- c(colMeans(y_predict_values_SGP))
y_mean_values_SGP <- y_mean_values_SGP[1:(length(y_mean_values_SGP)-1)]

#Computing Standard Deviation
pred_gp_summary_SGP <- summary(pred_gp_SGP, sd=c("sd"))$summary
pred_gp_sd_SGP <- pred_gp_summary_SGP[, c("sd")]
y_sd_values_SGP <- pred_gp_sd_SGP[1:(length(pred_gp_sd_SGP)-1)]
```

3-2: Computing Means bs

```
#Computing Mean
y_predict_values_bs <- extract(pred_gp_bs,permuted=FALSE)
y_mean_values_bs <- c(colMeans(y_predict_values_bs))
y_mean_values_bs <- y_mean_values_bs[1:(length(y_mean_values_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_bs <- summary(pred_gp_bs, sd=c("sd"))$summary
pred_gp_sd_bs <- pred_gp_summary_bs[, c("sd")]
y_sd_values_bs <- pred_gp_sd_bs[1:(length(pred_gp_sd_bs)-1)]
```

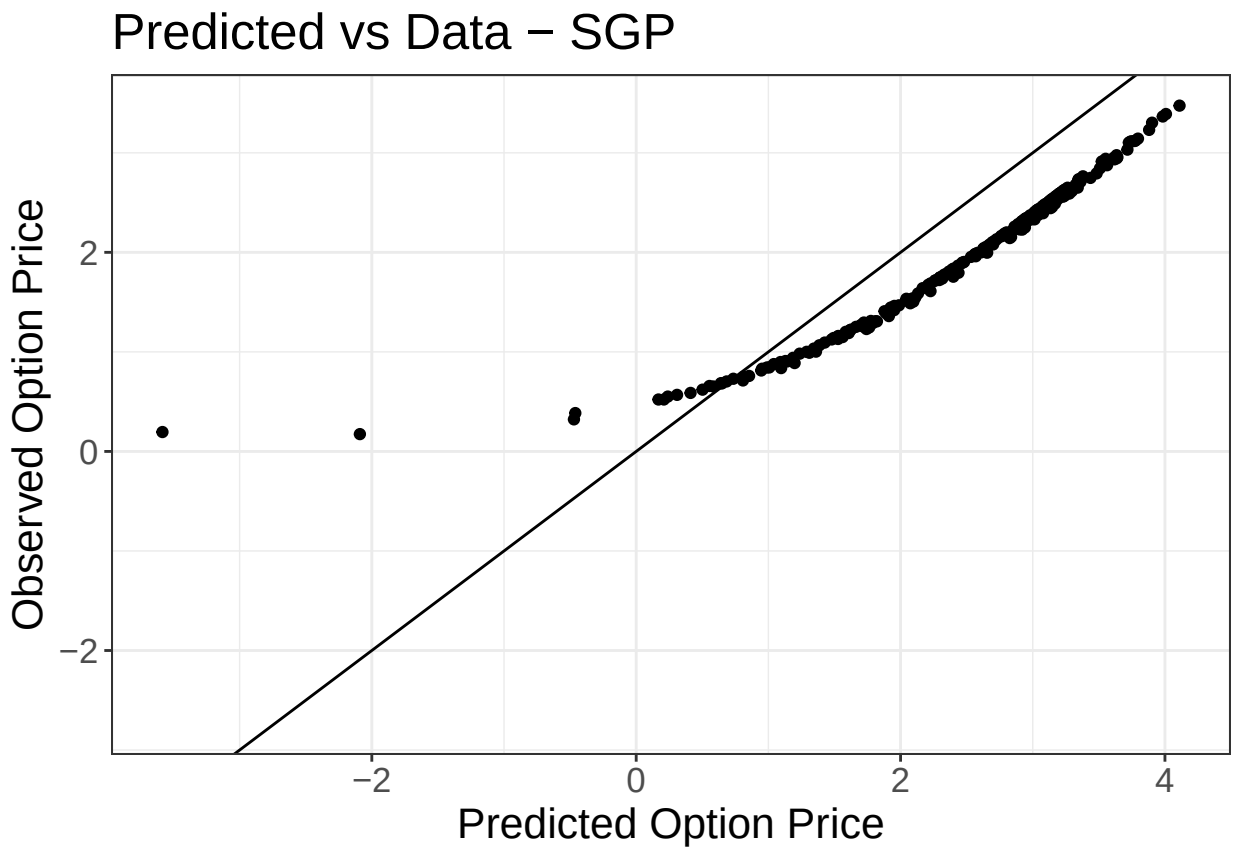
3-3: Plotting Predicted Values against Truth

```
par(mfrow=c(1,3))
#Plotting Standard GP
ggplot(mapping=aes(x=log(y_mean_values_SGP),y=log(stan_dat$total_puts_American$mid_price[test_start:tes
  geom_point() +
  labs(title = "Predicted vs Data - SGP",
       x = "Predicted Option Price",
       y = "Observed Option Price") +
  theme_bw() +
  theme(text=element_text(size=16)) +
  geom_abline(intercept = 0, slope = 1)
```

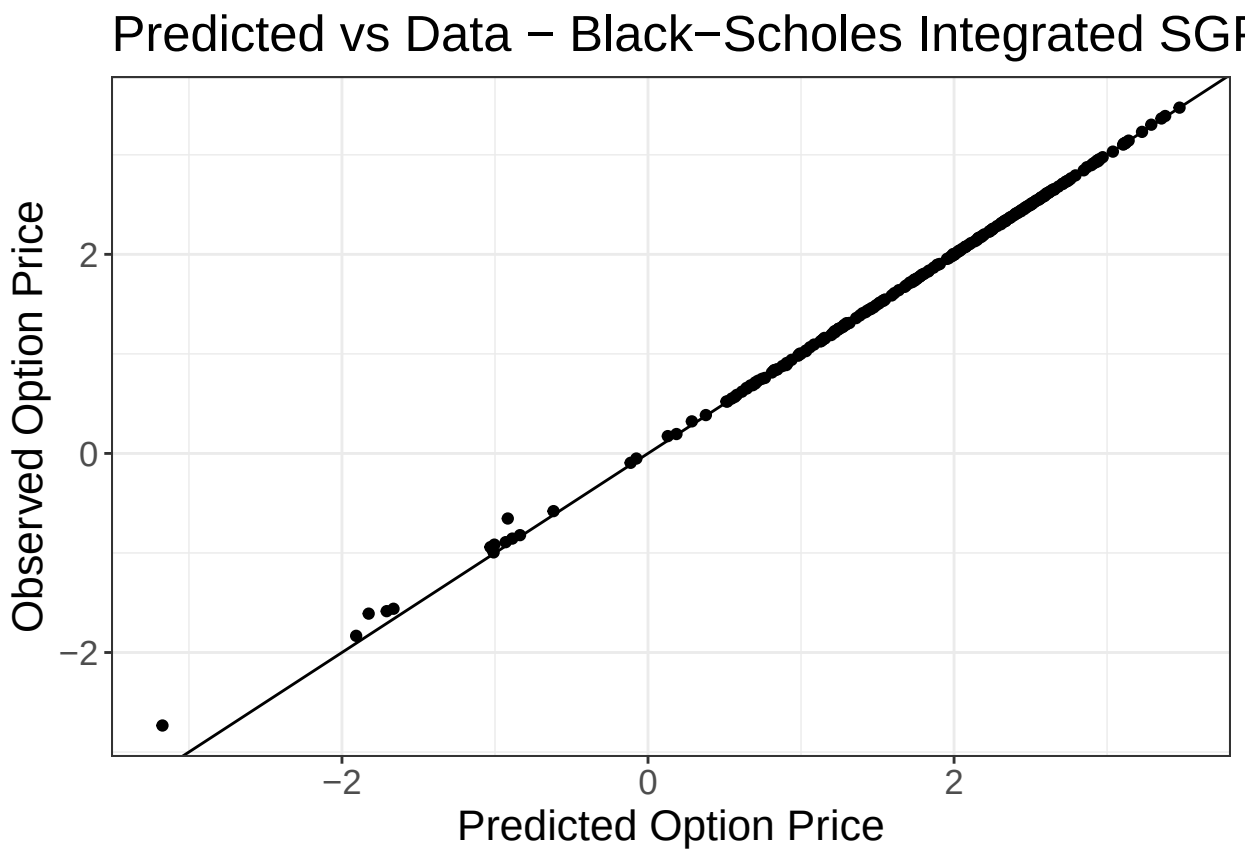
```
## Warning in log(y_mean_values_SGP): NaNs produced
```

```
## Warning in log(y_mean_values_SGP): NaNs produced
```

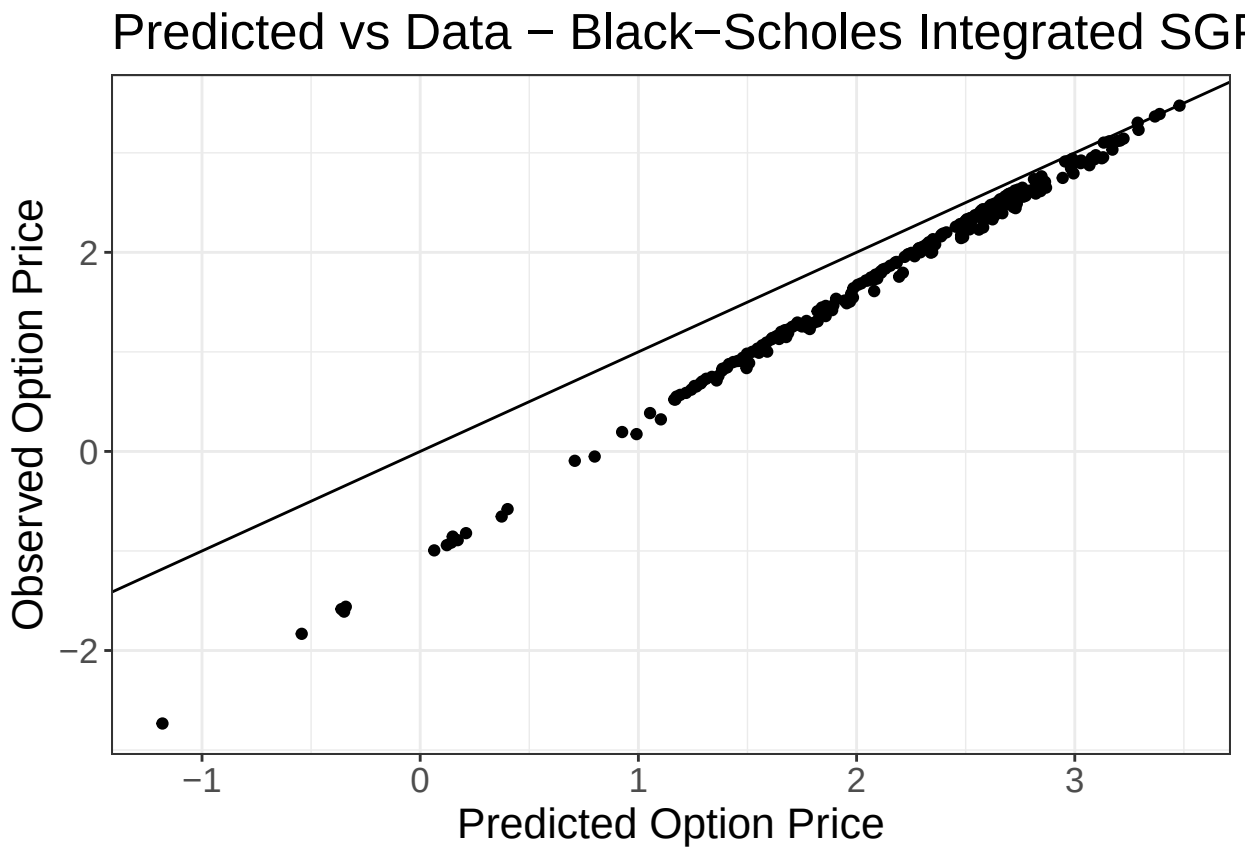
```
## Warning: Removed 15 rows containing missing values (geom_point).
```




```
#Plotting bs
ggplot(mapping=aes(x=log(y_mean_values_bs),y=log(stan_dat$total_puts_American$mid_price[test_start:test_
  geom_point() +
  labs(title = "Predicted vs Data - Black-Scholes Integrated SGP",
       x = "Predicted Option Price",
       y = "Observed Option Price") +
  theme_bw() +
  theme(text=element_text(size=16))+
  geom_abline(intercept = 0, slope = 1)
```




```
#Plotting Blackscholes
ggplot(mapping=aes(x=log(blackscholes_test),y=log(stan_dat$total_puts_American$mid_price[test_start:tes
  geom_point() +
  labs(title = "Predicted vs Data - Black-Scholes Integrated SGP",
       x = "Predicted Option Price",
       y = "Observed Option Price") +
  theme_bw() +
  theme(text=element_text(size=16))+
  geom_abline(intercept = 0, slope = 1)
```




```
#MSE
library('MLmetrics')
```

```
##
## Attaching package: 'MLmetrics'
```

```
## The following object is masked from 'package:base':
##
##     Recall
```

```
MSE(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 85.07085
```

```
MSE(y_mean_values_bs,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 0.002621515
```

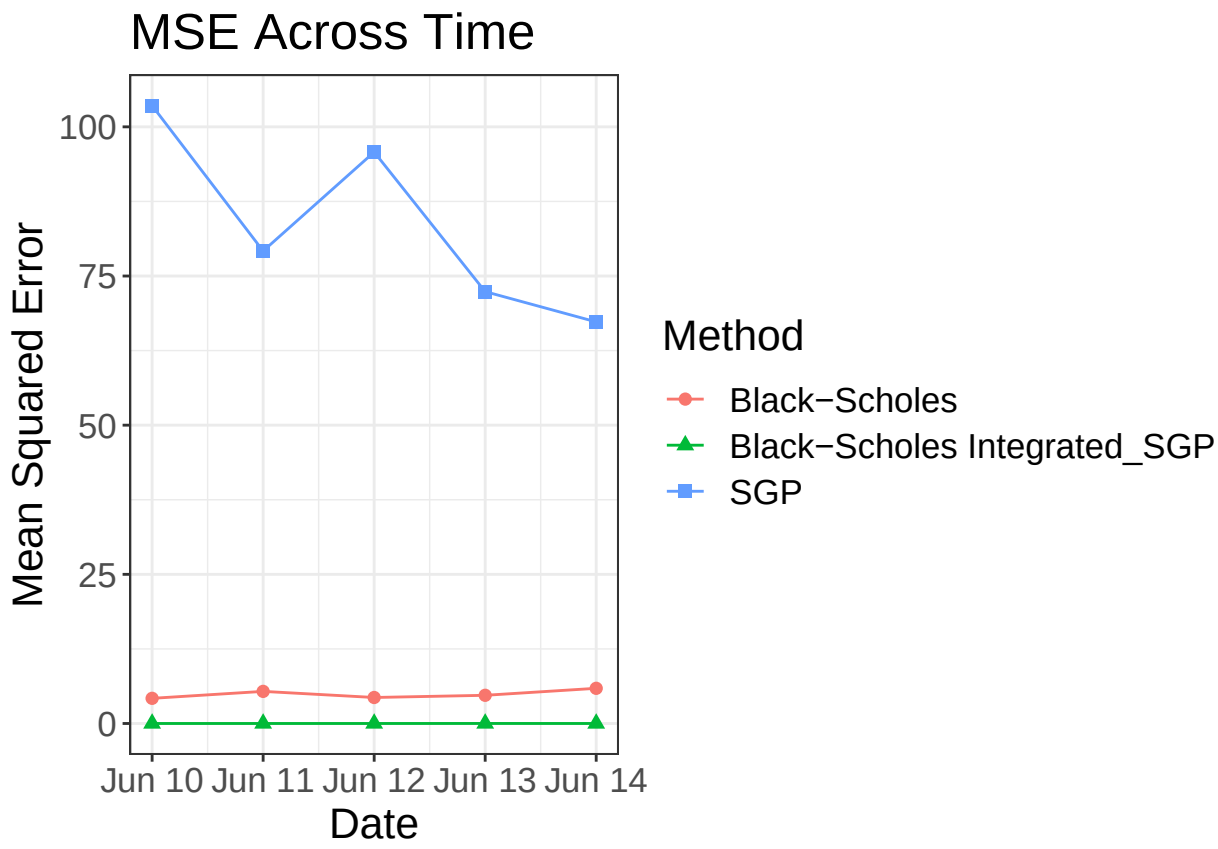
```
MSE(blackscholes_test,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 4.923348
```

##Part 4 MSE over Time

4-1: Plotting MSE over time

```
#MSE
library('MLmetrics')
SGP_MSE_0610 <- MSE(y_mean_values_SGP[1:71],stan_dat$total_puts_American$mid_price[323:393])
SGP_MSE_0611 <- MSE(y_mean_values_SGP[72:143],stan_dat$total_puts_American$mid_price[394:465])
SGP_MSE_0612 <- MSE(y_mean_values_SGP[144:164],stan_dat$total_puts_American$mid_price[466:486])
SGP_MSE_0613 <- MSE(y_mean_values_SGP[165:200],stan_dat$total_puts_American$mid_price[487:522])
SGP_MSE_0614 <- MSE(y_mean_values_SGP[201:237],stan_dat$total_puts_American$mid_price[523:559])
SGP_MSE <- rbind(SGP_MSE_0610,SGP_MSE_0611,SGP_MSE_0612,SGP_MSE_0613,SGP_MSE_0614)

BS_SGP_MSE_0610 <- MSE(y_mean_values_bs[1:71],stan_dat$total_puts_American$mid_price[323:393])
BS_SGP_MSE_0611 <- MSE(y_mean_values_bs[72:143],stan_dat$total_puts_American$mid_price[394:465])
BS_SGP_MSE_0612 <- MSE(y_mean_values_bs[144:164],stan_dat$total_puts_American$mid_price[466:486])
BS_SGP_MSE_0613 <- MSE(y_mean_values_bs[165:200],stan_dat$total_puts_American$mid_price[487:522])
BS_SGP_MSE_0614 <- MSE(y_mean_values_bs[201:237],stan_dat$total_puts_American$mid_price[523:559])
BS_SGP_MSE <- rbind(BS_SGP_MSE_0610,BS_SGP_MSE_0611,BS_SGP_MSE_0612,BS_SGP_MSE_0613,BS_SGP_MSE_0614)

BS_MSE_0610 <- MSE(blackscholes_test[1:71],stan_dat$total_puts_American$mid_price[323:393])
BS_MSE_0611 <- MSE(blackscholes_test[72:143],stan_dat$total_puts_American$mid_price[394:465])
BS_MSE_0612 <- MSE(blackscholes_test[144:164],stan_dat$total_puts_American$mid_price[466:486])
BS_MSE_0613 <- MSE(blackscholes_test[165:200],stan_dat$total_puts_American$mid_price[487:522])
BS_MSE_0614 <- MSE(blackscholes_test[201:237],stan_dat$total_puts_American$mid_price[523:559])
BS_MSE <- rbind(BS_MSE_0610,BS_MSE_0611,BS_MSE_0612,BS_MSE_0613,BS_MSE_0614)

MSE_data <- as.data.frame(rbind(SGP_MSE,BS_SGP_MSE,BS_MSE))

MSE_data$date <- c(as.Date('00/00/0000', format = '%m/%d/%Y'))
MSE_data$date[c(1,6,11)] <- as.Date('06/10/2019', format = '%m/%d/%Y')
MSE_data$date[c(2,7,12)] <- as.Date('06/11/2019', format = '%m/%d/%Y')
MSE_data$date[c(3,8,13)] <- as.Date('06/12/2019', format = '%m/%d/%Y')
MSE_data$date[c(4,9,14)] <- as.Date('06/13/2019', format = '%m/%d/%Y')
MSE_data$date[c(5,10,15)] <- as.Date('06/14/2019', format = '%m/%d/%Y')

MSE_data$Method <- "Method"
MSE_data$Method[1:5] <- "SGP"
MSE_data$Method[6:10] <- "Black-Scholes Integrated_SGP"
MSE_data$Method[11:15] <- "Black-Scholes"

MSE_data <- MSE_data %>%
  rename(MSE_values = V1)

#Plotting Blackscholes
ggplot(data=MSE_data, mapping=aes(x=date, y=MSE_values, colour = Method, shape = Method)) +
  geom_point(size = 2) +
  geom_line() +
  labs(title = "MSE Across Time",
       x = "Date",
       y = "Mean Squared Error") +
  theme_bw() +
  theme(text=element_text(size=16))
```



##Part 5 Interpretations

5-1: Contour Plots of Forward Price & Strike Price

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$forward_price_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility_scaled[test_start:tes
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_en
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility[test_start:test_en
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end]))
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
```

```
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)

x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_test_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_test_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,1],K=x2_cont_bs[row,2],r=
}

post_data_cont_SGP <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_theta_3_SGP,pos

post_data_cont_bs <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,post_m


# post_data

pred_gp_cont_SGP <- stan(file="Predictive GP_6dimension_SGP.stan", data=post_data_cont_SGP,iter=200, wa
```

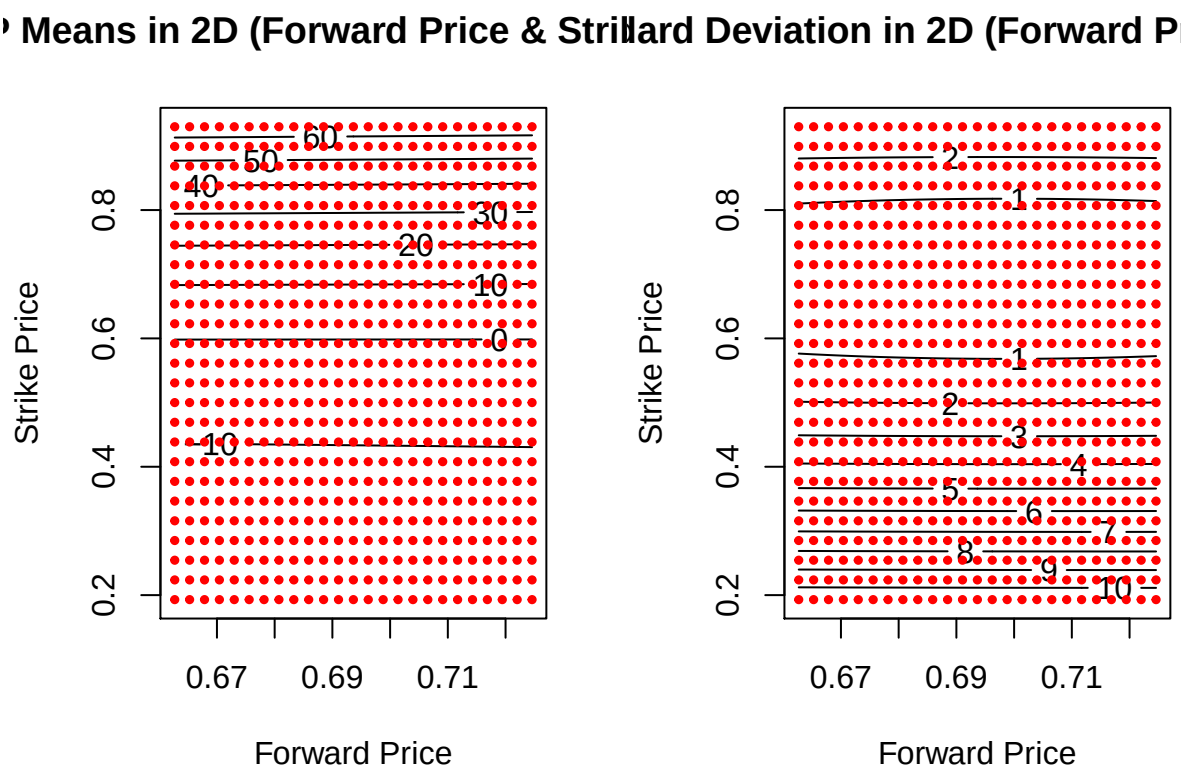
```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                76.571 seconds (Sampling)
## Chain 1:                76.571 seconds (Total)
## Chain 1:
```

```
pred_gp_cont_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_cont_bs,iter=20
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                72.13 seconds (Sampling)
## Chain 1:                72.13 seconds (Total)
## Chain 1:
```

```
#Computing Mean
y_predict_values_cont_SGP <- extract(pred_gp_cont_SGP,permuted=FALSE)
```

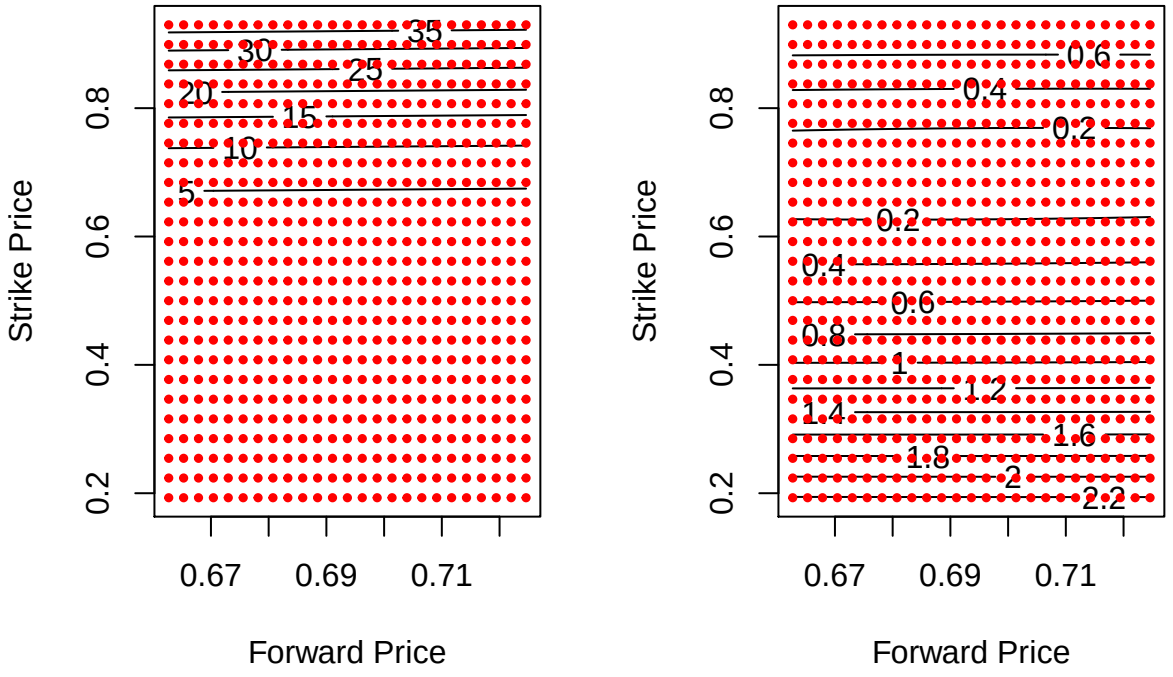
```
y_mean_values_cont_SGP <- c(colMeans(y_predict_values_cont_SGP))
y_mean_values_cont_SGP <- y_mean_values_cont_SGP[1:(length(y_mean_values_cont_SGP)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_SGP <- summary(pred_gp_cont_SGP, sd=c("sd"))$summary
pred_gp_sd_cont_SGP <- pred_gp_summary_cont_SGP[, c("sd")]
y_sd_values_cont_SGP <- pred_gp_sd_cont_SGP[1:(length(pred_gp_sd_cont_SGP)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont_SGP, length(dim1), length(dim2)), labcex =1, main = "SGP M
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont_SGP, length(dim1), length(dim2)), labcex =1, main = "SGP Sta
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```




```
#Computing Mean
y_predict_values_cont_bs <- extract(pred_gp_cont_bs,permuted=FALSE)
y_mean_values_cont_bs <- c(colMeans(y_predict_values_cont_bs))
y_mean_values_cont_bs <- y_mean_values_cont_bs[1:(length(y_mean_values_cont_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_bs <- summary(pred_gp_cont_bs, sd=c("sd"))$summary
pred_gp_sd_cont_bs <- pred_gp_summary_cont_bs[, c("sd")]
y_sd_values_cont_bs <- pred_gp_sd_cont_bs[1:(length(pred_gp_sd_cont_bs)-1)]
```

```r
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont_bs, length(dim1), length(dim2)), labcex =1, main = "BS_GP
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont_bs, length(dim1), length(dim2)), labcex =1, main = "BS_GP S
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

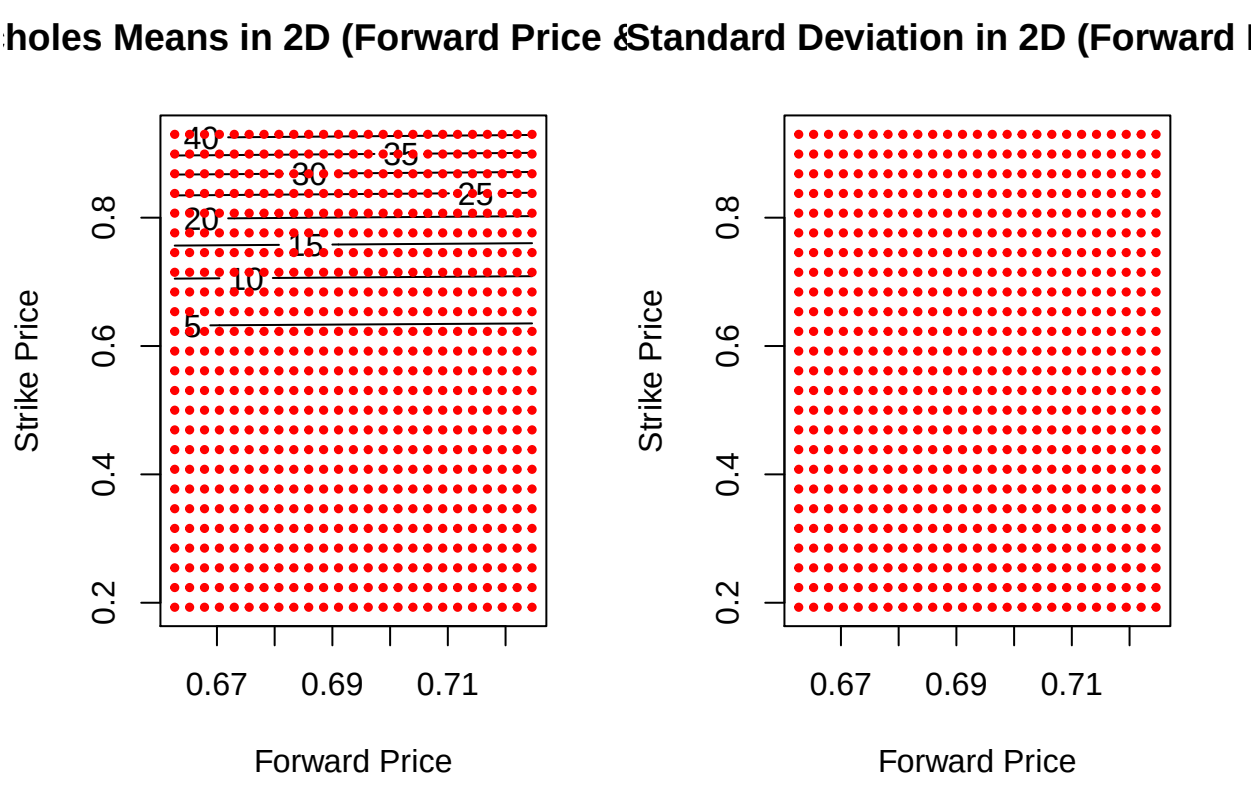
;P Means in 2D (Forward Price & Stndard Deviation in 2D (Forward Pric




```r
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(blackscholes_test_cont, length(dim1), length(dim2)), labcex = 1, main = "Bla
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(sd(blackscholes_test_cont), length(dim1), length(dim2)), labcex =1, main = "
```

```
## Warning in contour.default(dim1, dim2, matrix(sd(blackscholes_test_cont), :
## all z values are equal
```

```r
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



5-2: Contour Plots of Implied Volatility & Time to Expiration

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price_scaled[test_start:test_
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price_scaled[test_start:test_e
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end]
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)
```

```r
x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_test_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_test_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,3],K=x2_cont_bs[row,4],r=
}


post_data_cont_SGP <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_theta_3_SGP,pos

post_data_cont_bs <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,post_m


# post_data

pred_gp_cont_SGP <- stan(file="Predictive GP_6dimension_SGP.stan", data=post_data_cont_SGP,iter=200, wa
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                64.892 seconds (Sampling)
## Chain 1:                64.892 seconds (Total)
## Chain 1:
```

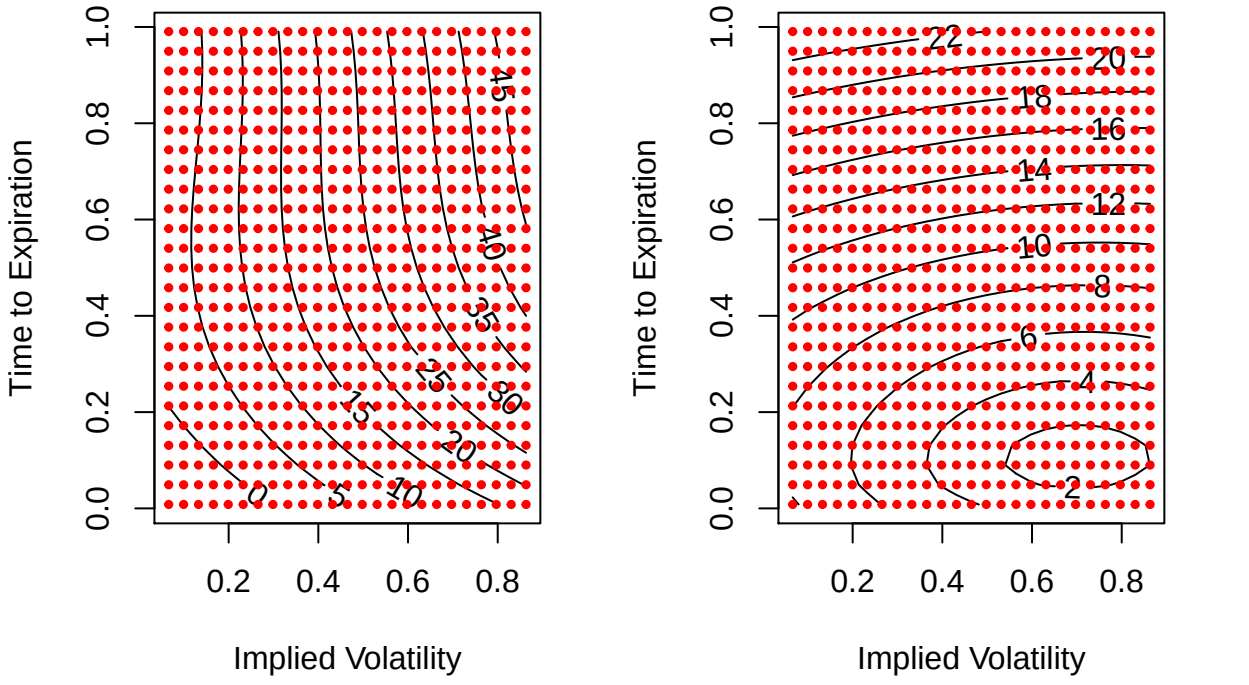
```r
pred_gp_cont_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_cont_bs,iter=20
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                67.856 seconds (Sampling)
## Chain 1:                67.856 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont_SGP <- extract(pred_gp_cont_SGP,permuted=FALSE)
y_mean_values_cont_SGP <- c(colMeans(y_predict_values_cont_SGP))
```

```r
y_mean_values_cont_SGP <- y_mean_values_cont_SGP[1:(length(y_mean_values_cont_SGP)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_SGP <- summary(pred_gp_cont_SGP, sd=c("sd"))$summary
pred_gp_sd_cont_SGP <- pred_gp_summary_cont_SGP[, c("sd")]
y_sd_values_cont_SGP <- pred_gp_sd_cont_SGP[1:(length(pred_gp_sd_cont_SGP)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont_SGP, length(dim1), length(dim2)), labcex =1, main = "SGP M
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont_SGP, length(dim1), length(dim2)), labcex =1, main = "SGP Mea
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

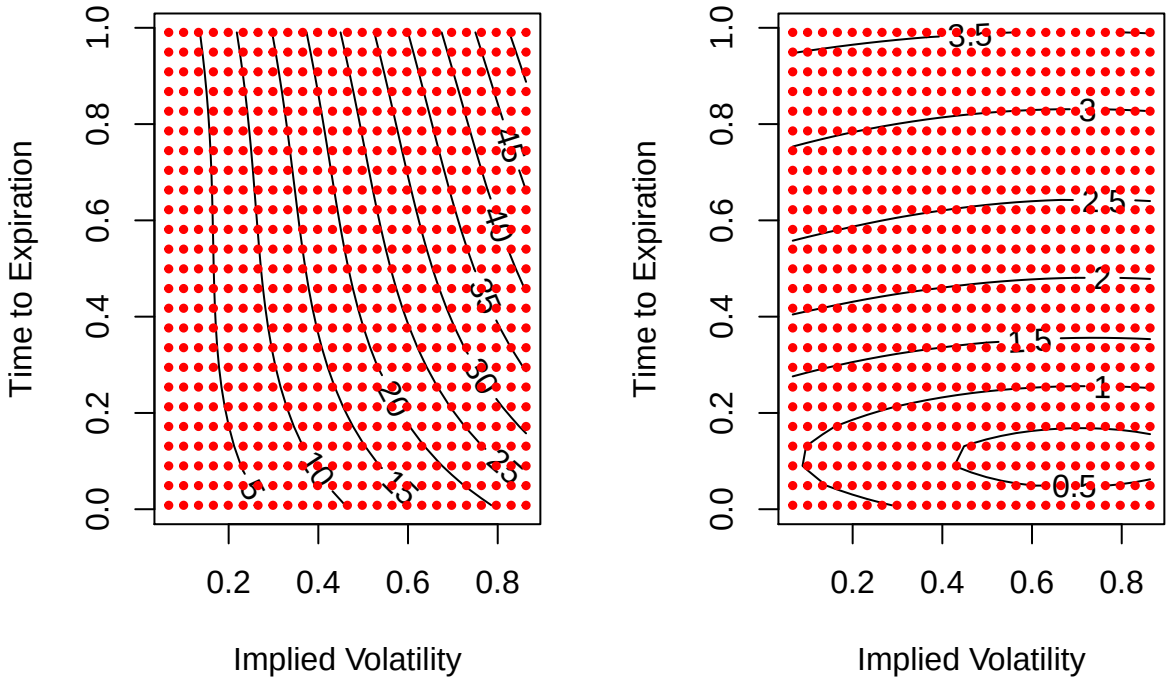
GP Means in 2D (Impl_vol & Time_tGP Means in 2D (Impl_vol & Time_tc




```r
#Computing Mean
y_predict_values_cont_bs <- extract(pred_gp_cont_bs,permuted=FALSE)
y_mean_values_cont_bs <- c(colMeans(y_predict_values_cont_bs))
y_mean_values_cont_bs <- y_mean_values_cont_bs[1:(length(y_mean_values_cont_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_bs <- summary(pred_gp_cont_bs, sd=c("sd"))$summary
pred_gp_sd_cont_bs <- pred_gp_summary_cont_bs[, c("sd")]
y_sd_values_cont_bs <- pred_gp_sd_cont_bs[1:(length(pred_gp_sd_cont_bs)-1)]
```

```r
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont_bs, length(dim1), length(dim2)), labcex =1, main = "BSGP M
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont_bs, length(dim1), length(dim2)), labcex =1, main = "BSGP Sta
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

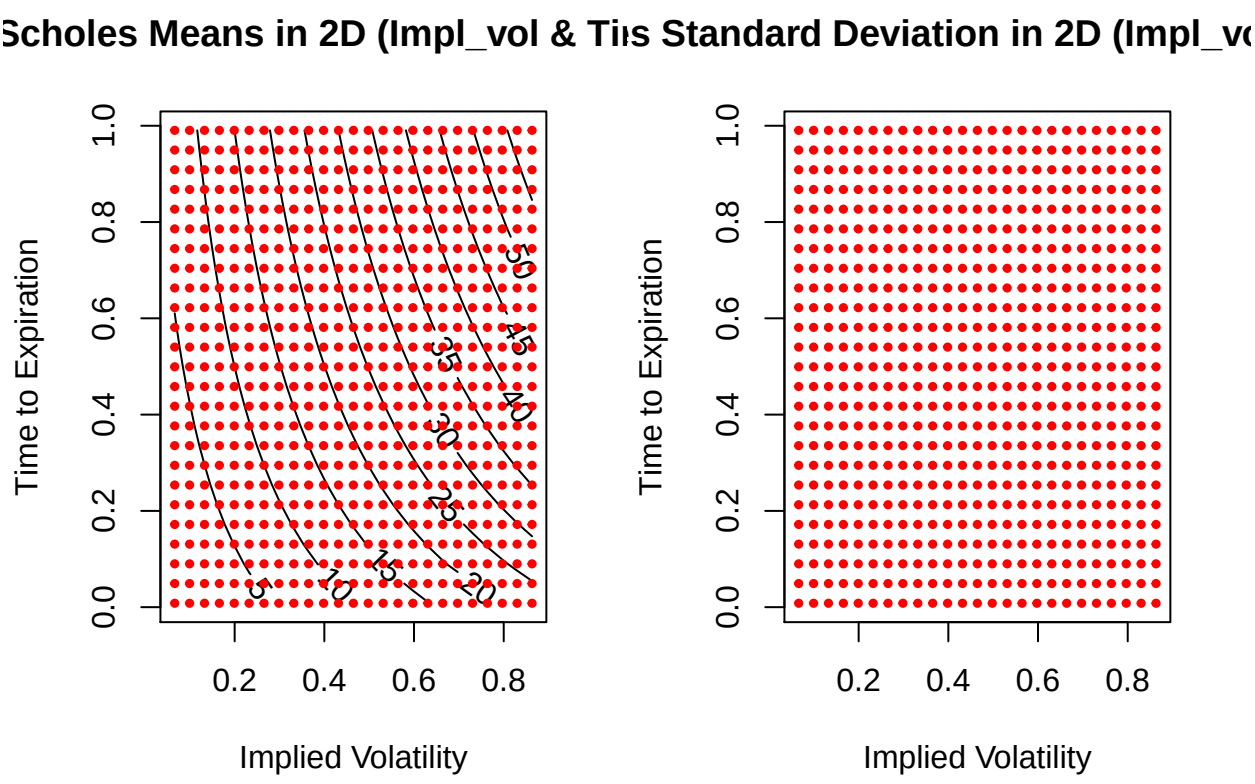
GP Means in 2D (Impl_vol & Time_to_Expiration) Standard Deviation in 2D (Impl_vol & T




```r
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(blackscholes_test_cont, length(dim1), length(dim2)), labcex =1, main = "Black
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(sd(blackscholes_test_cont), length(dim1), length(dim2)), labcex =1, main = "B
```

```
## Warning in contour.default(dim1, dim2, matrix(sd(blackscholes_test_cont), :
## all z values are equal
```

```r
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



5-3: Contour Plots of Strike Price and Implied Volatility

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price_scaled[test_start:test_
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_en
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end]
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end]))
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)
```

```r
x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_test_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_test_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,3],K=x2_cont_bs[row,1],r=
}


post_data_cont_SGP <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_theta_3_SGP,pos

post_data_cont_bs <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,post_m


# post_data

pred_gp_cont_SGP <- stan(file="Predictive GP_6dimension_SGP.stan", data=post_data_cont_SGP,iter=200, wa
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                69.763 seconds (Sampling)
## Chain 1:                69.763 seconds (Total)
## Chain 1:
```

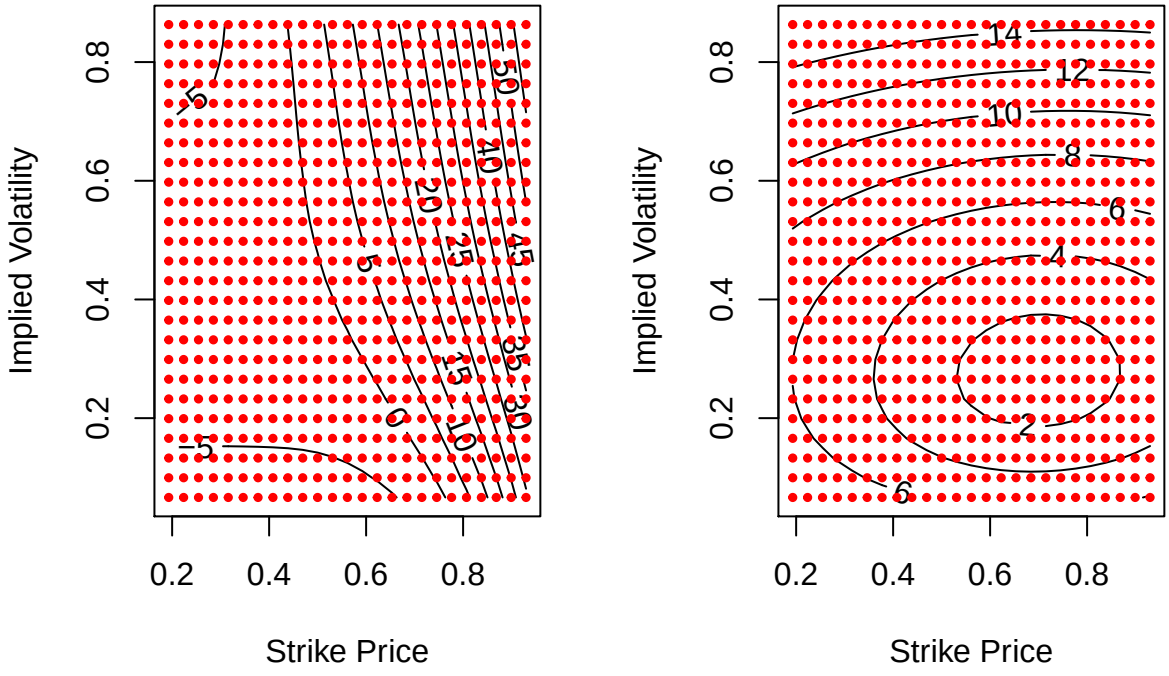
```r
pred_gp_cont_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_cont_bs,iter=20
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                68.8 seconds (Sampling)
## Chain 1:                68.8 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont_SGP <- extract(pred_gp_cont_SGP,permuted=FALSE)
y_mean_values_cont_SGP <- c(colMeans(y_predict_values_cont_SGP))
```

```
y_mean_values_cont_SGP <- y_mean_values_cont_SGP[1:(length(y_mean_values_cont_SGP)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_SGP <- summary(pred_gp_cont_SGP, sd=c("sd"))$summary
pred_gp_sd_cont_SGP <- pred_gp_summary_cont_SGP[, c("sd")]
y_sd_values_cont_SGP <- pred_gp_sd_cont_SGP[1:(length(pred_gp_sd_cont_SGP)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont_SGP, length(dim1), length(dim2)), labcex =1, main = "SGP M
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont_SGP, length(dim1), length(dim2)), labcex =1, main = "SGP Sta
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

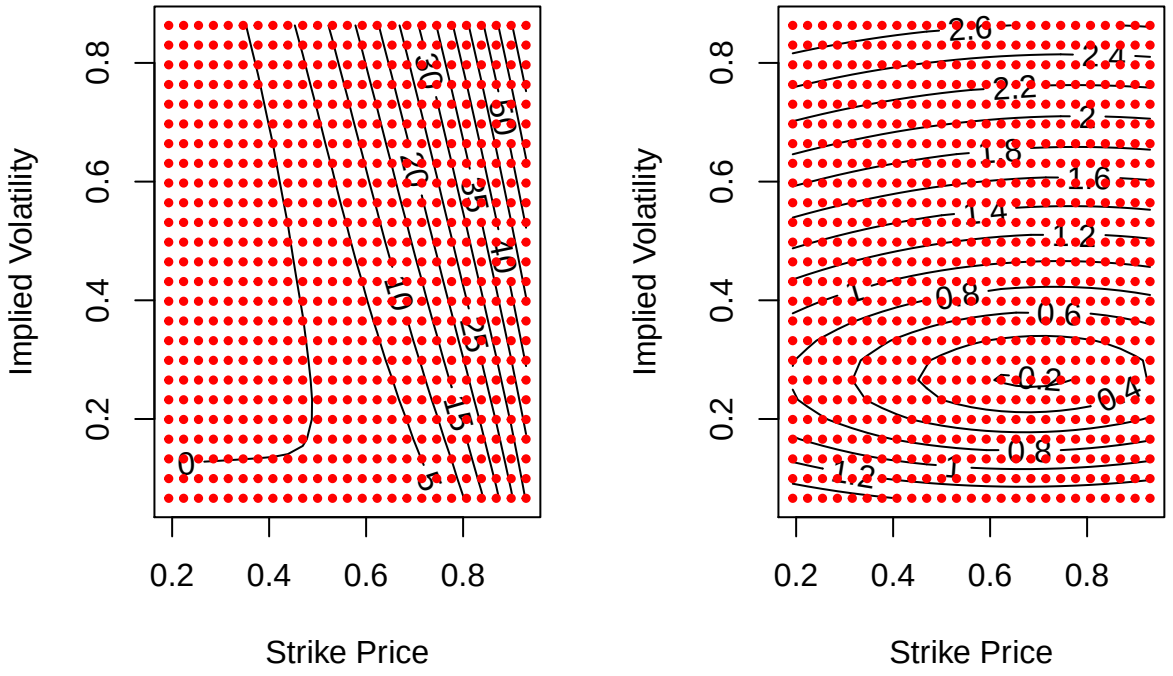
GP Means in 2D (Strike Price & Imp andard Deviation in 2D (Strike Price




```
#Computing Mean
y_predict_values_cont_bs <- extract(pred_gp_cont_bs,permuted=FALSE)
y_mean_values_cont_bs <- c(colMeans(y_predict_values_cont_bs))
y_mean_values_cont_bs <- y_mean_values_cont_bs[1:(length(y_mean_values_cont_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_bs <- summary(pred_gp_cont_bs, sd=c("sd"))$summary
pred_gp_sd_cont_bs <- pred_gp_summary_cont_bs[, c("sd")]
y_sd_values_cont_bs <- pred_gp_sd_cont_bs[1:(length(pred_gp_sd_cont_bs)-1)]
```

```
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont_bs, length(dim1), length(dim2)), labcex =1, main = "BSGP M
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont_bs, length(dim1), length(dim2)), labcex =1, main = "BSGP Sta
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

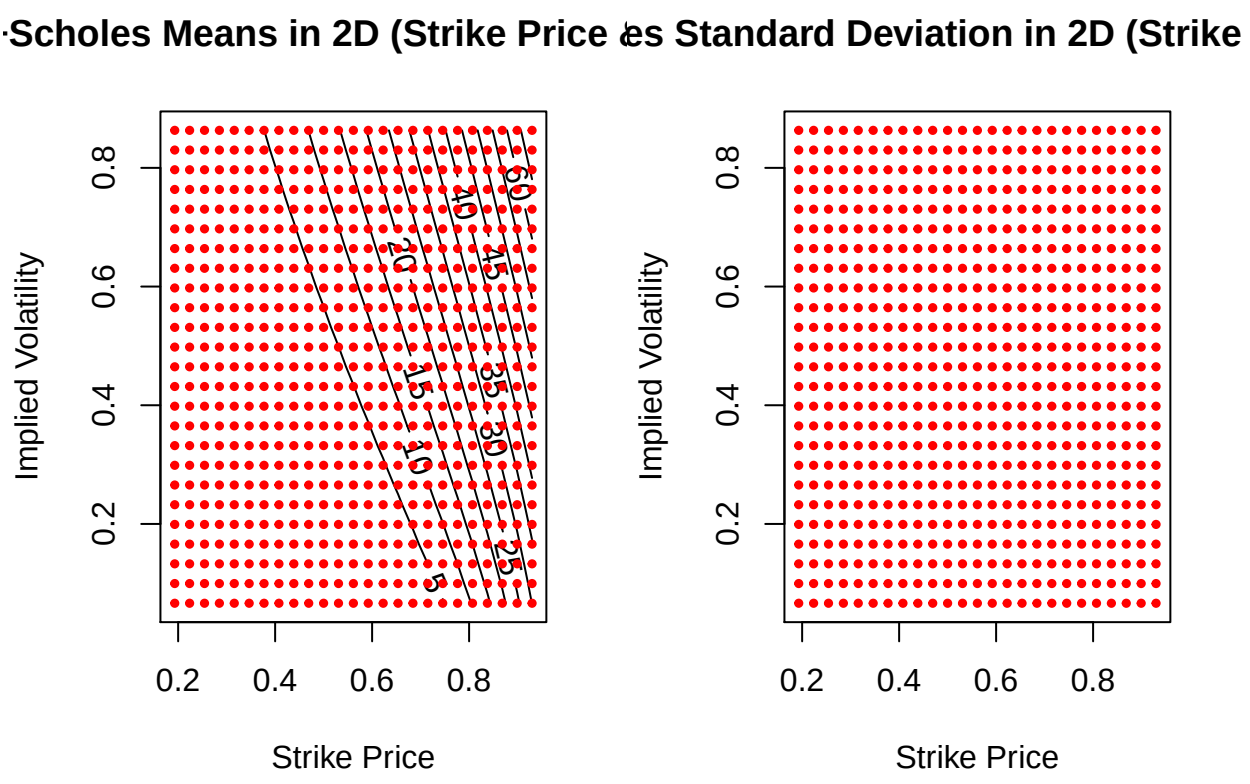
SGP Means in 2D (Strike Price & Implied Volatility) | Standard Deviation in 2D (Strike Price & Implied Volatility)




```
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(blackscholes_test_cont, length(dim1), length(dim2)), labcex =1, main = "Black
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(sd(blackscholes_test_cont), length(dim1), length(dim2)), labcex =1, main = "B
```

```
## Warning in contour.default(dim1, dim2, matrix(sd(blackscholes_test_cont), :
## all z values are equal
```

```
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

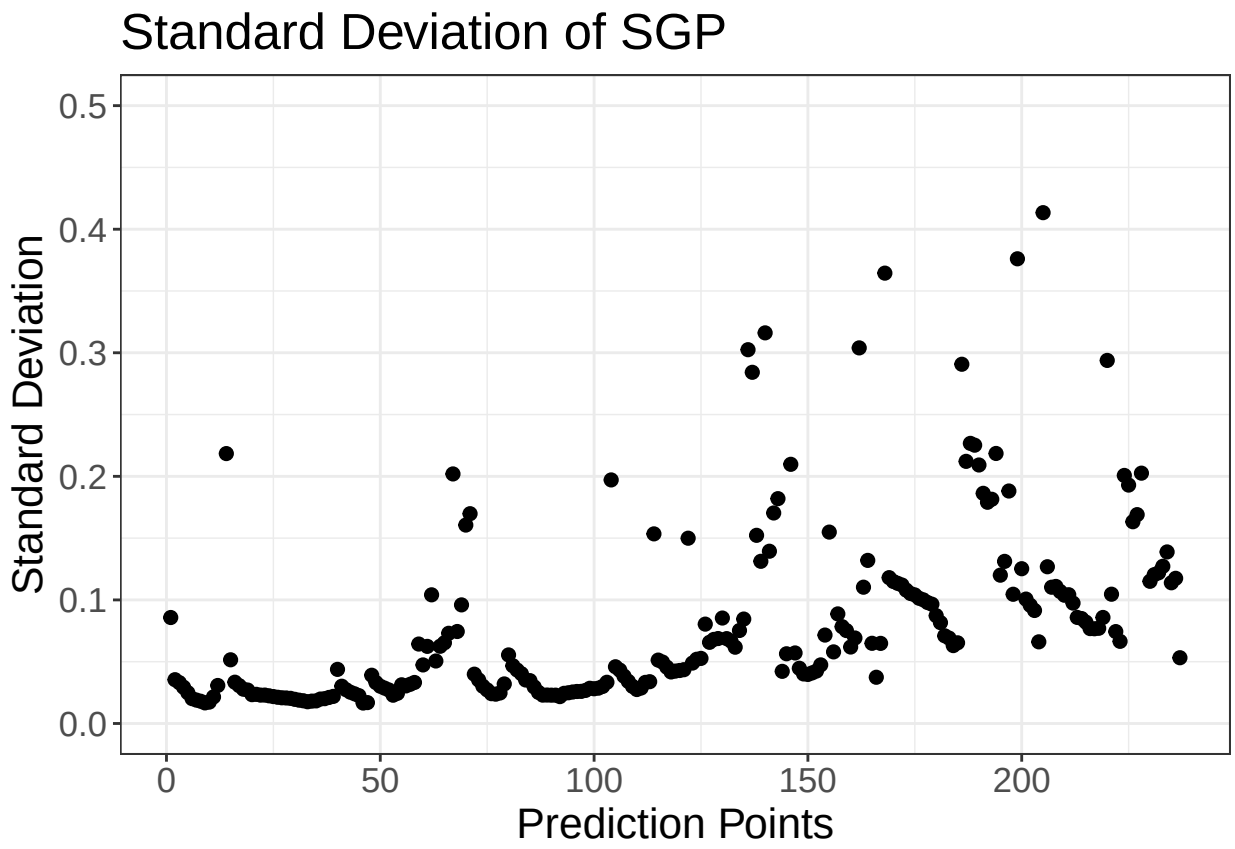


Part 6: Standard Deviation of Predicted Values

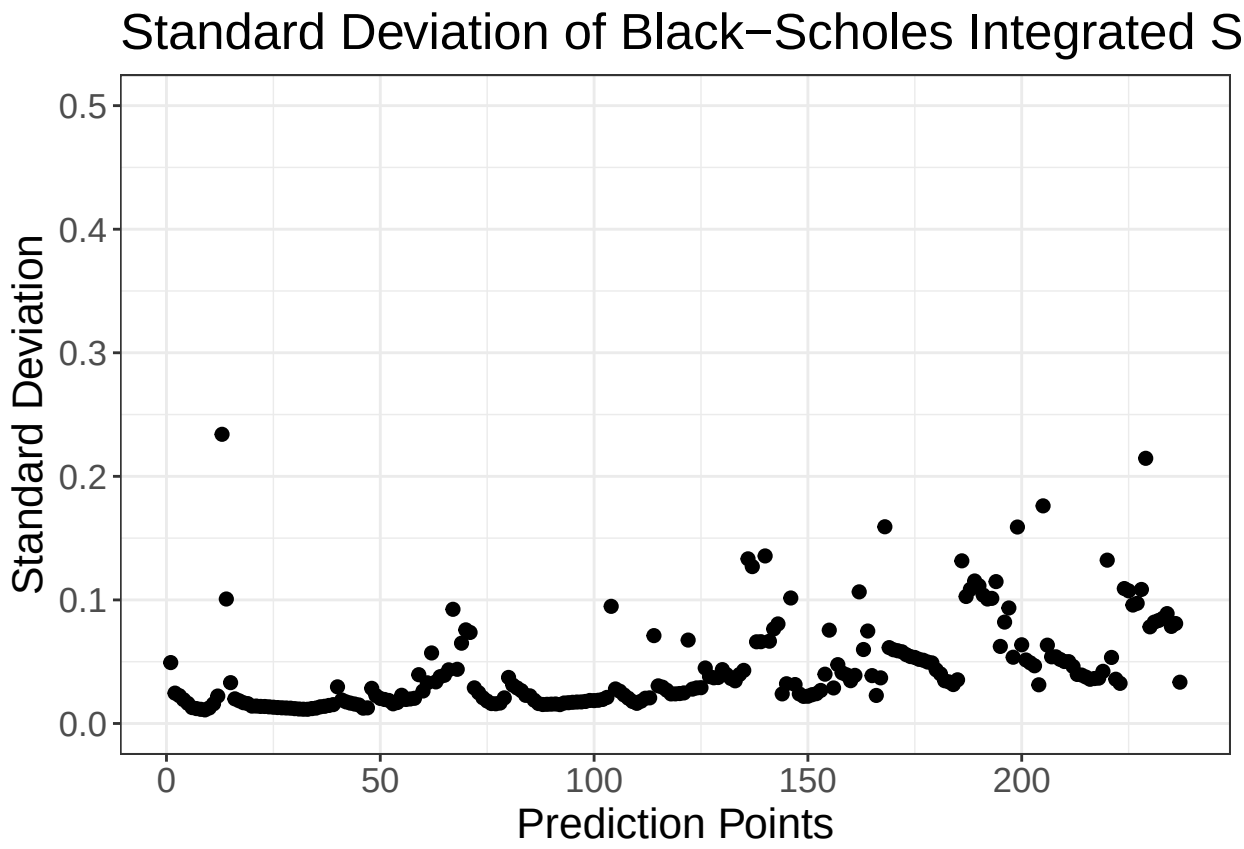
6-1: Plotting Standard Deviation of Predictions

```
par(mfrow=c(1,3))
ggplot(mapping = aes(x = seq(1:237),y=y_sd_values_SGP)) + geom_point(size = 2)+
  ylim(0,0.5)+ labs(title = "Standard Deviation of SGP",
       x = "Prediction Points",
       y = "Standard Deviation") +
  theme_bw() +
  theme(text=element_text(size=16))
```

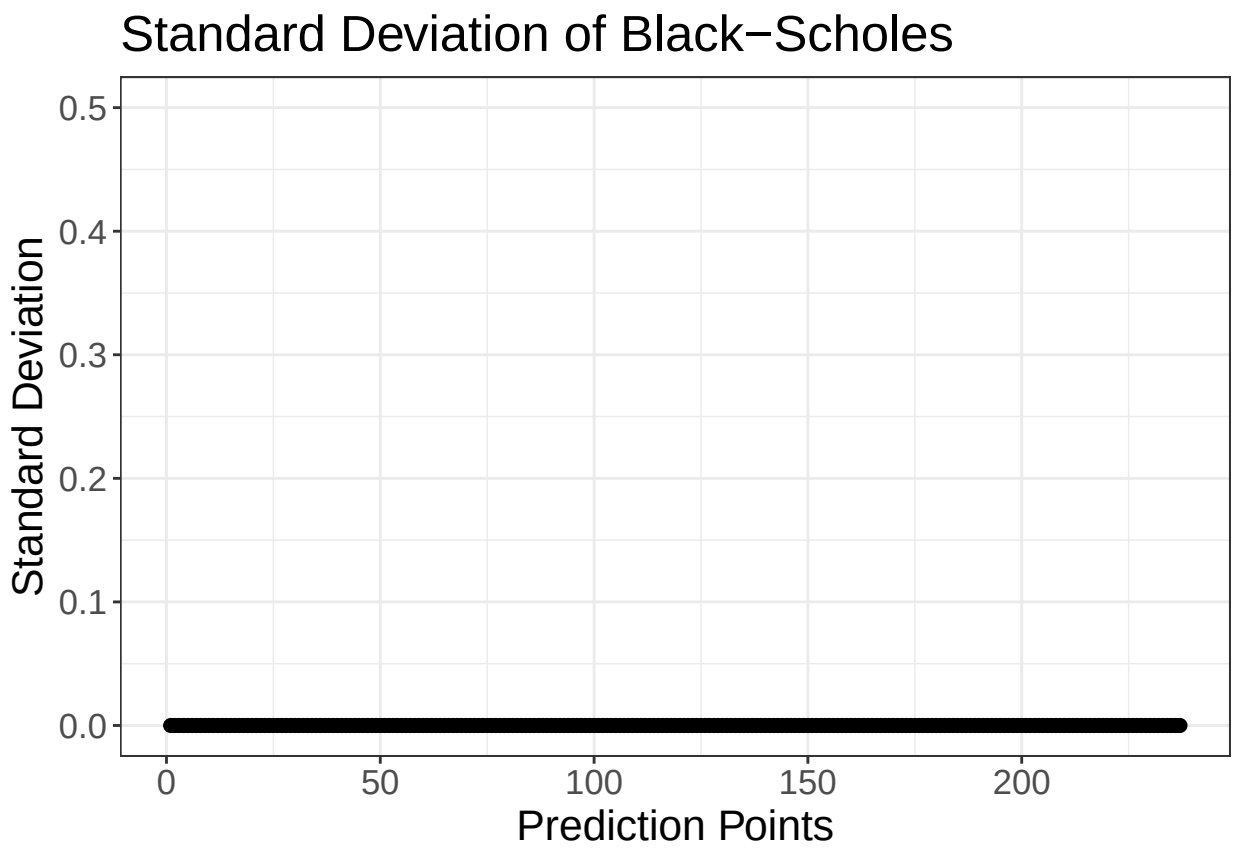
```
## Warning: Removed 2 rows containing missing values (geom_point).
```




```
ggplot(mapping = aes(x = seq(1:237),y=y_sd_values_bs)) + geom_point(size = 2)+
  ylim(0,0.5)+
  labs(title = "Standard Deviation of Black-Scholes Integrated SGP",
       x = "Prediction Points",
       y = "Standard Deviation") +
  theme_bw() +
  theme(text=element_text(size=16))
```




```
ggplot(mapping = aes(x = seq(1:237),y=rep(0,237))) + geom_point(size = 2)+
  ylim(0,0.5)+
  labs(title = "Standard Deviation of Black-Scholes",
       x = "Prediction Points",
       y = "Standard Deviation") +
  theme_bw() +
  theme(text=element_text(size=16))
```




```
mean(y_sd_values_SGP)
```

```
## [1] 0.08607074
```

```
mean(y_sd_values_bs)
```

```
## [1] 0.04442394
```

```
mean(0)
```

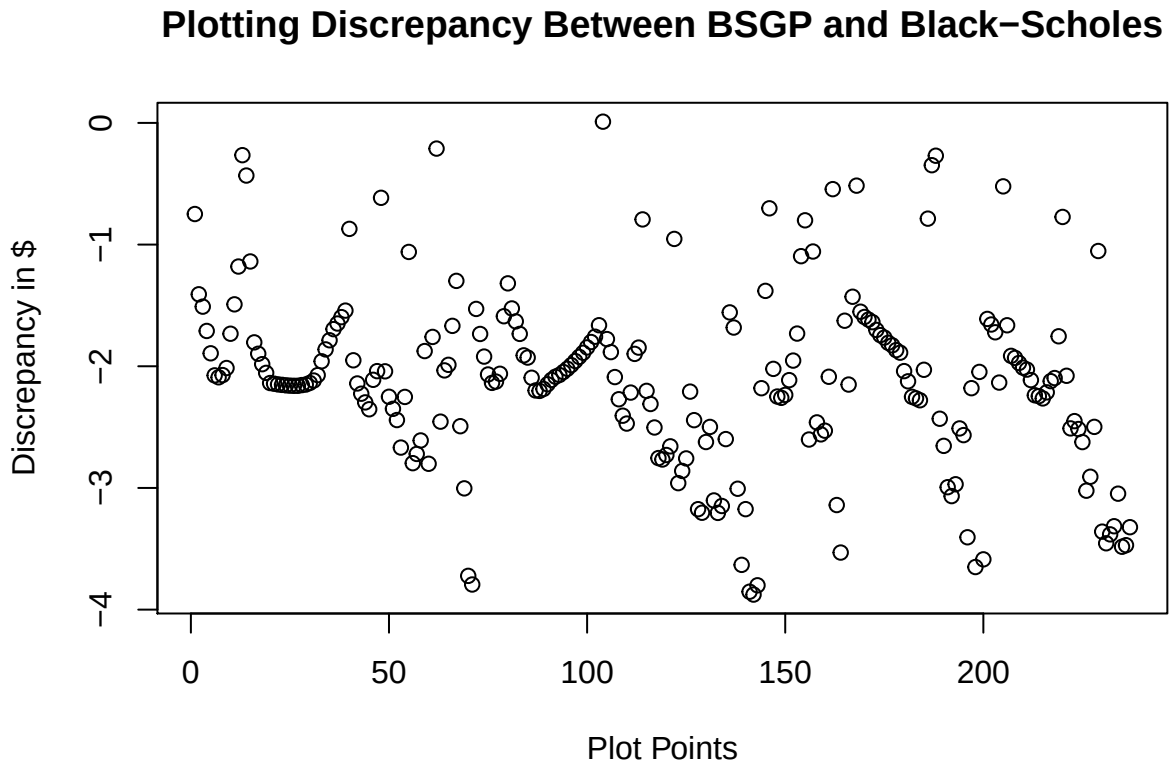
```
## [1] 0
```

##Part 7 Discrepancy Modeling

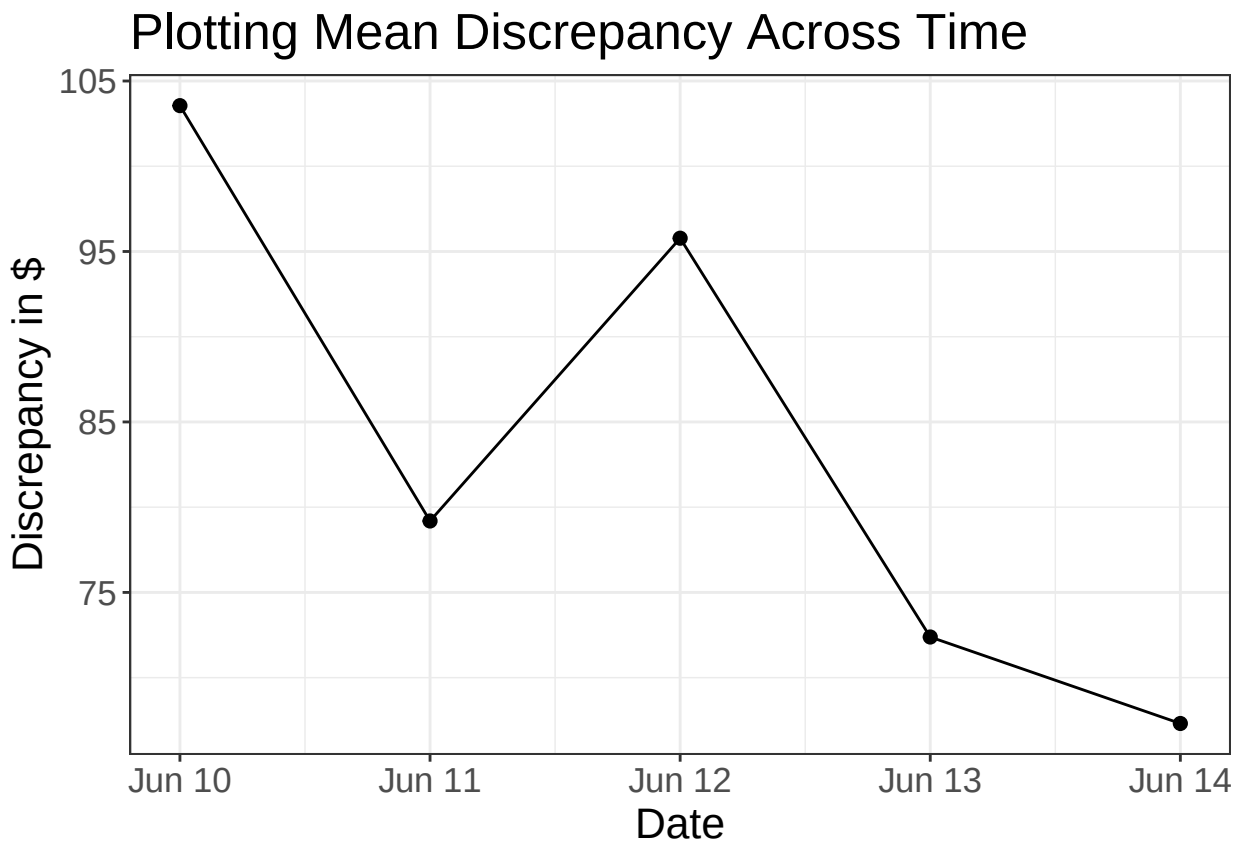
7-1: Computing Discrepancy

```
Discrepancy <- y_mean_values_bs - blackscholes_test

library('MLmetrics')
Discrepnacy_0610 <- mean(Discrepancy[1:71])
Discrepancy_0611 <- mean(Discrepancy[72:143])
Discrepancy_0612 <- mean(Discrepancy[144:164])
Discrepancy_0613 <- mean(Discrepancy[165:200])
Discrepancy_0614 <- mean(Discrepancy[201:237])
```

```r
Discrepancy_data <- as.data.frame(rbind(SGP_MSE_0610,SGP_MSE_0611,SGP_MSE_0612,SGP_MSE_0613,SGP_MSE_061

Discrepancy_data$date <- c(as.Date('00/00/0000', format = '%m/%d/%Y'))
Discrepancy_data$date[1] <- as.Date('06/10/2019', format = '%m/%d/%Y')
Discrepancy_data$date[2] <- as.Date('06/11/2019', format = '%m/%d/%Y')
Discrepancy_data$date[3] <- as.Date('06/12/2019', format = '%m/%d/%Y')
Discrepancy_data$date[4] <- as.Date('06/13/2019', format = '%m/%d/%Y')
Discrepancy_data$date[5] <- as.Date('06/14/2019', format = '%m/%d/%Y')

Discrepancy_data <- Discrepancy_data %>%
  rename(Discrepancy_values = V1)

plot(Discrepancy, main = "Plotting Discrepancy Between BSGP and Black-Scholes", xlab = "Plot Points", yl
```




```r
#Plotting Blackscholes
ggplot(data=Discrepancy_data, mapping=aes(x=date, y=Discrepancy_values)) +
  geom_point(size = 2) +
  geom_line() +
  labs(title = "Plotting Mean Discrepancy Across Time",
       x = "Date",
       y = "Discrepancy in $") +
  theme_bw() +
  theme(text=element_text(size=16))
```



7-2: Discrepancy Contour Plots of Forward Price & Strike Price

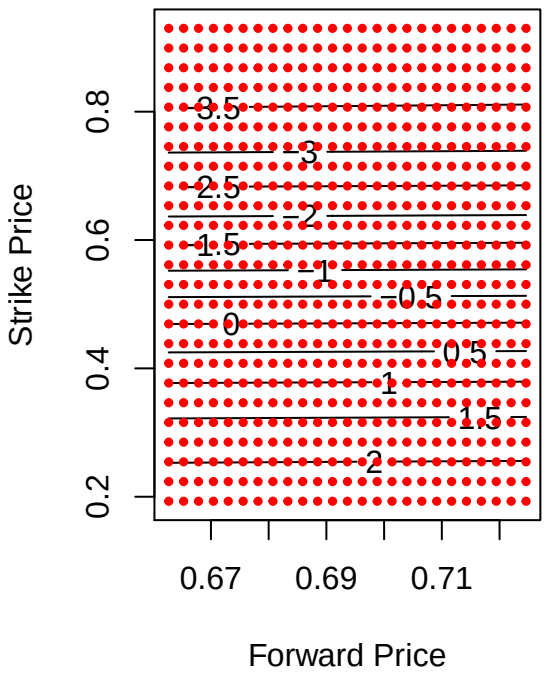
```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$forward_price_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility_scaled[test_start:tes
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_en
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility[test_start:test_en
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end]))
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)
```

```r
x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_test_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_test_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,1],K=x2_cont_bs[row,2],r=
}

post_data_cont_bs <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,post_m


# post_data


pred_gp_cont_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_cont_bs,iter=20
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                69.465 seconds (Sampling)
## Chain 1:                69.465 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont_bs <- extract(pred_gp_cont_bs,permuted=FALSE)
y_mean_values_cont_bs <- c(colMeans(y_predict_values_cont_bs))
y_mean_values_cont_bs <- y_mean_values_cont_bs[1:(length(y_mean_values_cont_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_bs <- summary(pred_gp_cont_bs, sd=c("sd"))$summary
pred_gp_sd_cont_bs <- pred_gp_summary_cont_bs[, c("sd")]
y_sd_values_cont_bs <- pred_gp_sd_cont_bs[1:(length(pred_gp_sd_cont_bs)-1)]

#Discrepancy
Discrepancy_function <- y_mean_values_cont_bs - blackscholes_test_cont

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(Discrepancy_function, length(dim1), length(dim2)), labcex =1, main = "Discre
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

ncy Modeled in 2D (Forward Price &



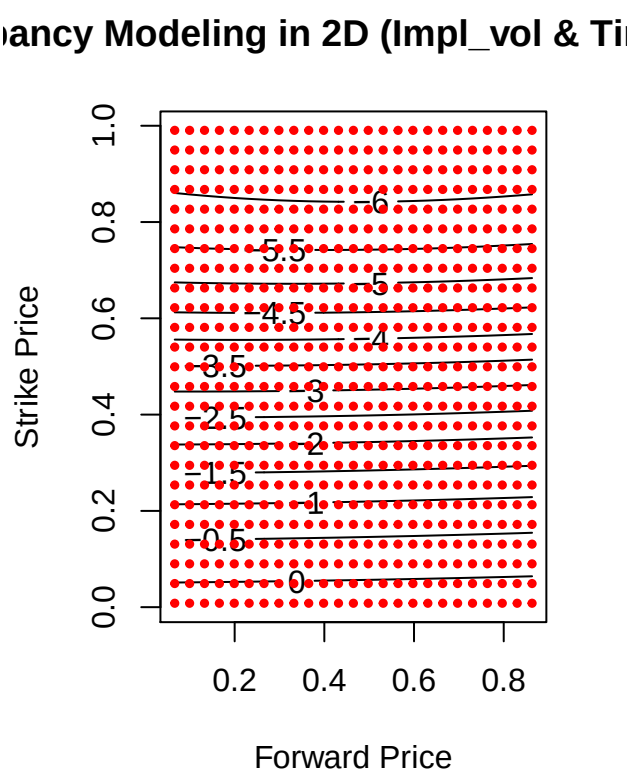
7-3: Discrepancy Contour Plots of Forward Price & Strike Price

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price_scaled[test_start:test_
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price_scaled[test_start:test_e
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end]
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)
```

```r
x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_test_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_test_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,3],K=x2_cont_bs[row,4],r=
}


post_data_cont_bs <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,post_m


# post_data

pred_gp_cont_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_cont_bs,iter=20
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                69.104 seconds (Sampling)
## Chain 1:                69.104 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont_bs <- extract(pred_gp_cont_bs,permuted=FALSE)
y_mean_values_cont_bs <- c(colMeans(y_predict_values_cont_bs))
y_mean_values_cont_bs <- y_mean_values_cont_bs[1:(length(y_mean_values_cont_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_bs <- summary(pred_gp_cont_bs, sd=c("sd"))$summary
pred_gp_sd_cont_bs <- pred_gp_summary_cont_bs[, c("sd")]
y_sd_values_cont_bs <- pred_gp_sd_cont_bs[1:(length(pred_gp_sd_cont_bs)-1)]

#Discrepancy
Discrepancy_function <- y_mean_values_cont_bs - blackscholes_test_cont

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(Discrepancy_function, length(dim1), length(dim2)), labcex =1, main = "Discre
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



7-4: Discrepancy Contour Plots of Forward Price & Strike Price

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price_scaled[test_start:test_
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_en
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end]
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end]))
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)
```

```r
x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_test_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_test_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,3],K=x2_cont_bs[row,1],r=
}


post_data_cont_bs <- list(theta=c(post_mean_theta_1_bs,post_mean_theta_2_bs,post_mean_theta_3_bs,post_m


# post_data

pred_gp_cont_bs <- stan(file="Predictive GP_6dimension_withBS_SGP.stan", data=post_data_cont_bs,iter=20


## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_SGP' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                74.883 seconds (Sampling)
## Chain 1:                74.883 seconds (Total)
## Chain 1:

#Computing Mean
y_predict_values_cont_bs <- extract(pred_gp_cont_bs,permuted=FALSE)
y_mean_values_cont_bs <- c(colMeans(y_predict_values_cont_bs))
y_mean_values_cont_bs <- y_mean_values_cont_bs[1:(length(y_mean_values_cont_bs)-1)]

#Computing Standard Deviation
pred_gp_summary_cont_bs <- summary(pred_gp_cont_bs, sd=c("sd"))$summary
pred_gp_sd_cont_bs <- pred_gp_summary_cont_bs[, c("sd")]
y_sd_values_cont_bs <- pred_gp_sd_cont_bs[1:(length(pred_gp_sd_cont_bs)-1)]

#Discrepancy
Discrepancy_function <- y_mean_values_cont_bs - blackscholes_test_cont

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(Discrepancy_function, length(dim1), length(dim2)), labcex =1, main = "Discre
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```

pancy Modeling in 2D (Strike Price