# Gaussian_Process_Code

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##Part 1: Standard Gaussian Process

1-1: Fitting

```r
library(rstan)
```

```
## Loading required package: StanHeaders

## Loading required package: ggplot2

## rstan (Version 2.19.2, GitRev: 2e1f913d3ca3)

## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)

## For improved execution time, we recommend calling
## Sys.setenv(LOCAL_CPPFLAGS = '-march=native')
## although this causes Stan to throw an error on a few processors.
```

```r
source("gp.utility.R")

# Fitting GP model
stan_dat <- read_rdump('Financial_Data_Put_American.R')
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

## Loading required package: limSolve

##
## Attaching package: 'limSolve'

## The following object is masked from 'package:ggplot2':
##
##     resolution
```

```
## Loading required package: futile.logger

## Welcome to ragtop.  Logging can be enabled with commands such as
##   futile.logger::flog.threshold(futile.logger::INFO, name='ragtop.calibration')

## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )

## See spec(...) for full column specifications.
```

```r
fit_gp_SGP_American <- stan(file="gp-fit-6dimension_withBS.stan", data=stan_dat,
               iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.073 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 730 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
```

```
## Chain 1:  Elapsed Time: 47.761 seconds (Warm-up)
## Chain 1:                52.859 seconds (Sampling)
## Chain 1:                100.62 seconds (Total)
## Chain 1:
```

```
print(fit_gp_SGP_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##           mean se_mean    sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
## theta[1]  2.13    0.16  0.88  0.90  1.46  2.00  2.66  3.83    29 0.98
## theta[2]  1.76    0.11  0.66  0.82  1.41  1.72  2.02  2.90    35 1.00
## theta[3]  3.43    0.09  0.87  1.98  2.82  3.37  3.83  5.30    85 0.98
## theta[4]  1.67    0.03  0.24  1.31  1.49  1.61  1.86  2.11    48 1.01
## theta[5]  1.71    0.17  0.95  0.50  1.04  1.52  2.10  4.36    33 0.98
## theta[6]  0.66    0.02  0.17  0.42  0.57  0.62  0.75  1.04    84 0.98
## sigma2    0.00    0.00  0.00  0.00  0.00  0.00  0.00  0.00    47 0.98
## gamma2   28.75    1.95 12.77 11.75 19.43 24.64 36.24 55.42    43 0.99
##
## Samples were drawn using NUTS(diag_e) at Sun Mar 29 18:26:51 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_SGP_American <- extract(fit_gp_SGP_American,permuted=FALSE)
```

```
# Predicting from GP model
post_mean_theta_1_SGP <- mean(sum_gp_SGP_American[,1,1]) #theta
post_mean_theta_2_SGP <- mean(sum_gp_SGP_American[,1,2]) #theta
post_mean_theta_3_SGP <- mean(sum_gp_SGP_American[,1,3]) #theta
post_mean_theta_4_SGP <- mean(sum_gp_SGP_American[,1,4]) #theta
post_mean_theta_5_SGP <- mean(sum_gp_SGP_American[,1,5]) #theta
post_mean_theta_6_SGP <- mean(sum_gp_SGP_American[,1,6]) #theta
post_mean_sigma2_SGP <- mean(sum_gp_SGP_American[,1,7]) #sigma2
post_mean_gamma2_SGP <- mean(sum_gp_SGP_American[,1,8]) #gamma2
post_mean_mu_SGP <- stan_dat$blackscholes
```

```
test_start <- 323 #06/10
test_end <- 559 #06/14

# test_start <- 560 #06/17
# test_end <- 852 #06/20


x2_bs <- cbind(as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end]),as.numeric(s

library('qrmtools')
```

```
## Registered S3 method overwritten by 'xts':
##   method     from
##   as.zoo.xts zoo
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library('ragtop')
blackscholes_2 <- rep(NA,length(x2_bs[,1]))
for (row in 1:nrow(data.frame(x2_bs))){
  blackscholes_2[row] <- as.numeric(blackscholes(-1,S0=as.numeric(stan_dat$total_puts_American$forward_p
}

x.grid_1 <- as.numeric(stan_dat$total_puts_American$forward_price_scaled[test_start:test_end])
x.grid_2 <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])
x.grid_3 <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])
x.grid_4 <- as.numeric(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_end])
x.grid_5 <- as.numeric(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_end])
x.grid_6 <- as.numeric(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_end])
x2 <- cbind(x.grid_1,x.grid_2,x.grid_3,x.grid_4,x.grid_5,x.grid_6)
```

1-2: Predictions

```
post_data_SGP_American <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_theta_3_SGP
# post_data

pred_gp_SGP <- stan(file="Predictive GP_6dimension_withBS.stan", data=post_data_SGP_American,iter=200, w
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                23.835 seconds (Sampling)
## Chain 1:                23.835 seconds (Total)
## Chain 1:
```

##Part2: Bdrycov Gaussian Process

2-1: Fitting

```
# Fitting GP model for Bdrycov
stan_dat <- read_rdump('Financial_Data_Put_American.R')
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
```

```
##   exdate = col_character(),
##   cp_flag = col_character(),
##   ticker = col_character(),
##   exercise_style = col_character()
## )

## See spec(...) for full column specifications.
```

```r
fit_gp_Bdrycov_American <- stan(file="gp-fit-6dimension_withBS_Bdrycov.stan", data=stan_dat,
               iter=100, chains=1);
```

```
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.392 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 3920 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 323.78 seconds (Warm-up)
## Chain 1:                276.541 seconds (Sampling)
## Chain 1:                600.321 seconds (Total)
## Chain 1:
```

```r
print(fit_gp_Bdrycov_American, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS_Bdrycov.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
##          mean se_mean   sd 2.5%  25%  50%  75% 97.5% n_eff Rhat
```

```
## theta[1] 1.10    0.07 0.49 0.59 0.77 0.95 1.16  2.36    51 0.99
## theta[2] 1.27    0.11 0.64 0.56 0.82 1.15 1.47  2.82    35 1.00
## theta[3] 1.31    0.10 0.64 0.48 0.79 1.16 1.82  2.56    41 0.99
## theta[4] 1.33    0.13 0.82 0.49 0.79 1.06 1.58  3.73    41 0.98
## theta[5] 1.28    0.15 0.89 0.45 0.67 0.92 1.51  3.58    34 1.02
## theta[6] 1.19    0.06 0.52 0.54 0.84 1.01 1.41  2.45    70 1.01
## sigma2   2.61    0.81 1.87 0.05 0.64 2.98 4.33  5.02     5 1.11
## gamma2   1.95    0.83 1.90 0.02 0.06 1.64 4.06  4.72     5 1.13
##
## Samples were drawn using NUTS(diag_e) at Sun Mar 29 18:39:32 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```r
sum_gp_Bdrycov_American <- extract(fit_gp_Bdrycov_American,permuted=FALSE)
# saveRDS(fit_gp,file ="fit_gp_vol50_within50spot_7to19days")
```

```r
# Predicting from GP model - 2 dimensional case
post_mean_theta_1_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,1]) #theta
post_mean_theta_2_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,2]) #theta
post_mean_theta_3_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,3]) #theta
post_mean_theta_4_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,4]) #theta
post_mean_theta_5_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,5]) #theta
post_mean_theta_6_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,6]) #theta
post_mean_sigma2_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,7]) #sigma2
post_mean_gamma2_Bdrycov <- mean(sum_gp_Bdrycov_American[,1,8]) #gamma2
post_mean_mu_Bdrycov <- stan_dat$blackscholes


# x2 <- as.numeric(unlist(spx_spy_2019_06_30_put_2017_06_500rows_test['strike_price']))
# x2<- cbind(spy_2013_01_01_2013_01_31_put$strike_price[201:300],spy_2013_01_01_2013_01_31_put$impl_vol
# x2 <- seq(from=-2,to=2,by=0.01)

# x2 <- cbind(seq(from=0,to=1,by=0.01),seq(from=0,to=1,by=0.01))
```

2-2: Predictions

```r
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_Bdrycov_American <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean
# post_data

pred_gp_Bdrycov <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_Bdrycov_Amer
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
```

```
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                53.561 seconds (Sampling)
## Chain 1:                53.561 seconds (Total)
## Chain 1:
```

##Part 3 Predictions Versus Truth

3-1: Computing Means Standard GP

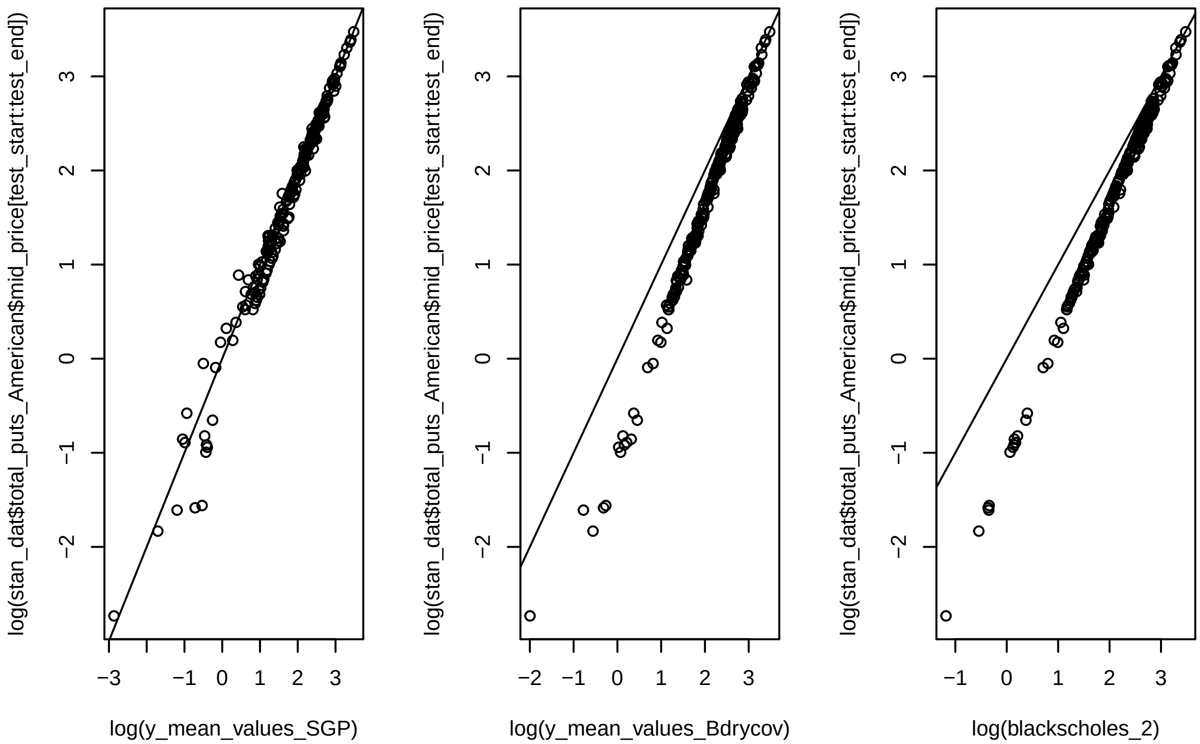
```
#Computing Mean
y_predict_values_SGP <- extract(pred_gp_SGP,permuted=FALSE)
y_mean_values_SGP <- c(colMeans(y_predict_values_SGP))
y_mean_values_SGP <- y_mean_values_SGP[1:(length(y_mean_values_SGP)-1)]

#Computing Standard Deviation
pred_gp_summary_SGP <- summary(pred_gp_SGP, sd=c("sd"))$summary
pred_gp_sd_SGP <- pred_gp_summary_SGP[, c("sd")]
y_sd_values_SGP <- pred_gp_sd_SGP[1:(length(pred_gp_sd_SGP)-1)]
```

3-2: Computing Means Bdrycov

```
#Computing Mean
y_predict_values_Bdrycov <- extract(pred_gp_Bdrycov,permuted=FALSE)
y_mean_values_Bdrycov <- c(colMeans(y_predict_values_Bdrycov))
y_mean_values_Bdrycov <- y_mean_values_Bdrycov[1:(length(y_mean_values_Bdrycov)-1)]

#Computing Standard Deviation
pred_gp_summary_Bdrycov <- summary(pred_gp_Bdrycov, sd=c("sd"))$summary
pred_gp_sd_Bdrycov <- pred_gp_summary_Bdrycov[, c("sd")]
y_sd_values_Bdrycov <- pred_gp_sd_Bdrycov[1:(length(pred_gp_sd_Bdrycov)-1)]
```

3-3: Plotting Predicted Values against Truth

```
par(mfrow=c(1,3))
#Plotting Standard GP
plot(log(y_mean_values_SGP),log(stan_dat$total_puts_American$mid_price[test_start:test_end]),xlim = c(mi
abline(0,1)

#Plotting BDrycov
plot(log(y_mean_values_Bdrycov),log(stan_dat$total_puts_American$mid_price[test_start:test_end]), xlim =
abline(0,1)

#Plotting Blackscholes
plot(log(blackscholes_2),log(stan_dat$total_puts_American$mid_price[test_start:test_end]), xlim = c(min
abline(0,1)
```




```r
#MSE
library('MLmetrics')
```

```
##
## Attaching package: 'MLmetrics'

## The following object is masked from 'package:base':
##
##     Recall
```

```r
MSE(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 0.3419418
```

```r
MSE(y_mean_values_Bdrycov,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 4.955886
```

```r
MSE(blackscholes_2,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 4.923348
```

##Part 4 Visualizations

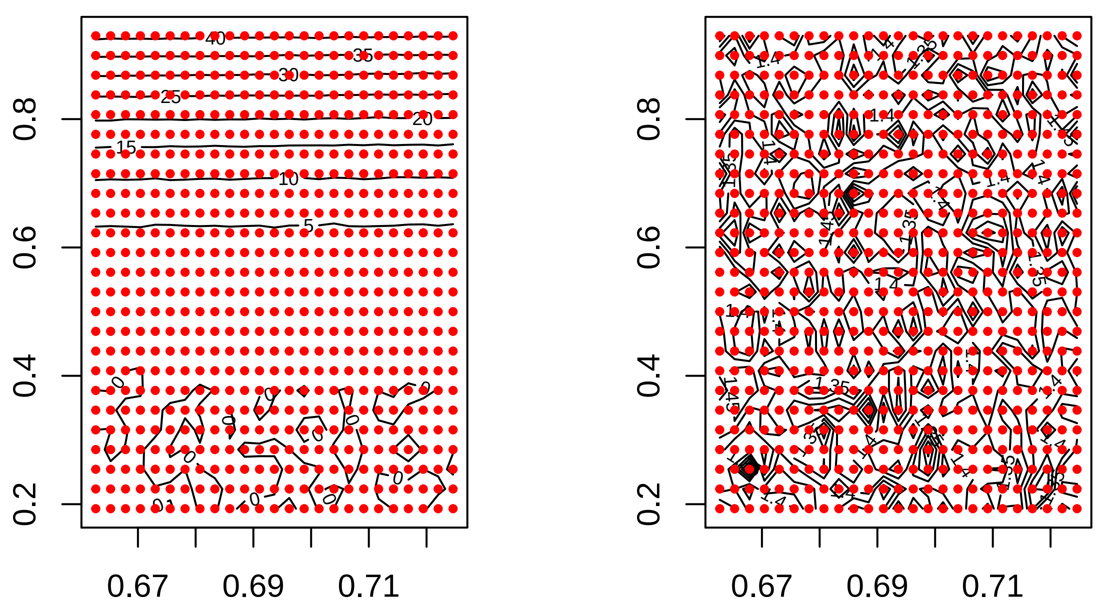
4-1: Contour Plots of Forward Price & Strike Price

```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$forward_price_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility_scaled[test_start:tes
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_en
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility[test_start:test_en
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end]))
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)

x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_2_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_2_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,1],K=x2_cont_bs[row,2],r=x2_c
}


post_data_cont <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean_theta_3_Bd
# post_data

pred_gp_cont <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_cont,iter=200,
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                150.453 seconds (Sampling)
```

```
## Chain 1:                150.453 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont <- extract(pred_gp_cont,permuted=FALSE)
y_mean_values_cont <- c(colMeans(y_predict_values_cont))
y_mean_values_cont <- y_mean_values_cont[1:(length(y_mean_values_cont)-1)]

#Computing Standard Deviation
pred_gp_summary_cont <- summary(pred_gp_cont, sd=c("sd"))$summary
pred_gp_sd_cont <- pred_gp_summary_cont[, c("sd")]
y_sd_values_cont <- pred_gp_sd_cont[1:(length(pred_gp_sd_cont)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```




4-2: Contour Plots of Implied VOlatility & Time to Expiration

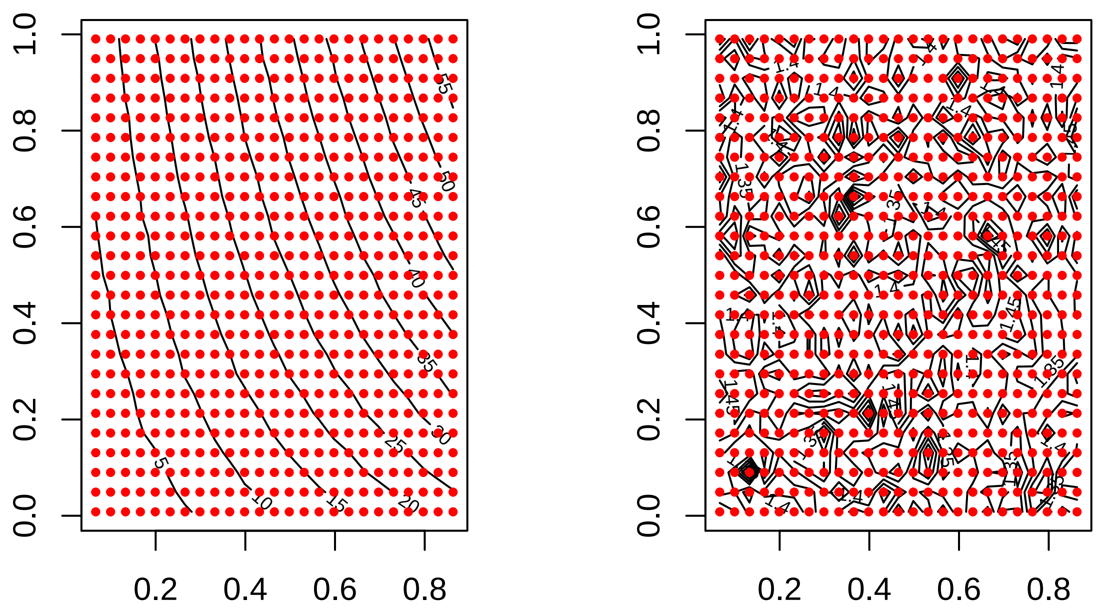
```r
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_end])
```

```
dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price_scaled[test_start:test_
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end])),n
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$impl_volatility[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$time_to_exp[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end]
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end])
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$dividend_yield[test_start:test_end
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$interest_rate[test_start:test_end]

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)

x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_2_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_2_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,3],K=x2_cont_bs[row,4],r=x2_
}


post_data_cont <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean_theta_3_Bd
# post_data

pred_gp_cont <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_cont,iter=200,
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                152.694 seconds (Sampling)
## Chain 1:                152.694 seconds (Total)
## Chain 1:
```

```
#Computing Mean
y_predict_values_cont <- extract(pred_gp_cont,permuted=FALSE)
y_mean_values_cont <- c(colMeans(y_predict_values_cont))
```

```
y_mean_values_cont <- y_mean_values_cont[1:(length(y_mean_values_cont)-1)]

#Computing Standard Deviation
pred_gp_summary_cont <- summary(pred_gp_cont, sd=c("sd"))$summary
pred_gp_sd_cont <- pred_gp_summary_cont[, c("sd")]
y_sd_values_cont <- pred_gp_sd_cont[1:(length(pred_gp_sd_cont)-1)]

par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



4-3: Contour Plots of Interest Rate & Dividend Yield

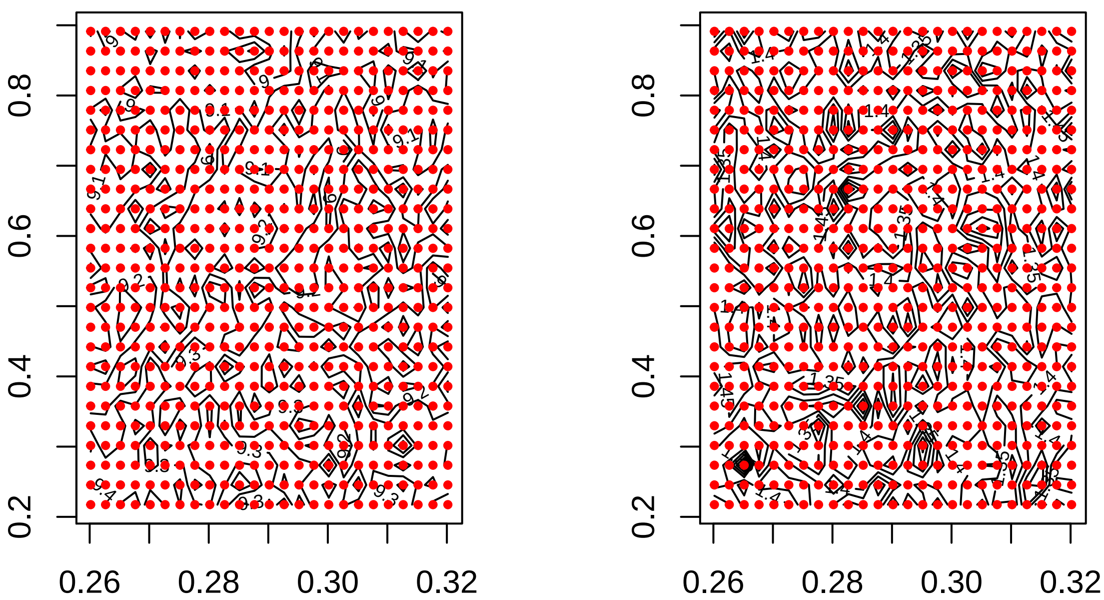
```
x.grid_1_cont <- as.numeric(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_end])
x.grid_2_cont <- as.numeric(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_end])

dim1 <- seq(min(x.grid_1_cont),max(x.grid_1_cont),length.out = 25)
dim2 <- seq(min(x.grid_2_cont),max(x.grid_2_cont),length.out = 25)
X.grid <- expand.grid(x1 = dim1, x2 = dim2)

x.grid_3_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility_scaled[test_start:tes
x.grid_4_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_en
```

```r
x.grid_5_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price_scaled[test_start:test_e
x.grid_6_cont <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price_scaled[test_start:test_en

x2_cont <- cbind(X.grid,x.grid_3_cont,x.grid_4_cont,x.grid_5_cont,x.grid_6_cont)

x.grid_1_cont_bs <- as.numeric(stan_dat$total_puts_American$dividend_yield[test_start:test_end])
x.grid_2_cont_bs <- as.numeric(stan_dat$total_puts_American$interest_rate[test_start:test_end])
x.grid_3_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$impl_volatility[test_start:test_end
x.grid_4_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$time_to_exp[test_start:test_end]))
x.grid_5_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$forward_price[test_start:test_end])
x.grid_6_cont_bs <- as.numeric(rep(mean(stan_dat$total_puts_American$strike_price[test_start:test_end]))

dim1_bs <- seq(min(x.grid_1_cont_bs),max(x.grid_1_cont_bs),length.out = 25)
dim2_bs <- seq(min(x.grid_2_cont_bs),max(x.grid_2_cont_bs),length.out = 25)
X.grid_bs <- expand.grid(x1 = dim1_bs, x2 = dim2_bs)

x2_cont_bs <- cbind(X.grid_bs,x.grid_3_cont_bs,x.grid_4_cont_bs,x.grid_5_cont_bs,x.grid_6_cont_bs)

blackscholes_2_cont <- rep(NA,length(x2_cont_bs[,1]))
for (row in 1:nrow(data.frame(x2_cont_bs))){
  blackscholes_2_cont[row] <- as.numeric(blackscholes(-1,S0=x2_cont_bs[row,5],K=x2_cont_bs[row,6],r=x2_c
}


post_data_cont <- list(theta=c(post_mean_theta_1_Bdrycov,post_mean_theta_2_Bdrycov,post_mean_theta_3_Bdr
# post_data

pred_gp_cont <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_cont,iter=200,
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                152.014 seconds (Sampling)
## Chain 1:                152.014 seconds (Total)
## Chain 1:
```

```r
#Computing Mean
y_predict_values_cont <- extract(pred_gp_cont,permuted=FALSE)
y_mean_values_cont <- c(colMeans(y_predict_values_cont))
y_mean_values_cont <- y_mean_values_cont[1:(length(y_mean_values_cont)-1)]

#Computing Standard Deviation
pred_gp_summary_cont <- summary(pred_gp_cont, sd=c("sd"))$summary
pred_gp_sd_cont <- pred_gp_summary_cont[, c("sd")]
y_sd_values_cont <- pred_gp_sd_cont[1:(length(pred_gp_sd_cont)-1)]
```

```r
par(mfrow = c(1, 2))
#Contour for Predictions aka mean values of predicitons
contour(dim1, dim2, matrix(y_mean_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
#Contour of Variance
contour(dim1, dim2, matrix(y_sd_values_cont, length(dim1), length(dim2)))
points(x2_cont[,1], x2_cont[,2], pch = 19, cex = 0.5, col = "red")
```



##Part 5: Improving the model by incorporating discrepancy

5-1: Computing Predicted European Option Prices

```r
library(rstan)
source("gp.utility.R")

# Fitting GP model
stan_dat_European <- read_rdump('Financial_Data_Put_European.R')
```

```
## Parsed with column specification:
## cols(
##   .default = col_double(),
##   date = col_character(),
##   symbol = col_character(),
##   exdate = col_character(),
##   cp_flag = col_character(),
```

```
##   ticker = col_character(),
##   exercise_style = col_character()
## )

## See spec(...) for full column specifications.
```

```r
fit_gp_SGP_European <- stan(file="gp-fit-6dimension_withBS.stan", data=stan_dat_European,
               iter=100, chains=1);
```

```
## DIAGNOSTIC(S) FROM PARSER:
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'gp-fit-6dimension_withBS' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.011 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 110 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: There aren't enough warmup iterations to fit the
## Chain 1:          three stages of adaptation as currently configured.
## Chain 1:          Reducing each adaptation stage to 15%/75%/10% of
## Chain 1:          the given number of warmup iterations:
## Chain 1:            init_buffer = 7
## Chain 1:            adapt_window = 38
## Chain 1:            term_buffer = 5
## Chain 1:
## Chain 1: Iteration:  1 / 100 [  1%]  (Warmup)
## Chain 1: Iteration: 10 / 100 [ 10%]  (Warmup)
## Chain 1: Iteration: 20 / 100 [ 20%]  (Warmup)
## Chain 1: Iteration: 30 / 100 [ 30%]  (Warmup)
## Chain 1: Iteration: 40 / 100 [ 40%]  (Warmup)
## Chain 1: Iteration: 50 / 100 [ 50%]  (Warmup)
## Chain 1: Iteration: 51 / 100 [ 51%]  (Sampling)
## Chain 1: Iteration: 60 / 100 [ 60%]  (Sampling)
## Chain 1: Iteration: 70 / 100 [ 70%]  (Sampling)
## Chain 1: Iteration: 80 / 100 [ 80%]  (Sampling)
## Chain 1: Iteration: 90 / 100 [ 90%]  (Sampling)
## Chain 1: Iteration: 100 / 100 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 8.793 seconds (Warm-up)
## Chain 1:                6.532 seconds (Sampling)
## Chain 1:                15.325 seconds (Total)
## Chain 1:
```

```r
print(fit_gp_SGP_European, pars = c('theta','sigma2','gamma2'))
```

```
## Inference for Stan model: gp-fit-6dimension_withBS.
## 1 chains, each with iter=100; warmup=50; thin=1;
## post-warmup draws per chain=50, total post-warmup draws=50.
##
```

```
##              mean se_mean      sd    2.5%     25%     50%     75%   97.5%
## theta[1]     0.26    0.01    0.05    0.19    0.22    0.26    0.29    0.38
## theta[2]     4.64    0.28    1.31    2.63    3.73    4.38    5.57    6.84
## theta[3]    10.06    0.68    3.47    5.55    7.81    9.02   12.04   18.58
## theta[4]     0.37    0.01    0.08    0.23    0.31    0.36    0.42    0.53
## theta[5]     1.26    0.21    0.64    0.65    0.79    1.05    1.52    2.98
## theta[6]     0.30    0.01    0.08    0.20    0.24    0.28    0.36    0.50
## sigma2       0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
## gamma2    2778.18  252.00 1152.63 1620.56 2011.17 2617.92 3054.29 6466.34
##          n_eff Rhat
## theta[1]    83 0.99
## theta[2]    22 1.05
## theta[3]    26 1.14
## theta[4]    82 1.00
## theta[5]     9 1.23
## theta[6]    35 0.99
## sigma2      70 0.98
## gamma2      21 1.01
##
## Samples were drawn using NUTS(diag_e) at Sun Mar 29 18:49:37 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
sum_gp_SGP_European <- extract(fit_gp_SGP_European,permuted=FALSE)
```

```
# Predicting from GP model
post_mean_theta_1_SGP <- mean(sum_gp_SGP_European[,1,1]) #theta
post_mean_theta_2_SGP <- mean(sum_gp_SGP_European[,1,2]) #theta
post_mean_theta_3_SGP <- mean(sum_gp_SGP_European[,1,3]) #theta
post_mean_theta_4_SGP <- mean(sum_gp_SGP_European[,1,4]) #theta
post_mean_theta_5_SGP <- mean(sum_gp_SGP_European[,1,5]) #theta
post_mean_theta_6_SGP <- mean(sum_gp_SGP_European[,1,6]) #theta
post_mean_sigma2_SGP <- mean(sum_gp_SGP_European[,1,7]) #sigma2
post_mean_gamma2_SGP <- mean(sum_gp_SGP_European[,1,8]) #gamma2
post_mean_mu_SGP <- stan_dat_European$blackscholes
```

```
x.grid_1 <- as.numeric(stan_dat$total_puts_American$forward_price_scaled[test_start:test_end])
x.grid_2 <- as.numeric(stan_dat$total_puts_American$strike_price_scaled[test_start:test_end])
x.grid_3 <- as.numeric(stan_dat$total_puts_American$impl_volatility_scaled[test_start:test_end])
x.grid_4 <- as.numeric(stan_dat$total_puts_American$time_to_exp_scaled[test_start:test_end])
x.grid_5 <- as.numeric(stan_dat$total_puts_American$dividend_yield_scaled[test_start:test_end])
x.grid_6 <- as.numeric(stan_dat$total_puts_American$interest_rate_scaled[test_start:test_end])
x2 <- cbind(x.grid_1,x.grid_2,x.grid_3,x.grid_4,x.grid_5,x.grid_6)
```

```
# X.grid <- expand.grid(x1 = x.grid_1, x2 = x.grid_2)

post_data_Bdrycov_American_disc <- list(theta=c(post_mean_theta_1_SGP,post_mean_theta_2_SGP,post_mean_t
# post_data

pred_gp_Bdrycov_disc <- stan(file="Predictive GP_6dimension_withBS_Bdrycov.stan", data=post_data_Bdryco
```

```
## DIAGNOSTIC(S) FROM PARSER:
```

```
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
## Info: Comments beginning with # are deprecated.  Please use // in place of # for line comments.
##
##
## SAMPLING FOR MODEL 'Predictive GP_6dimension_withBS_Bdrycov' NOW (CHAIN 1).
## Chain 1: Iteration:   1 / 200 [  0%]  (Sampling)
## Chain 1: Iteration: 100 / 200 [ 50%]  (Sampling)
## Chain 1: Iteration: 200 / 200 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0 seconds (Warm-up)
## Chain 1:                26.572 seconds (Sampling)
## Chain 1:                26.572 seconds (Total)
## Chain 1:
```

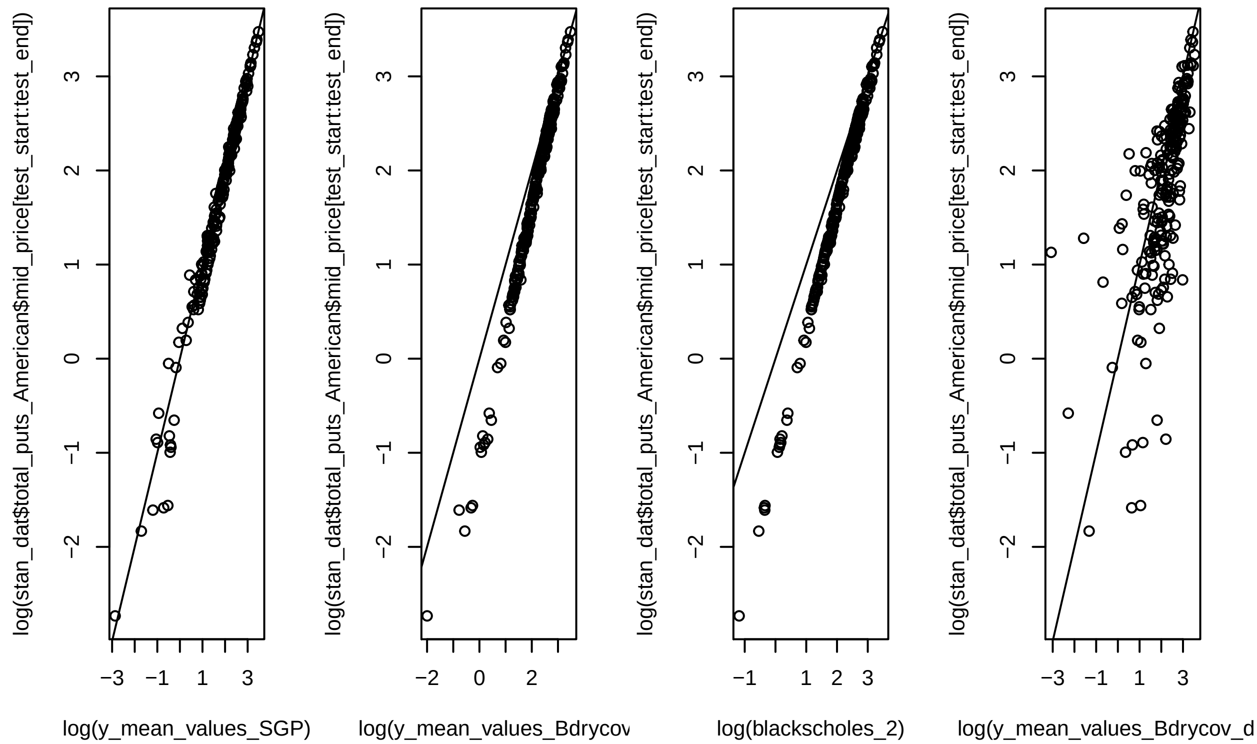
```r
#Computing Mean
y_predict_values_Bdrycov_disc <- extract(pred_gp_Bdrycov_disc,permuted=FALSE)
y_mean_values_Bdrycov_disc <- c(colMeans(y_predict_values_Bdrycov_disc))
y_mean_values_Bdrycov_disc <- y_mean_values_Bdrycov_disc[1:(length(y_mean_values_Bdrycov_disc)-1)]

#Computing Standard Deviation
pred_gp_summary_Bdrycov_disc <- summary(pred_gp_Bdrycov_disc, sd=c("sd"))$summary
pred_gp_sd_Bdrycov_disc <- pred_gp_summary_Bdrycov_disc[, c("sd")]
y_sd_values_Bdrycov_disc <- pred_gp_sd_Bdrycov_disc[1:(length(pred_gp_sd_Bdrycov_disc)-1)]
```

3-3: Plotting Predicted Values against Truth

```r
par(mfrow=c(1,4))
#Plotting Standard GP
plot(log(y_mean_values_SGP),log(stan_dat$total_puts_American$mid_price[test_start:test_end]),xlim = c(mi
abline(0,1)

#Plotting BDrycov
plot(log(y_mean_values_Bdrycov),log(stan_dat$total_puts_American$mid_price[test_start:test_end]), xlim =
abline(0,1)

#Plotting Blackscholes
plot(log(blackscholes_2),log(stan_dat$total_puts_American$mid_price[test_start:test_end]), xlim = c(min


#Plotting Discrepancy Model
plot(log(y_mean_values_Bdrycov_disc),log(stan_dat$total_puts_American$mid_price[test_start:test_end]), x
```

```
## Warning in log(y_mean_values_Bdrycov_disc): NaNs produced

## Warning in log(y_mean_values_Bdrycov_disc): NaNs produced

## Warning in log(y_mean_values_Bdrycov_disc): NaNs produced
```

```r
abline(0,1)
```




```
#MSE
library('MLmetrics')
MSE(y_mean_values_SGP,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 0.3419418
```

```
MSE(y_mean_values_Bdrycov,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 4.955886
```

```
MSE(blackscholes_2,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 4.923348
```

```
MSE(y_mean_values_Bdrycov_disc,stan_dat$total_puts_American$mid_price[test_start:test_end])
```

```
## [1] 21.8571
```