BIOS 6643 - Interpreting covariate effects

Linear link (e.g. normal) $Y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$

- Each additional unit of x is associated with an increase of β_1 units in the mean of Y.
- $E(Y_i | x_i + 1) - E(Y_i | x_i) = [\beta_0 + \beta_1 (x_i + 1)] - [\beta_0 + \beta_1 x_i] = \beta_1$

Log link (e.g. Poisson) $Y_i \sim Pois(e^{\beta_0 + \beta_1 x_i})$

- Each additional unit of x is associated with an increase of a factor of e^{β_1} in the mean rate of Y.
- $\dfrac{E(Y_i | x_i + 1)}{E(Y_i | x_i)} = \dfrac{e^{\beta_0 + \beta_1 (x_i + 1)}}{e^{\beta_0 + \beta_1 x_i}} = e^{\beta_1}$ = rate ratio

Logistic link (e.g. Binomial) $Y_i \sim Bin\left(p_i = \dfrac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}}, \underbrace{n_i}_{\text{often } = 1}\right)$

- Each additional unit of x is associated with an increase of a factor of e^{β_1} in the odds of $Y = 1$
- $\dfrac{P(Y_i = 1 | x_i + 1) / P(Y_i = 0 | x_i + 1)}{P(Y_i = 1 | x_i) / P(Y_i = 0 | x_i)} = \dfrac{\dfrac{e^{\beta_0 + \beta_1 (x_i + 1)}}{1 + e^{\beta_0 + \beta_1 (x_i + 1)}} \Big/ \dfrac{1}{1 + e^{\beta_0 + \beta_1 (x_i + 1)}}}{\dfrac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \Big/ \dfrac{1}{1 + e^{\beta_0 + \beta_1 x_i}}} =$

$$= \frac{e^{\beta_0 + \beta_1 (x_i + 1)}}{e^{\beta_0 + \beta_1 x_i}} = e^{\beta_1} = \text{odds ratio}$$

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