**Lecture 2 (Forwards Markets) Assignment, MTH 9865**

Due start of class, September 16, 2015.

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**Question 1 (4 marks)**

Why are correlations of daily returns of spot vs daily returns of forward prices so high in the FX markets? What are the two requirements a market must support to enforce a high correlation across the forward curve?

Most of the time the correlation of daily changes in spot and the 1y all-in forward is very high, north of 99.5%. This is because of the spot/forward arbitrage, interest rate markets for G7 currencies are all well-developed so people can really execute the arbitrage conveniently, therefore the spot and forwards will move together resulting in high correlation.

To enforce a high correlation across the forward curve, the two requirements of the market is

1. You can store currencies (and receive an interest rate for them)
2. You can borrow & short currencies (and pay an interest rate)

**Question 2 (2 marks)**

Why is risk management more complex for an FX forwards risk manager than for an FX spot risk manager?

In general, pricing and risk management for forwards becomes more complex than for spot because of the tenor dimension of a forward contract, i.e.

1. Not every forward contract is fungible with every other.
2. An FX forwards portfolio might contain forwards settling on many different dates.
3. Extra risk like the difference between all-in forward prices and the spot price. Or equivalently, non-USD interest rates, but risk displayed in notional of FX swap equivalents. Forwards traders also always monitor delta (risk to spot) but hedge that with their friends on the spot desk

**Question 3 (2 marks)**

Explain why risk to FX forward points can be expressed as risk to non-USD interest rates.

FX forward points depend on three market variables: the spot rate (though only weakly); the asset-currency interest rate; and the denominated-currency interest rate. One of the currencies is USD, since a cross currency forward can be represented in terms of the product or ratio of USD-pair forwards.

FX forward risk management is often viewed as equivalent to risk to the non-USD interest rate, because USD interest rate risk is normally managed separately, and the spot risk is managed through the portfolio's delta position. The only dimension of risk that isn't represented in other risk reports, then, is the non-USD interest rate risk.

**Question 4 (4 marks)**

Assume a portfolio has just one FX forward position in it, settling on a date T which lies between two benchmark settlement dates $T_1$ and $T_2$. Derive the notionals $N_1$ and $N_2$ of the benchmark forwards which hedge the portfolio risk assuming triangle shocks to the benchmark non-USD interest rates, as shown on page 21 of the lecture notes.

Start with expression for the price of a forward contract with settlement time T and strike K, then take partial derivative with respect to Q:

$$v = Se^{-QT} - Ke^{-RT}$$

$$\frac{\partial v}{\partial Q} = -STe^{-QT}$$

In order to hedge the risk to zero coupon bond rate for asset currency Q, we have following formulas

$$\frac{\partial v}{\partial Q} = N_1 \frac{\partial v_1}{\partial Q} + N_2 \frac{\partial v_2}{\partial Q}$$

Plug in the first formula into the formula above, we will be able to derive

$$-STe^{-QT} = N_1 \cdot (-ST_1 e^{-QT_1}) + N_2 \cdot (-ST_2 e^{-QT_2})$$

In order to let this formula hold, we can match the terms at both sides by setting two notionals. Firstly, to eliminate the exponential terms, there would be $e^{Q(T_1-T)}$ inside $N_1$ and $e^{Q(T_2-T)}$ inside $N_2$. Then what we will have would be

$$-T = n_1 \cdot (-T_1) + n_2 \cdot (-T_2)$$

where $n_1 = N_1 / e^{Q(T_1-T)}$ and similarly to $n_2$. Obviously next step we could see

$$n_1 = \frac{T_2 - T}{T_2 - T_1}\frac{T}{T_1}, n_2 = \frac{T - T_1}{T_2 - T_1}\frac{T}{T_2}$$

Therefore,

$$N_1 = \frac{T_2 - T}{T_2 - T_1}\frac{T}{T_1} e^{Q(T_1-T)}$$

$$N_2 = \frac{T - T_1}{T_2 - T_1}\frac{T}{T_2} e^{Q(T_2-T)}$$

is what expected on page 21 of the lecture notes.

**Question 5 (4 marks)**

Explain principal component analysis and factor models, focusing on the differences between the two approaches to reduce dimensionality.

PCA: Look for most important (non-parametric) shocks that tend to drive moves in the whole curve, e.g. parallel shift, tilt move, bend move, etc.

Factor Model: A formal model to predict observed variables from theoretical latent factors, i.e. Brownian motions is often viewed as more natural models for interest rates.

Difference –

1. PCA is picking off the component shocks (eigenfunctions of the covariance matrix) corresponding to the largest eigenvalues of the covariance matrix, while factor model is formulate a model with common factors that scientists think are important.
2. In PCA non-parametric shocks are hard to understand properly and can have unusual shapes due to specific data points in the history you're using. While in factor model parameters can be calculated once and stored so that you get consistent shocks, or recalculated each day.
3. In PCA non-parametric shock shapes change over time, which make traders unsure what their risk numbers mean. In factor model, it is usually using a fixed set of parameters, then the factor shocks are consistent from day to day, since that makes it easier for traders to understand the risk they're looking at.

**Question 6 (10 marks)**

This programming question will try to determine whether using a factor-based approach to reducing dimensionality is better than an ad hoc method.

We start by assuming a toy market: spot = 1, asset currency interest rate curve = Q(T) = flat at 3%, and denominated currency interest rate curve = R(T) = flat at 0%. We assume two benchmark dates, $T_1$ = 0.25y and $T_2$ = 1y; we will use forwards to those settlement dates to hedge the forward rate risk (or equivalently, the risk to moves in the asset currency interest rate) of our portfolio.

In the toy market, we assume that we know the dynamics of the asset currency interest rate:

$$dQ = \sigma_1 e^{-\beta_1 T} dz_1 + \sigma_2 e^{-\beta_2 T} dz_2$$
$$E[dz_1 dz_2] = \rho \, dt$$

where $\sigma_1$=1%/sqrt(yr), $\sigma_2$=0.8%/sqrt(yr), $\beta_1$=0.5/yr, $\beta_2$=0.1/yr, and $\rho$=-0.4.

The portfolio to hedge has one position: a unit asset-currency notional of a forward contract settling at time T. You'll try this for values of T in [0.1,0.25,0.5,0.75,1,2] to see how performance changes for portfolios with risk to different tenors.

You will try three different hedging strategies: one where you choose the hedge notionals (of forwards settling at times $T_1$ and $T_2$) based on the triangle shock we discussed in class (though as there are only two benchmarks here, the T1 shock will be flat before T1 and the T2 shock will be flat after T2); one where the notionals are set to hedge the actual two shocks from the factors described above; and lastly, one where you don't hedge at all.

The result should show that setting hedge notionals based on the true factor shocks should provide a better hedge performance than based on the ad hoc triangle shocks. You should analyze just how much better that performance is.

Run a Monte Carlo simulation where you do the following on each run, for each value of T, for each of the three hedging strategies described above:

1. Construct a portfolio long 1 unit of the forward settling at time T
2. Add in the hedges: two forwards, settling at times T1 and T2, with notionals set to hedge the portfolio (either against the two triangle shocks or against the two factor shocks). Don't bother adding the hedges in the third hedge scenario where we leave the portfolio unhedged.
3. Simulate the portfolio forward a time dt=0.001y. That will result in the asset-currency rates moving according to the factor model described above, which shocks the benchmark rates for tenors T1 and T2, and for the portfolio's risk tenor T. Determine the PNL realized.

Then construct the PNL distributions for the three hedging approaches. The unhedged version is the benchmark: you should compare how much more effectively the PNL standard deviation is reduced by hedging according to the true factors vs hedging according to the ad hoc triangle shocks.

Do this for all the values of T listed above, and discuss your results.

Below are the outputs of the program –

```
Value of Tenor: 0.1
     Non-hedging strategy PNL std: 0.303593126738
     Triangle-hedging strategy PNL std: 0.0203926849675
     Factor-hedging strategy PNL std: 3.74297299435e-06
Value of Tenor: 0.25
     Non-hedging strategy PNL std: 0.719778670233
     Triangle-hedging strategy PNL std: 0.0
     Factor-hedging strategy PNL std: 2.19904427193e-16
Value of Tenor: 0.5
     Non-hedging strategy PNL std: 1.30794039015
     Triangle-hedging strategy PNL std: 0.0179234319382
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    Factor-hedging strategy PNL std: 1.34569899252e-05
Value of Tenor: 0.75
    Non-hedging strategy PNL std: 1.81743722708
    Triangle-hedging strategy PNL std: 0.0257088746378
    Factor-hedging strategy PNL std: 1.73707830776e-05
Value of Tenor: 1
    Non-hedging strategy PNL std: 2.26298300851
    Triangle-hedging strategy PNL std: 0.0
    Factor-hedging strategy PNL std: 2.00560585855e-15
Value of Tenor: 2
    Non-hedging strategy PNL std: 3.63306417728
    Triangle-hedging strategy PNL std: 1.31276451678
    Factor-hedging strategy PNL std: 0.000333222588875
```

Conclusion: Obviously, at any tenors, setting hedge notionals based on the true factor shocks provides the best hedge performance, the ad hoc triangle shocks seconds, and non-hedging is the worst.

Analysis: According to the number above, the standard deviation of non-hedging strategy is from 0.3 to 3.6 respectively to the tenor. It will reduce to 10% when using a triangle-hedging strategy. But the most amazing difference happens on factor-hedging strategy, which reduces to almost zero compared to the standard deviations of previous two strategies.