

## Lecture 1 (Spot Markets) Assignment, MTH 9865

Due start of class, September 9, 2015

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### Question 1 (2 marks)

Describe the four factors that contribute to the bid and ask prices a market maker will show to a client during voice trading?

The four factors are inter-dealer market, current risk position, market views, and client behavior.

### Question 2 (2 marks)

Why has the daily turnover in the FX markets increased so much in the past fifteen years? Give some statistics.

**Statistics:** The 2013 Triennial Survey shows a significant pickup in global FX market activity to \$5.3 trillion per day in 2013, up from \$4.0 trillion in 2010. With growth in global FX turnover of about 35% at current exchange rates, the 2013 survey results continue the trend of strong turnover growth evidenced in past Triennial Surveys. FX turnover computed at constant exchange rates grew roughly by the same magnitude. The growth in global FX market activity between 2010 and 2013 outpaced the 19% rise from 2007 to 2010 reported in the prior survey, but falls short of the record 72% increase (at current exchange rates) between 2004 and 2007.

**Conclusion and Reason:** Significant growth in the last 15y as the FX market turned from a mostly voice trading market to an electronic trading market. Electronic trading allows for a lot of churn, especially by high frequency trading funds. That churn is also supported by a drop in bid/ask spreads, which reduces trading friction for high frequency trading. Bid/ask spreads in the FX markets have gone from 2-3bp for a \$5M-ish sized trade to 0.2-0.5bp today as the volumes have increased and trading has moved electronic.

**Source:** BIS Survey 2013, <http://www.bis.org/publ/rpfx13fx.pdf>.

### Question 3 (4 marks)

Describe the OTC market structure and the different roles involved in executing a trade. Describe the steps involved in executing a trade for voice trading and then for electronic trading.

### Question 4 (4 marks)

Today is October 27<sup>th</sup>, 2015. Tomorrow (October 28<sup>th</sup>) is a good business day for all three currencies. October 29<sup>th</sup> is a JPY currency settlement holiday. October 30<sup>th</sup> (a Friday) is a USD settlement holiday and November 2<sup>nd</sup> (a Monday) is a EUR settlement holiday. November 3<sup>rd</sup> is a good business day for all three currencies.

The EURUSD mid-market spot rate is 1.1300 (the price of a EUR in USD) and the USDJPY mid-market spot rate is 120.00 (the price of a USD in JPY). The USD interest rate is 0.25%, the EUR interest rate is 0.50%, and the JPY interest rate is 0.10%.

What are the spot dates for EURUSD, for USDJPY, and for EURJPY? What is the EURJPY mid-market spot rate implied from the triangle arbitrage?

Spot date for EURUSD: Oct 29<sup>th</sup>

Spot date for USDJPY: Nov 1<sup>st</sup>

Spot date for EURJPY: Nov 3<sup>rd</sup>

$$\begin{aligned}
 \text{EURJPY spot rate} &= \text{EURUSD spot rate} * \text{USDJPY spot rate} \\
 &= F_1 e^{(Q1-R1)T1} * F_2 e^{(Q2-R2)T2} \\
 &= 1.1300 * e^{(0.50\% - 0.25\%) * 2} * 120.00 * e^{(0.25\% - 0.10\%) * 5} \\
 &= 137.31
 \end{aligned}$$

### Question 5 (2 marks)

Same market as Question 4. Assume zero bid/ask spread in interest rates.

Take the bid/ask for EURUSD as 1.1299/1.1301, and the bid/ask for USDJPY as 119.99/120.01. What is the bid/ask for EURJPY implied from the triangle arbitrage?

**Bid:**

$$\begin{aligned}
 \text{EURJPY spot rate} &= F_1 e^{(Q1-R1)T1} * F_2 e^{(Q2-R2)T2} \\
 &= 1.1299 * e^{(0.50\% - 0.25\%) * 2} * 119.99 * e^{(0.25\% - 0.10\%) * 5} \\
 &= 137.28
 \end{aligned}$$

**Ask:**

$$\begin{aligned}
 \text{EURJPY spot rate} &= F_1 e^{(Q1-R1)T1} * F_2 e^{(Q2-R2)T2} \\
 &= 1.1301 * e^{(0.50\% - 0.25\%) * 2} * 120.01 * e^{(0.25\% - 0.10\%) * 5} \\
 &= 137.33
 \end{aligned}$$

### Question 6 (10 marks)

In Python, implement a variation of the “toy simulation algorithm” we discussed in class. Model parameters to assume:

- Spot starts at 1
- Volatility is 10%/year
- Poisson frequency  $\lambda$  for client trade arrival is 1 trade/second
- Each client trade that happens delivers a position of either +1 unit of the asset or -1 unit of the asset, with even odds
- Bid/ask spread for client trades is 1bp
  - Receive PNL equal to  $1\text{bp} \times \text{spot} \times 50\%$  on each client trade (since client trades always have unit notional in this simulation)
- Bid/ask spread for inter-dealer hedge trades is 2bp
  - Pay PNL equal to  $2\text{bp} \times \text{spot} \times \text{hedge notional} \times 50\%$  on each hedge trade
- A delta limit of 3 units before the algorithm executes a hedge in the inter-dealer market.

Use a time step  $\Delta t$  equal to  $0.1/\lambda$  and assume that only a single client trade can happen in each time step (with probability equal to  $1 - e^{-\lambda \Delta t}$ ). Use 500 time steps and a number of simulation runs to give sufficient convergence.

When converted between seconds and years, assume 260 (trading) days per year.

The variation in the algorithm: when the algorithm decides to hedge, it can do a partial hedge, where it trades such that the net risk is equal to the delta limit (either positive or negative depending on whether the original position was above the delta limit or below  $-1 \times \text{delta limit}$ ); or it can do a full hedge, like in the algorithm we discussed in class, where the net risk is reduced to zero.

Use the Sharpe ratio of the simulation PNL distribution to determine which of those two hedging approaches is better.

Your solution should deliver the Python script that implements the simulation, and you should explain your answer by giving numerical results from the simulation as well as some qualitative intuition behind the result. Include data that shows that you have used sufficient Monte Carlo simulation runs to show that your final result is not affected by statistical noise.

Marks will be given for both the numerical results generated from the simulation as well as the quality of your Python code. Remember to include lots of explanatory comments in your code and use variable names that are meaningful. Use external packages like numpy/scipy where applicable rather than rolling your own low-level numerical functions like random number generators. For top marks, use only vectorized operations across the Monte Carlo paths to speed up execution.

Here is one sample result of the output:

*Full Hedge approach*

<i>PNL Sharpe ratio:</i>	<i>1.49046508153</i>
<i>PNL Mean:</i>	<i>0.000839054899793</i>
<i>PNL Std dev:</i>	<i>0.000562948377786</i>

*Partial Hedge approach*

<i>PNL Sharpe ratio:</i>	<i>3.32160686914</i>
<i>PNL Mean:</i>	<i>0.00175617131148</i>
<i>PNL Std dev:</i>	<i>0.000528711367921</i>

The second approach is better, quantitative intuition is that if you look into the different of hedge trades and client trades between these two approaches, their client trades are similar, but the hedge trades of the second approach is much greater, which cost more but makes no difference when spots go up and down.

To replicate the result, please run *test\_simulator.py* via

```
python3 test_simulator.py
```

where *test\_simulator()* is a linear run with Monte Carlo paths = 100,000 and *test\_simulators()* is a parallel run on 100 buckets, i.e.  $100 * 100,000 = 10,000,000$  Monte carlo paths. Parallel run is designed for top marks.