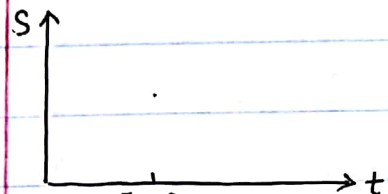
MTH 9821 - L11

2017/11/30

Dec. 14th. : Another Class Dec. 16th: Final Exam.

Derivatives Valuation for Discrete Divident Paying Assets

Asset pays divident D at time t_{div}.



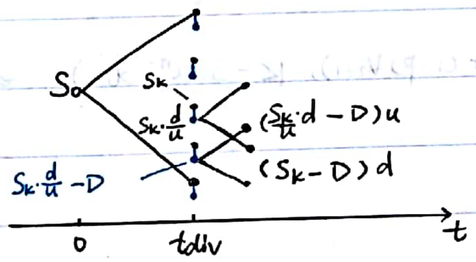
t_{div}^- t_{div}^+ $S(t_d^+) = S(t_d^-) - D$ (Here "t_d" is "t_{div}").

Proportional Divident: $D = d \cdot S(t_d^-)$

Fixed Divident: D constant not depending on $S(t_d^-)$

Binomial tree model for evolution of a discrete divident paying asset

$u = e^{\sigma\sqrt{\delta t}}$. $d = e^{-\sigma\sqrt{\delta t}}$

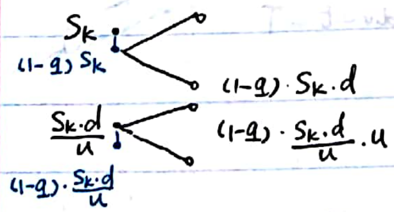


$(S_k \cdot \frac{d}{u} - D) \cdot u = (S_k - D) \cdot d$ (Fix divident)

for any $S_k = S_0 u^{n-k} d^k$. (X)

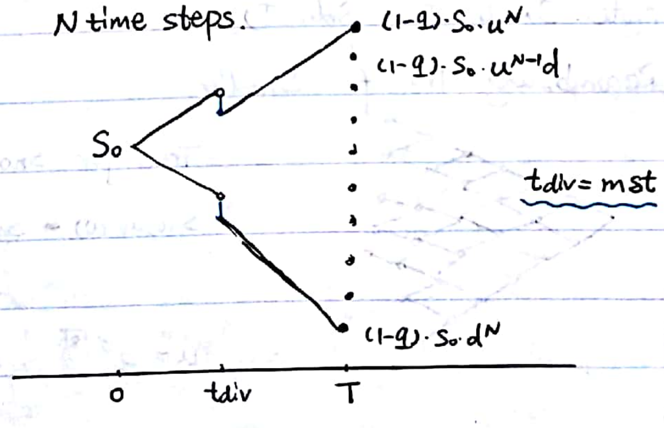
Not a recombining tree!

Proportional Divident.



In this case, tree recombining!

N time steps.



for $i = 0 : N$ $\quad V_i = \max(K - (1-q) S_0 u^{N-i} d^i, 0)$ $\quad$ end ;

for $n = (N-1) : 0$

$\quad$ for $i = 0 : n$ $\quad V_i = e^{-r\delta t}[pV_i + (1-p) \cdot V_{i+1}]$

$\quad$ end

end

Exactly the same as start from : $S_0(1-q)$.

D_l at t_l, with $D_l = q_l \cdot S(t_l^-)$ for $l = 1 : p$.

$V_{Eur}(S_{\text{discrete div}}) = V_{Eur}(\prod_{l=1}^{p} S_{\text{no divs}})$

$\downarrow$ Starting at S_0 $\qquad\qquad$ $\downarrow$ Starting at $\prod_{l=1}^{p}(1-q_l) \cdot S_0$

American Options : (Put)

for $n = (N-1) : m$ $\rightarrow m+1$ (for call)

$\quad$ for $i = 0 : n$ $\quad V_i = \max\left(e^{-r\delta t}[pV_i + (1-p)V_{i+1}],\ K - (1-q) S_0 u^{n-i} d^i\right)$ $\quad$ end

end

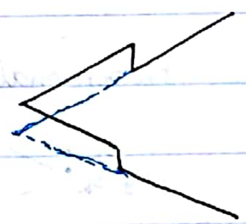
for $n = (m-1) : 0$ $\rightarrow m$ (for call)

$\quad$ for $i = 0 : n$ $\quad V_i = \max(e^{-r\delta t}[pV_i + (1-p)V_{i+1}),\ K - S_0 u^{n-i} d^i)$ $\quad$ end

end

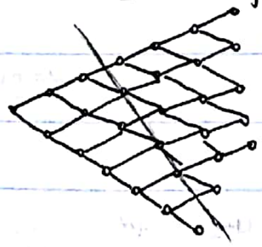
Fixed divident D at t_{div}.

$$\text{Let } S_{Nodiv}(t) = \begin{cases} S_{div}(t) - De^{-r(t_{div}-t)}, & 0 \le t \le t_{div} \\ S_{div}(t), & t_{div} < t \le T \end{cases}$$



Note: $S_{Nodiv}(T) = S_{div}(T)$

Recombining Tree for S_{nodiv}

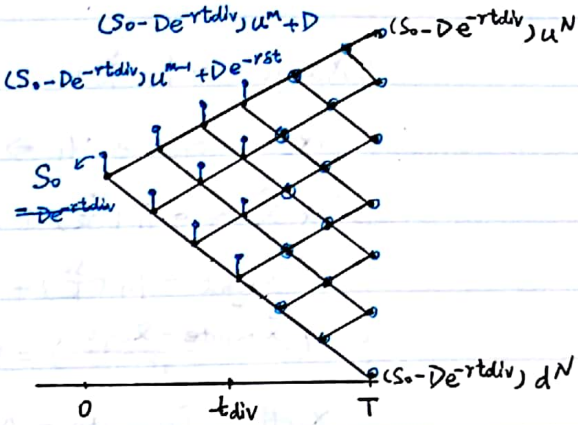


Tree for $S_{nodiv}(t)$

$S_{nodiv}(0) = \underset{\underset{S_0}{\parallel}}{S_{div}(0)} - De^{-rt_{div}}$

$u = e^{\sigma\sqrt{\delta t}}$. $\quad d = e^{-\sigma\sqrt{\delta t}}$

$$S_{div}(t) = \begin{cases} S_{nodiv}(t) + De^{-r(t_{div}-t)}, & 0 \le t \le t_{div} \\ S_{nodiv}(t), & t < t_{div} \le t \end{cases}$$



For HW Question:

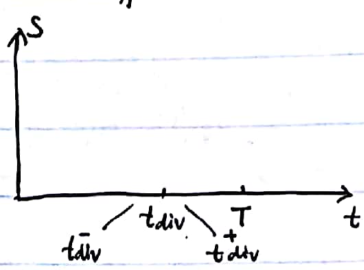
<fixed 2 dividends>

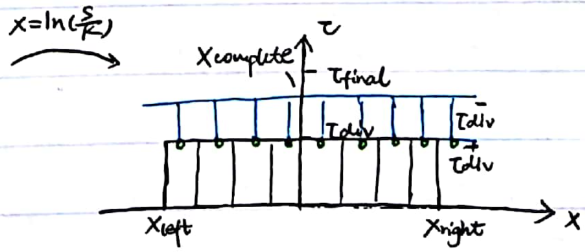
$$S_{nodiv}(t) \begin{cases} S_{div}(t), & \frac{6}{12} < t \le \frac{7}{12} \\ S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)}, & \frac{2}{12} < t \le \frac{6}{12} \\ S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)} - 0.50e^{-r(\frac{2}{12}-t)}, & 0 \le t \le \frac{2}{12} \end{cases}$$

$$S_{nodiv}(t) \begin{cases} 0.99\, S_{div}(t), & \frac{6}{12} < t \le \frac{7}{12} \\ 0.99\,[S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)}], & \frac{4}{12} < t \le \frac{6}{12} \\ S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)}, & \frac{2}{12} < t \le \frac{4}{12} \\ S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)} - 0.50e^{-r(\frac{2}{12}-t)}, & 0 \le t \le \frac{2}{12} \end{cases}$$

$$S_{nodiv}(t) = \begin{cases} S_{div}(t), & \frac{6}{12} < t \le \frac{7}{12} \\ S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)}, & \frac{4}{12} < t \le \frac{6}{12} \\ 0.99(S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)}), & \frac{2}{12} < t \le \frac{4}{12} \\ 0.99(S_{div}(t) - 0.75e^{-r(\frac{6}{12}-t)}) - 0.50e^{-r(\frac{2}{12}-t)}, & 0 \le t \le \frac{2}{12} \end{cases}$$

Finite Difference





$\tau_{div} = \frac{(T - t_{div})}{2}\sigma^2$

$S(t_{div}^+) = (1-q)\cdot S(t_{div}^-)$ $\qquad X(t_{div}^+) = \ln(1-q) + X(\tau_{div}^-)$.

$X(\tau_{div}^-) = X(\tau_{div}^+) - \ln(1-q) > X(\tau_{div}^+)$.

Scanned by CamScanner

$x_{compute} = \ln(\frac{S_0}{K})$. $\bar{x}_{compute} = \ln(1-q) + \ln(\frac{S_0}{K})$. ← require this to be on the grid.

Fix α_1. choose $M_1 \Rightarrow \delta\tau_1 = \frac{\tau_{div}}{M_1} \Rightarrow \delta x = \sqrt{\frac{\delta\tau_1}{\alpha_1}}$

Let $\tilde{x}_{left} = \ln(\frac{S_0}{K}) + (r-\frac{\sigma^2}{2})T - 3\sigma\sqrt{T}$

$\tilde{x}_{right} = \ln(\frac{S_0}{K}) + (r-\frac{\sigma^2}{2})T + 3\sigma\sqrt{T}$

$\text{ceil}(\frac{\bar{x}_{compute} - \tilde{x}_{left}}{\delta x}) = N_{left}$. $\text{ceil}(\frac{-\bar{x}_{compute} + \tilde{x}_{right}}{\delta x}) = N_{right}$. $N = N_{left} + N_{right}$

$x_{left} = \bar{x}_{compute} - N_{left}\,\delta x$. $x_{right} = \bar{x}_{compute} + N_{right}\,\delta x$.

$V(S,t) = e^{-ax-b\tau} u(x,\tau)$. $V(S(t_{div}^-), t_{div}^-) = V(S(t_{div}^+), t_{div}^+)$

$e^{-ax_{div}^- - b\tau_{div}^-} u(x_{div}^-, \tau_{div}^-) = e^{-ax_{div}^+ - b\tau_{div}^+} u(x_{div}^+, \tau_{div}^+)$.

<Want this new BCs on $\tau_{div}^- < \tau < \tau_{final}$> <known from solving on $0 < \tau < \tau_{div}^+$>

$u(x_{div}^-, \tau_{div}^-) = e^{-a(x_{div}^+ - x_{div}^-)} u(x_{div}^+, \tau_{div}^+) = e^{-a\ln(1-q)} u(x_{div}^+, \tau_{div}^+)$

$= (1-q)^{-a} u(x^+_{div}, \tau^+_{div})$.

Computational domain on $\tau_{div}^- < \tau < \tau_{final}$.

$x_{left,new} = x_{left} - \ln(1-q)$. $x_{right,new} = x_{right} - \ln(1-q)$.

Let $\alpha_{temp} = \alpha$. $\delta\tau_{2,temp} = \alpha_{temp}(\delta x)^2$. $M_2 = \text{ceil}(\frac{\tau_{final} - \tau_{div}}{\delta\tau_{2,temp}})$.

$\delta\tau_2 = \frac{\tau_{final} - \tau_{div}}{M_2}$. $\alpha_2 = \frac{\delta\tau_2}{(\delta x)^2} \le \alpha_{temp} = \alpha_1$.