Lecture 25: Jump Diffusion Models Continued

Final Exam: Room 303, Wed May 13: 1:10pm - 4:00 pm

170 minutes and 170 points

No cheat sheet, just a calculator and pen

Similar in style to the midterm, aims to test your understanding rather than whether you can plug the right number into the right formula.

Modeling Jumps Alone

Pure Jump Processes: Calibration and Compensation Always Important

Discrete binomial approximation to a diffusion process over time Δt:

$$\begin{tikzcd} & \mu\Delta t + \sigma\sqrt{\Delta t} \\ \ln(S/S_0) \quad 0 \arrow[ur, no head, "0.5"] \arrow[dr, no head, "0.5"'] & \\ & \mu\Delta t - \sigma\sqrt{\Delta t} \end{tikzcd}$$

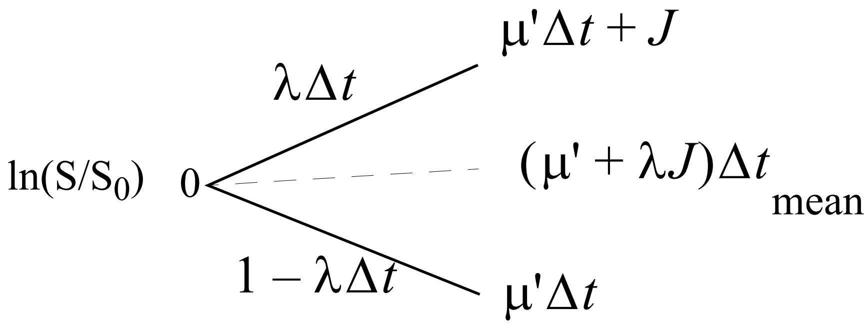
The **probabilities** of both up and down moves are **finite**, but the **moves** themselves are **small**, of order $\sqrt{\Delta t}$.

The net variance is $\sigma^2 \Delta t$ and the drift is μ. In continuous time this represents the process

$$d\ln S = \mu dt + \sigma dZ$$

Jumps are fundamentally different.

There the **probability of a jump *J* is small**, of order Δt, but the **jump itself is finite**.



3 parameters μ', J, λ

Mean:

$$\begin{aligned} E[\ln S] &= \lambda\Delta t[\mu'\Delta t + J] + (1 - \lambda\Delta t)\mu'\Delta t \\ &= (\mu' + \lambda J)\Delta t \end{aligned}$$

Variance

$$\begin{aligned} \text{var} &= \lambda\Delta t[J(1 - \lambda\Delta t)]^2 + (1 - \lambda\Delta t)[J\lambda\Delta t]^2 \\ &= (1 - \lambda\Delta t)J^2\lambda\Delta t[1 - \lambda\Delta t + \lambda\Delta t] \\ &= (1 - \lambda\Delta t)J^2\lambda\Delta t \\ &\to J^2\lambda\Delta t \qquad \text{as } \Delta t \to 0 \end{aligned}$$

$$\textit{Observed}\text{ drift } \mu = (\mu' + \lambda J)$$

$$\textit{Observed}\text{ volatility } \sigma^2 = J^2\lambda.$$

Calibration: If we *observe* a drift μ and a volatility σ, we must calibrate the jump process so that

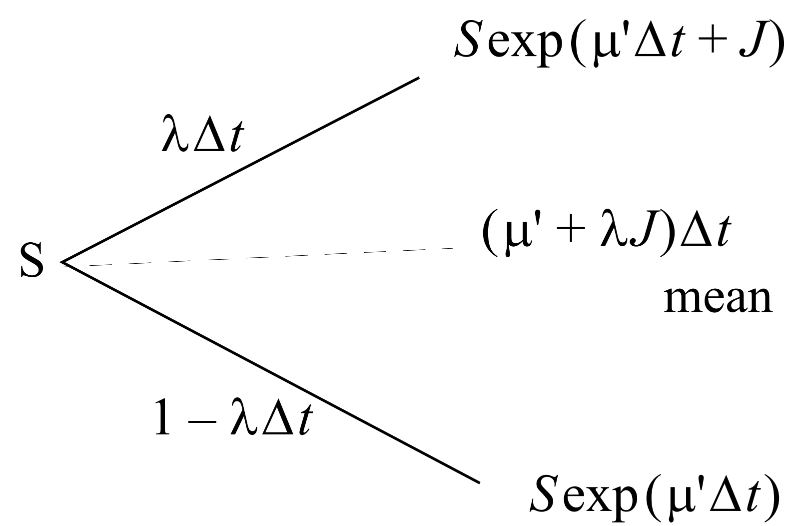
$$J = \frac{\sigma}{\sqrt{\lambda}}$$

$$\mu' = \mu - \sqrt{\lambda}\sigma$$

The one unknown is λ which is the probability of a jump in return of J in $\ln S$ per unit time.

This described how $\ln(S)$ evolves. How does S evolve?

$$\begin{aligned} E[S] &= (1-\lambda\Delta t)S\exp(\mu'\Delta t) + \lambda\Delta t S\exp(\mu'\Delta t + J) \\ &= S\exp(\mu'\Delta t)[1 + \lambda\Delta t(e^J - 1)] \\ &\approx S\exp\left[\left\{\mu' + \lambda(e^J - 1)\right\}\Delta t\right] \end{aligned}$$



Thus risk neutral growth means

$$r = \mu' + \lambda(e^{J} - 1)$$

$$\mu' = r - \lambda(e^{J} - 1)$$

We have to **compensate the drift for the jump contribution to calibrate to a total return r**.

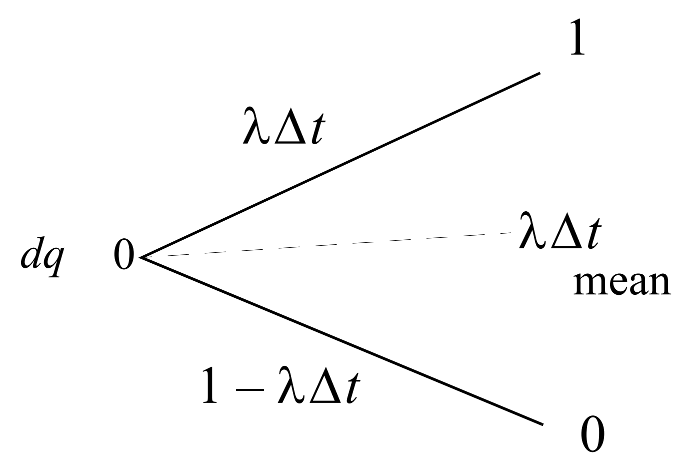
In continuous-time notation the jump can be written as a Poisson process

$$d\ln S = \mu' dt + J dq$$

Here dq is a jump or Poisson process that is modeled as follows:

The increment dq takes the valueS:

1 with probability λdt if a jump occurs
0 with probability $1 - \lambda dt$ if no jump occurs
expected value $E[dq] = \lambda dt$.



The Poisson Distribution of Jumps

λ = the constant probability of a jump J occurring per unit time.

$P(n, t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$ the probability of n jumps occurring during time t.

The mean number of jumps during time t is λt so λ is the probability per unit time of one jump.

Merton's jump-diffusion model and its PDE

Poisson jumps + GBM diffusion, $$\frac{dS}{S} = \mu dt + \sigma dZ + J dq$$

$$E[dq] = \lambda dt$$
$$var[dq] = \lambda dt$$

J is like a random dividend, **not** paid to stockholder.. Later we'll make J a normal random variable.

You can derive a partial differential equation for options valuation:

Option $C(S, t)$ and usual hedged portfolio $\pi = C - nS$

$$n dS = nS(\mu dt + \sigma dZ + J dq)$$

$$\Delta C = \left[C_t + \frac{1}{2} C_{SS} (\sigma S)^2\right] dt + C_S(\mu S dt + \sigma S dZ) + [C(S + JS, t) - C(S, t)] dq$$

$$\begin{aligned}\Delta\pi &= \Delta C - n[\mu S dt + \sigma S dZ + JS dq] \\ &= \left[C_t + C_S \mu S + \frac{1}{2} C_{SS} (\sigma S)^2 - n\mu S\right] dt + (C_S - n)\sigma S dZ \\ &\quad + [C(S + JS, t) - C(S, t) - nSJ] dq\end{aligned}$$

Choose n to hedge the diffusion: $n = C_S$.

$$\Delta\pi = \left[C_t + \frac{1}{2}C_{SS}(\sigma S)^2\right]dt + [C(S + JS, t) - C(S, t) - C_S SJ]dq$$

The partially hedged portfolio is still risky because of the possibility of jumps.

Imagine that we can diversity our portfolio over many different stocks and their options, where the stocks have uncorrelated jumps, so that jump risk becomes diversifiable and can be eliminated?

$$E[\Delta V] = rV\Delta t \qquad\qquad E[dq] = \lambda\Delta t$$

$$C_t + \frac{1}{2}C_{SS}(\sigma S)^2 + E[C((1 + J)S, t) - C(S, t) - C_S SJ]\lambda = (C - SC_S)r$$

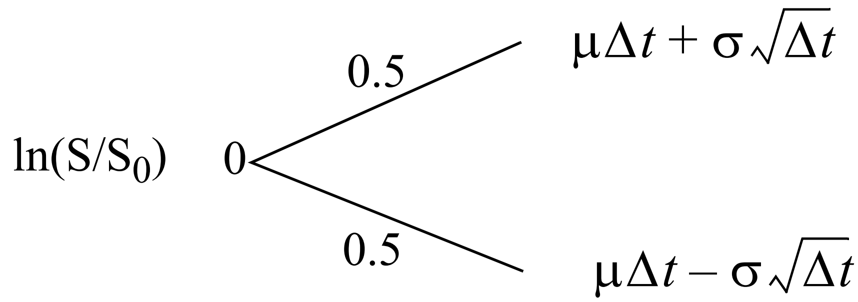
averaging over all jump sizes.

$$C_t + \frac{1}{2}C_{SS}(\sigma S)^2 + rSC_S - rC + E[C((1 + J)S, t) - C(S, t) - C_S SJ]\lambda = 0$$

This is a mixed difference/partial-differential equation for a standard call with terminal payoff $C_T = max(S_T - K, 0)$. For $\lambda = 0$ it reduces to the Black-Scholes equation. We will solve it a little later by the Feynman-Kac method as an expected discounted value of the payoffs.

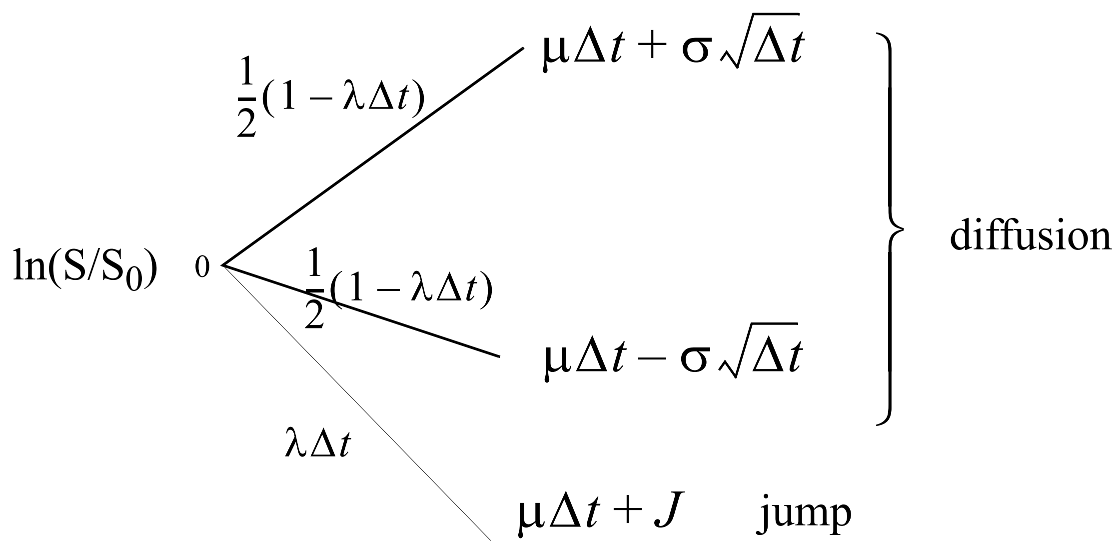
Trinomial Jump-Diffusion and Compensation

Diffusion can be modeled binomially, as in



The volatility σ of the log returns adds an Ito $\sigma^2/2$ term to the drift of the stock price S itself, so that for pure risk-neutral diffusion one must choose $\mu = r - \sigma^2/2$.

To add jumps one J needs a third, trinomial, leg in the tree:



Just as diffusion modifies the drift of the stock price, so do jumps.

The expected log return after time Δt:

$$E\left[\log\frac{S}{S_0}\right] = \left(\frac{1-\lambda\Delta t}{2}\right)2\mu\Delta t + \lambda\Delta t[\mu\Delta t + J]$$

$$= (\mu + J\lambda)\Delta t$$

Thus the effective drift of the log of the jump-diffusion process will be $\mu_{JD} = \mu + J\lambda$.

$$var \equiv \left(\frac{1-\lambda\Delta t}{2}\right)[\sigma\sqrt{\Delta t} - J\lambda\Delta t]^2 + \left(\frac{1-\lambda\Delta t}{2}\right)[\sigma\sqrt{\Delta t} + J\lambda\Delta t]^2$$

$$+ \lambda\Delta t[J(1-\lambda\Delta t])]^2$$

The variance at time Δt is

$$= \left(\frac{1-\lambda\Delta t}{2}\right)[2\sigma^2\Delta t + 2J^2\lambda^2(\Delta t)^2] + \lambda\Delta t J^2(1-\lambda\Delta t)^2$$

$$= (1-\lambda\Delta t)[\sigma^2\Delta t] + (1-\lambda\Delta t)J^2\lambda\Delta t(\lambda\Delta t + 1 - \lambda\Delta t)$$

$$= (1-\lambda\Delta t)[\sigma^2 + J^2\lambda]\Delta t$$

so that, as $\Delta t \to 0$, the variance of the jump diffusion process is $\sigma_{JD}^2 = [\sigma^2 + J^2\lambda]$

the sum of the diffusion variance plus the jump variance. The drift and variance are both affected by the fractional jump J and its probability λ of occurring per unit time.

The Compensated Process

How must we choose/calibrate the diffusion and jumps so that $E[dS] = Srdt$?

First let's compute the stock growth rate under jump diffusion.

$$E\left[\frac{S}{S_0}\right] = \frac{(1-\lambda\Delta t)}{2}e^{\mu\Delta t + \sigma\sqrt{\Delta t}} + \frac{(1-\lambda\Delta t)}{2}e^{\mu\Delta t - \sigma\sqrt{\Delta t}} + \lambda\Delta t e^{\mu\Delta t + J}$$

$$= e^{\mu\Delta t}\left[\frac{(1-\lambda\Delta t)}{2}\left(e^{\sigma\sqrt{\Delta t}} + e^{-\sigma\sqrt{\Delta t}}\right) + \lambda\Delta t e^{J}\right]$$

One can show by expanding this to keep terms of order Δt that

$$E\left[\frac{S}{S_0}\right] = \exp\left(\left\{\mu + \frac{\sigma^2}{2} + \lambda(e^J - 1)\right\}\Delta t\right) + \text{higher order terms}$$

so that, if we want the stock to grow risk-neutrally, we must set $r = \mu + \frac{\sigma^2}{2} + \lambda(e^J - 1)$

$$\mu_{JD} = r - \underbrace{\frac{\sigma^2}{2}}_{\text{diffusion compensation}} - \underbrace{\lambda(e^J - 1)}_{\text{jump compensation}}$$

Valuing a Call in the Jump-Diffusion Model

The process we are considering is $\frac{dS}{S} = \mu dt + \sigma dZ + Jdq$

where
$$E[dq] = \lambda dt$$
$$var[dq] = \lambda dt$$

J is assumed to be a fixed jump size now, but will later be generalized to a normal variable.

Risk neutrality: $$\mu = r - \frac{\sigma^2}{2} - \lambda(e^J - 1)$$

The value of a standard call in this model is given by

$$C_{JD} = e^{-r\tau}E[(S_T - K, 0)]$$

The risk-neutral terminal value of the stock price is $S_T = Se^{\mu\tau + Jq + \sigma\sqrt{\tau}Z}$

Sum over 0, 1, ... n ... jumps plus the diffusion, where the probability of n jumps $\frac{(\lambda\tau)^n}{n!}e^{-\lambda\tau}$

Thus,

$$C_{JD}(S, K) = e^{-r\tau} \sum_{n=0}^{\infty} \frac{\lambda\tau^n}{n!} e^{-\lambda\tau} E\left[max\left(S_T^n - K, 0\right)\right]$$

where S_T^n is the terminal lognormal distribution of the stock price that started with initial price S and underwent *n* jumps as well as the diffusion. Note: $C_{JD}(S, 0) \equiv S$

Each term is an expectation over a lognormal stock price that, after time τ, has undergone *n* jumps, and therefore is simply related to a Black-Scholes expectation with a jump-shifted distribution or different forward price.

In the compensated world, the expected return on a stock that started at an initial price *S* and suffered *n* jumps is

$$\mu_n = r - \frac{\sigma^2}{2} - \lambda(e^J - 1) + \frac{nJ}{\tau}$$

where the last term in the above equation adds the drift corresponding to *n* jumps to the standard compensated risk-neutral drift $r - \frac{\sigma^2}{2}$, which appears in the Black-Scholes formula via the terms $d_{1,2}$.

Thus, since S_T is lognormal with a shifted center moved by *n* jumps,

$$E\left[max\left(S_T^n - K, 0\right)\right] \equiv e^{r_n \tau} C_{BS}(S, K, \tau, \sigma, r_n)$$

where $C_{BS}(S, K, \tau, \sigma, r_n)$ is the standard Black-Scholes formula for a call with strike K and volatility σ with the drift rate r_n given by

$$r_n \equiv \mu_n + \frac{\sigma^2}{2} = r - \lambda(e^J - 1) + \frac{nJ}{\tau}$$

The $\sigma^2/2$ term is omitted because the Black-Scholes formula for a stock with volatility σ already includes the term $\sigma^2/2$ in the $N(d_{1,2})$ terms as part of the definition of C_{BS}.

$$C_{JD} = e^{-r\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} e^{-\lambda\tau} e^{r_n\tau} C_{BS}(S, K, \tau, \sigma, r_n)$$

$$= e^{-r\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} e^{-\lambda\tau} e^{\left(r - \lambda(e^J - 1) + \frac{nJ}{\tau}\right)\tau} C_{BS}(S, K, \tau, \sigma, r_n)$$

$$= e^{-(\lambda e^J\tau)} \sum_{n=0}^{\infty} \frac{(\lambda e^J\tau)^n}{n!} C_{BS}\left(S, K, \tau, \sigma, r + \frac{nJ}{\tau} - \lambda(e^J - 1)\right)$$

Writing $\bar{\lambda} = \lambda e^J$ as the "effective" probability of jumps, we obtain

$$C_{JD} = e^{-\bar{\lambda}\tau} \sum_{n=0}^{\infty} \frac{(\bar{\lambda}\tau)^n}{n!} C_{BS}\left(S, K, \tau, \sigma, r + \frac{nJ}{\tau} - \lambda(e^J - 1)\right)$$

This is a **mixing formula**. The jump-diffusion price is a mixture of Black-Scholes options prices with compensated drifts. This is similar to the result we got for stochastic volatility models with zero correlation -- a mixing theorem -- but here we had to appeal to the diversification of jumps or actuarial pricing rather than perfect riskless hedging.

Generalize, as Merton did, to a distribution of normal jumps with

$$E[J] = \bar{J} \qquad var[J] = \sigma_J^2$$

Then

$$E[e^J] = e^{\bar{J} + \frac{1}{2}\sigma_J^2}$$

Incorporating the expectation over this distribution of jumps has two effects:

- J gets replaced by $\bar{J} + \frac{1}{2}\sigma_J^2$
- second, the variance of the jump process adds to the variance of the entire distribution in the Black-Scholes formula by blurring the mean of each subdistribution, so that we must replace σ^2 by $\sigma^2 + \frac{n\sigma_J^2}{\tau}$ because n jumps adds $\frac{n\sigma_J^2}{\tau}$ amount of variance. (The division by τ is necessary because variance is defined in terms of geometric Brownian motion and grows with time, but the variance of normally distributed J is independent of time.)

The general formula is therefore:

$$C_{JD} = e^{-\bar{\lambda}\tau} \sum_{n=0}^{\infty} \frac{(\bar{\lambda}\tau)^n}{n!} C_{BS}\left(S, K, \tau, \sqrt{\sigma^2 + \frac{n\sigma_J^2}{\tau}}, r + \frac{n\left(\bar{J} + \frac{1}{2}\sigma_J^2\right)}{\tau} - \lambda\left(e^{\bar{J} + \frac{1}{2}\sigma_J^2} - 1\right)\right)$$

where $\bar{\lambda} = \lambda e^{\bar{J} + \frac{1}{2}\sigma_J^2}$

If $\bar{J} = -\frac{1}{2}\sigma_J^2$ so that $E[e^J] = 1$ and the jumps add no drift to the process, then we get the simple intuitive formula

$$C_{JD} = e^{-\lambda\tau} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} C_{BS}\left(S, K, \tau, \sqrt{\sigma^2 + \frac{n\sigma_J^2}{\tau}}, r\right)$$

in which we simply sum over an infinite number of Black-Scholes distributions, each with identical riskless drift but differing volatility dependent on the number of jumps and their distribution.

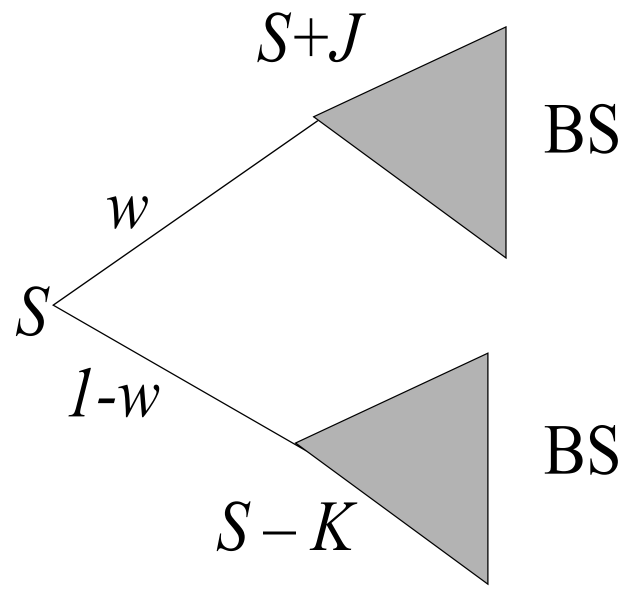
The Jump-Diffusion Smile (Qualitatively)

- Jump diffusion tends to produce a steep realistic very short-term smile in strike or delta, because the jump happens instantaneously and moves the stock price by a large amount.
- Stochastic volatility models, in contrast, have difficulty producing a very steep short-term smile unless volatility of volatility is very large.
- The long-term smile in a jump-diffusion model tends to be flat, because at large times the effect on the distribution of the diffusion of the stock price, whose variance grows like $\sigma^2\tau$, tends to overwhelm the diminishing Poisson probability of large moves via many jumps. Thus jumps produce steep short-term smiles and flat long-term smiles.
- Recall that mean-reverting stochastic volatility models also produce flat long-term smiles.
- Jumps of a fixed size tend to produce multi-modal densities centered around the jump size. Jumps of a higher frequency tend to wash out the multi modal density and produce a smoother distribution of multiply overlaid jumps at longer expirations.
- A higher jump frequency λ produces a steeper smile at expiration, because jumps are more probable and therefore are more likely to occur in the future as well.

Andersen and Andreasen claim that a jump-diffusion model can be fitted to the S&P 500 skew with a diffusion volatility of about 17.7%, a jump probability of $\lambda = 8.9\%$, an expected jump size of 45% and a variance of the jump size of 4.7%. A jump this size and with this probability seems excessive when compared to real markets, and suggests that the options market is paying a greater risk premium for protection against crashes.

An Intuitive Treatment of Jump Diffusion

Simple mixing: t
J a big instantaneous jump up with a small probability w
K a small move down with a large probability (1-w).
Thereafter by diffusion with volatility σ.



We assume that w is small, one jump or zero only.

Risk-neutrality $S = w(S+J) + (1-w)(S-K)$

$K = \frac{w}{1-w}J \approx wJ$ to leading order in w.

The mixing formula
$$\begin{aligned} C_{JD} &= w \times C(S+J, \sigma) + (1-w) \times C(S-K, \sigma) \\ &\approx w \times C(S+J, \sigma) + (1-w) \times C(S-wJ, \sigma) \end{aligned}$$
Mixing

to leading order in w, where $C(S, \sigma)$ is the Black-Scholes option price for strike K and volatility σ

Assume regime w, $\sigma\sqrt{\tau}$ and J/S satisfy $w \ll \sigma\sqrt{\tau} \ll (J/S)$

probability of a jump is much smaller than root variance much smaller than the percentage jump size.
Also assume strike is close to the at-money.

Since $J/S \gg \sigma\sqrt{\tau}$, the positive jump J takes the call deep into the money, so that the first call $C(S+J,\sigma)$ in the Mixing equation becomes equal to a forward whose value is

$$C(S+J,\sigma) \to (S+J) - Ke^{-r\tau}$$

For simplicity from now on we will also assume that $r = 0$

$$\begin{aligned} C_{JD} &\approx w \times \{(S+J)-K\} + (1-w) \times C(S-wJ,\sigma) \\ &\approx w \times (J+S-K) + (1-w) \times \left\{ C(S,\sigma) - \frac{\partial C}{\partial S} wJ \right\} \end{aligned}$$

We want to keep only terms of order *wS*, nothing smaller. For approximately at-the-money options, $C(S,\sigma) \sim S\sigma\sqrt{\tau}$, so we will neglect the term wC in the above equation since it is of order $w\sigma\sqrt{\tau}S$ which is smaller than wS.

Therefore

$$\begin{aligned} C_{JD} &\approx C(S,\sigma) + w \times \left(J + S - K - \frac{\partial C}{\partial S} J \right) \\ &\approx C(S,\sigma) + w \times [J + S - K - N(d_1)J] \\ &\approx C(S,\sigma) + w \times [S - K + J\{1 - N(d_1)\}] \end{aligned}$$

Now if K is close to at-the-money, we know that $N(d_1) \approx \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \frac{\ln S/K}{\sigma\sqrt{\tau}}$

Therefore
$$C_{JD} \approx C(S, \sigma) + w \times \left[\cancel{(S-K)} + J\left\{ \frac{1}{2} - \left[\frac{1}{\sqrt{2\pi}} \frac{\ln S/K}{\sigma\sqrt{\tau}} \right] \right\} \right]$$

Now close to at-the-money, the $S-K$ term in the above equation is negligible compared with J and the $J\ln S/K$ if $\sigma\sqrt{\tau}$ is small, because

$$J\frac{\ln S/K}{\sigma\sqrt{\tau}} = J\frac{\ln\left(1 + \frac{S-K}{K}\right)}{\sigma\sqrt{\tau}} \approx \frac{J}{K}\left\{ \frac{S-K}{(\sigma\sqrt{\tau})} \right\} \approx \frac{S-K}{(\sigma\sqrt{\tau})} \gg S-K$$

Therefore
$$C_{JD} \approx C(S, \sigma) + wJ\left[\frac{1}{2} - \frac{1}{\sqrt{2\pi}} \frac{\ln S/K}{\sigma\sqrt{\tau}} \right]$$

This is the approximate formula for the jump-diffusion call price in the case where we consider only one jump under the conditions $w \ll \sigma\sqrt{\tau} \ll (J/S)$, i.e. a small probability (relative to volatility) of a large one-sided jump. Recall that $C(S, \sigma)$ is the Black-Scholes option price.

Someone using the Black-Scholes model to interpret a jump-diffusion price will quote the price as $C(S, \Sigma)$ where Σ is the implied volatility smile function:

$$C(S, \Sigma) = C(S, \sigma + \Sigma - \sigma) \approx C(S, \sigma) + \frac{\partial C}{\partial \sigma}(\Sigma - \sigma)$$

Comparing equations we obtain

$$\Sigma \approx \sigma + \frac{wJ\left[\frac{1}{2} - \frac{1}{\sqrt{2\pi}}\frac{\ln S/K}{\sigma\sqrt{\tau}}\right]}{\frac{\partial C}{\partial \sigma}}$$

For options close to at the money, $\frac{\partial C}{\partial \sigma} = S\sqrt{\tau}N'(d_1) \approx \frac{S\sqrt{\tau}}{\sqrt{2\pi}}$, so that

$$\Sigma \approx \sigma + \frac{wJ}{S\sqrt{\tau}}\left[\sqrt{\frac{\pi}{2}} + \frac{\ln K/S}{\sigma\sqrt{\tau}}\right]$$

We see that the jump-diffusion smile is linear in $\ln S/K$ when the strike is close to being at-the money, and the implied volatility increases when the strike increases, as we would have expected for a positive jump J.

Improvement: In the Merton model $w = \bar{\lambda}\tau e^{-\bar{\lambda}\tau}$ where $\bar{\lambda} = \lambda e^{J}$ and λ was the probability of a jump per unit time. So

$$\Sigma \approx \sigma + \bar{\lambda}\sqrt{\tau}e^{-\bar{\lambda}\tau}\frac{J}{S}\left[\sqrt{\frac{\pi}{2}} + \frac{\ln K/S}{\sigma\sqrt{\tau}}\right]$$

$$= \sigma + \bar{\lambda}e^{-\bar{\lambda}\tau}\frac{J}{S}\left[\sqrt{\frac{\pi\tau}{2}} + \frac{\ln K/S}{\sigma}\right]$$

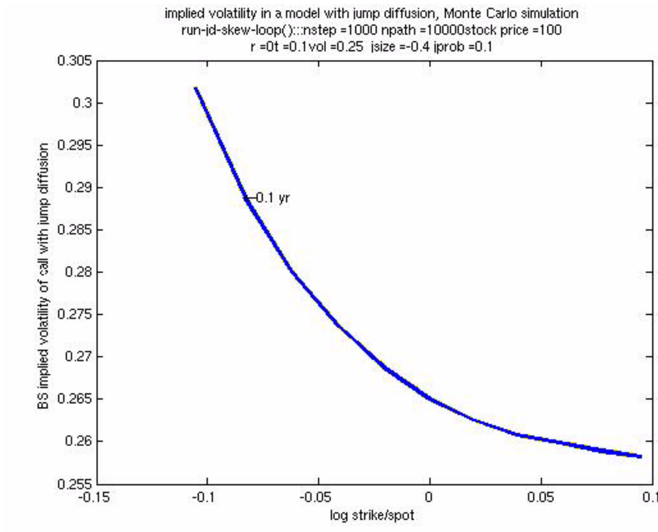
As $\tau \to 0$ for short expirations, the implied volatility smile becomes

$$\Sigma = \sigma + \bar{\lambda}\frac{J}{S}\left[\frac{\ln K/S}{\sigma}\right]$$

a finite smile proportional to the percentage jump and its probability, and linear in $\ln\frac{K}{S}$. The greater the expected jump, the greater the skew. This is a model suitable for explaining the short-term equity index skew.

For larger expirations the approximations required by are no longer valid.Nevertheless, the formula Equation illustrates that, as $\tau \to \infty$, the exponential time decay factor $e^{-\bar{\lambda}\tau}$ drives the skew to zero. Asymmetric jumps produce a steep short-term skew and a flat long-term skew, Here are two figures for the smile in a jump model, obtained from Monte Carlo simulation with a fixed jump size.

This is the skew for a jump probability of 0.1 and a percentage jump size of -0.4, with a diffusion volatility of 25%, for options with 0.1 years to expiration.



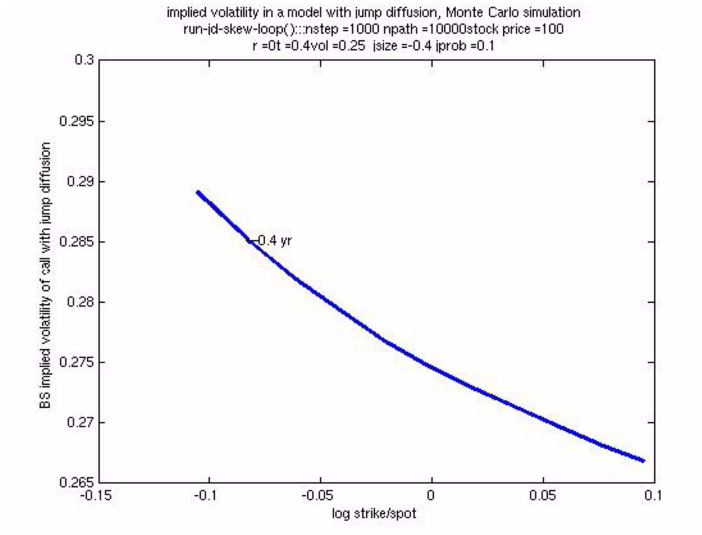
The Mixing Approximation gives the approximate formula

$$\Sigma \approx \sigma + \bar{\lambda} e^{-\bar{\lambda}\tau} \frac{J}{\bar{S}} \left[\sqrt{\frac{\pi\tau}{2}} + \frac{\ln K/S}{\sigma} \right]$$

$$\approx 0.266 + 0.16 \ln \frac{K}{S}$$

which matches the graph pretty well at the money.

Here is a similar skew for an option with 0.4 years to expiration.



It’s still not a bad fit, but because of the longer expiration our approximation of mixing between only zero and one instantaneous immediate jump is not as good. One would have to amend the approximation by allowing for jumps that occur throughout the life of the option.

Conclusion

First understand how volatility varies in your market.

Understand the effect it has on Black-Scholes values.

Look for static hedges.

Understand all these models that involve dynamics hedging.

Try to build the simplest model that can match most of the features of your market.

There is no easy solution that avoids thinking.