Nested Effects for mechanical strength of XXXX (the product):

We test the strength of 5 machines (A through E), and randomly sample 4 heads from each machine.
Because each head is dependent upon the machine from whence it came, this is nested model; as such:

$$Y_{ijk} = \mu + \tau_i + \beta_{j(i)} + \epsilon_{ijk}$$

Where Y is the strength; \mu is the grand mean; \tau_{i} is the mean effect of the i^{th} machine, it is a fixed effect; \beta_{j(i)} is the mean effect caused by j^{th} head nested in the i^{th} machine, it is a random effect; \epsilon_{ijk} is the random error associated with each observation, they are independent and normally distributed with mean 0 and constant variance (\epsilon_{ijk} iid N(0, \sigma^2)

HEAD	Machine A				Machine B				Machine C				Machine D				Machine E			
1	6	13	2	7	10	2	4	0	1	10	8	7	11	5	1	3	2	6	3	3
2	2	3	10	4	9	1	1	3	1	11	5	2	1	10	8	8	4	6	7	1
3	1	9	5	7	7	1	7	4	5	6	0	5	6	8	9	6	7	1	4	2
4	8	8	6	9	12	10	9	1	5	7	7	4	4	3	4	5	9	3	1	2

```
data glass;
do head=1 to 4;
do machine = "A", "B", "C", "D", "E";
do replica=1 to 4;
      input stress @@; output;
      end; end;end;
      cards;
6     13    2     7     10    2     4     0     1     10    8     7     11
      5     1     3     2     6     3     3
2     3     10    4     9     1     1     3     1     11    5     2     1
      10    8     8     4     6     7     1
1     9     5     7     7     1     7     4     5     6     0     5     6
      8     9     6     7     1     4     2
8     8     6     9     12    10    9     1     5     7     7     4     4
      3     4     5     9     3     1     2
run;

proc glm data=glass;
class machine head;
model stress = machine head(machine);
random head(machine)/test; run; quit;
```

Source	DF	Type III SS	Mean Square	F Value	Pr > F
machine	4	53.575000	13.393750	1.73	0.1965
Error	15	116.375000	7.758333		
Error: MS(head(machine))					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
head(machine)	15	116.375000	7.758333	0.69	0.7794
Error: MS(Error)	60	670.000000	11.166667		

Null hypothesis 1: $\tau_1 = \tau_2 = \ldots = \tau_5 = 0$
Alternative hypothesis 1: At least one (1) $\tau_i \neq 0$

$$F_1 = \frac{MSA}{MSB} = \frac{13.39375}{7.758333} = 1.73$$

This F statistic has an associated p-value of 0.1965. Therefore, we fail to reject the null hypothesis, and thus conclude that there are no statistically significant differences across machines

Null hypothesis 2: $\sigma_\beta^2 = 0$
Alternative hypothesis 2: $\sigma_\beta^2 > 0$

$$F_2 = \frac{MSA}{MSB} = \frac{7.758333}{11.166667} = 0.69$$

This F statistic has an associated p-value of 0.7794. Therefore, we fail to reject the null hypothesis, and thus conclude that the variability across heads (nested within machines) does not statistically differ from 0.